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Research paper



Time to Recruitment for a Two Grade Manpower System by Max Policy of Recruitment

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Abstract

In this paper an organization with two different grades, the grade wise depletion of manpower occurs due to its policy decisions is considered. Using max policy of recruitment the system characteristics namely mean and variance of time to recruitment are obtained by considering two different forms of wastages. The influence of the nodal parameters on the system characteristics is studied.

Keywords: *Grade wise wastage, Max Policy, system characteristics*

1. Introduction

Employees quit their job is an important aspect in the study of manpower planning. Many mathematical models have been discussed using different kinds of wastages in [1] and [3]. By using an univariate max policy, eexpected time to recruitment in a single graded system is obtained for several models under different conditions in [9] and [2]. In [4] the author has obtained the system characteristics for a single grade manpower system when the inter-decision times form an order statistics by using Univariate CUM policy. In [6],[7] and [8] the authors have extended the work in [4] for a two grade man power system, assuming different distributions for the thresholds.

"The present paper studies the results of [5] for the two grade manpower system using a univariate max policy of recruitment by considering grade wise wastages for two different forms of watages in the organization."

2. Mathematical Description

"Let X_{i1} and $X_{i2}, i = 1,2,3..k$ are the loss of manhours in grades 1 and 2 respectively for decision i , which forms a sequence of independent and identically distributed exponential random variables with parameters c_1 and c_2 (c_1,c_2 >0). Let g(.) be the density function of X_i where X_i =max{ X_{i1} , X_{i2}} or X_i =X_{i1} + X_{i2} . Let $U_i, i = 1,2,3...$ be the inter decision time between i^{th} and $(i-1)^{th}$ decisions. Let $U_{(1)}, U_{(2)}, ...U_{(k)}$ be the order statistics with respective density functions $f_{u(1)}(.), f_{u(2)}(.)....f_{u(k)}(.)$. Let S be the random variable denoting the time for recruitment in the organization with probability density function (distribution function) $l(.)(L(.))$. The univariate Max policy employed in this paper is as follows: Recruitment is done as and when the maximum loss of man-hours in the organization exceeds Y. Let $D_k(t)$ be the probability that there are exactly k-decision epochs in (0,t]. Let E(S) and V(S) be the mean and variance of time for recruitment respectively."

2.1. Main results

"By Univariate Max policy

$$P(S > t) = \sum_{k=0}^{\infty} D_k(t)\, P\left(\max_{1 \le i \le k} X_i \le Y \right) \quad (1)$$

$$P(S > t) = \sum_{k=0}^{\infty} D_k(t)\, C^k \quad (2)$$

where $C = p(X_i \le Y)$ (3)

Proceeding as in [**5**] we get

$$L(t) = [1-C] \sum_{k=1}^{\infty} F_k(t)\, C^{k-1} \quad (4)$$

from (4) it can be shown that

$$\ell^*(s) = \frac{[1-C]\, f^*(s)}{1 - f^*(s)\, C} \quad (5)$$

Suppose f(t)=fu(1)(t)

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$$f^*_{u(1)}(s) = \frac{k\lambda}{k\lambda + s} \tag{6}$$

It is known that

$$E(S) = -\left.\frac{d(\ell^*(s))}{ds}\right|_{s=0}, E(S^2) = \left.\frac{d^2(\ell^*(s)}{ds^2}\right|_{s=0} \tag{7}$$

and $V(S) = E(S^2) - (E(S))^2$ From (5), (6) and (7) we get

$$E(S) = \frac{1}{k\lambda[1-C]} \tag{8}$$

$$V(S) = \frac{1}{k^2\lambda^2[1-C]^2} \tag{9}$$

Suppose f(t)=fu(k)(t)

$$f^*_{u(k)}(s) = \frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)...(s+k\lambda)} \tag{10}$$

Therefore from (5), (7) and (10) we get

$$E(S) = \frac{\sum_{n=1}^{k} 1/n}{\lambda[1-C]} \tag{11}$$

$$V(S) = \frac{1}{\lambda^2}\left[\frac{\left(\sum_{n=1}^{k} 1/n\right)^2\left(\frac{1}{1-C}\right)^2 - \left[\left(\sum_{n=1}^{k} 1/n\right)^2 - \sum_{n=1}^{k} 1/n^2\right]}{\left(\frac{1}{1-C}\right)}\right] \tag{12}$$

Model - 1

For this model $Y = \max(Y_1, Y_2)$ The values of C are computed and given below for different cases on the threshold distribution

Case (i):

$Y_1 \sim \exp(\alpha_1)$ and $Y_2 \sim \exp(\alpha_2)$

In this case it is shown that

$$C = b_1 + b_2 - b_3 \tag{13}$$

Where

$$b_1 = g^*(\alpha_1), b_2 = g^*(\alpha_2) \text{ and } b_3 = g^*(\alpha_1 + \alpha_2) \tag{14}$$

$$g^*(\tau) = \left\{\begin{array}{ll} \frac{c_1}{c_1+\tau} + \frac{c_2}{c_2+\tau} - \frac{c_1+c_2}{c_1+c_2+\tau}, & Xi = \max\{Xi1, Xi2\} \\ \frac{c_1c_2}{(c_1+\tau)(c_2+\tau)}, & Xi = Xi1 + Xi2 \end{array}\right\}$$

where $\tau = \alpha_1, \alpha_2$ and $\alpha_1 + \alpha_2$ (15)

In (8), (9), (11) and (12) C is given by (15)

Case (ii) : $Y_1 \sim \exp(\alpha_1, 2)$ and $Y_2 \sim \exp(\alpha_2, 2)$

In this case it is found that

$$C = 2b_1 + 2b_2 - 4b_3 + 2b_4 + 2b_5 - b_6 - b_7 - b_8 \tag{16}$$

where $b_4 = g^*(2\alpha_1 + \alpha_2), b_5 = g^*(\alpha_1 + 2\alpha_2),$

$b_6 = g^*(2\alpha_1 + 2\alpha_2), b_{7=} g^*(2\alpha_1)$ and $b_8 = g^*(2\alpha_2)$ (17)

In (8), (9), (11) and (12) C is given by (15) and (16)

Case 3:

When $Y_1 \sim \exp(\alpha_1, 2)$ and $Y_2 \sim \exp(\alpha_2)$ it is found that

$$C = 2b_1^2 + b_2^2 + b_4^2 - 2b_3^2 - b_7^2 \tag{18}$$

In (8), (9), (11) and (12) C is given by (15) and (18).

Model – 2

In this model $Y = \min(Y_1, Y_2)$. The corresponding values of C are given below for the above cited cases

Case (i):

$$C = b_3 \tag{19}$$

In (8), (9), (11) and (12) C is given by (15) and (19).

Case (ii):

$$C = 4b_3 + b_6 - 2b_4 - 2b_5 \tag{20}$$

In (8), (9), (11) and (12) C is given by (15) and (20).

Case 3:

$$C = 2b_3 - b_4 \tag{21}$$

In (8), (9), (11) and (12) C is given by (15) and (21).

Model – 3

For this model $Y = Y_1 + Y_2$ The corresponding values of C is given below for the above cited cases

Case (i):

$$C = \left(\frac{\alpha_1}{\alpha_1 - \alpha_2}\right)b_2 - \left(\frac{\alpha_1}{\alpha_1 - \alpha_2}\right)b_1 \tag{22}$$

In (8), (9), (11) and (12) C is given by (15) and (22).

Case (ii):

$$\begin{aligned} C = &\left(\left(\frac{4\alpha_2}{\alpha_1 - 2\alpha_2}\right) - \left(\frac{4\alpha_2}{\alpha_1 - \alpha_2}\right)\right)b_1 + \\ &\left(\left(\frac{4\alpha_1}{\alpha_1 - \alpha_{\backslash 2}}\right) - \left(\frac{4\alpha_1}{2\alpha_A - \alpha_B}\right)\right)b_2 \\ &\left(\left(\frac{2\alpha_2}{2\alpha_1 - \alpha_2}\right) - \left(\frac{\alpha_2}{\alpha_1 - \alpha_2}\right)\right)b_7 + \\ &\left(\left(\frac{\alpha_1}{\alpha_1 - \alpha_2}\right) - \left(\frac{2\alpha_1}{\alpha_1 - 2\alpha_2}\right)\right)b_8 \end{aligned} \tag{23}$$

In (8), (9), (10) and (11) C is given by (15) and (23).

Case (iii):

$$C=\left(\frac{\alpha_1}{2\alpha_1-\alpha_2}\right)b_7-\left(\frac{2\alpha_1}{\alpha_1-\alpha_2}\right)b_1+\left(\left(\frac{2\alpha_1}{\alpha_1-\alpha_2}\right)-\left(\frac{2\alpha_1}{2\alpha_1-\alpha_2}\right)\right)b_2 \qquad (24)$$

In (8), (9), (11) and (12) C is given by (15) and (24)."

3. Numerical illustration

The influence of nodal parameters on the performance measures analyzed by varying the parameter 'k' and keeping $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, $\lambda = 0.5$, $c_1 = 1$ and $c_2 = 1.5$.

Table 1: Effect of ' k' on the performance measures for Model 1

	k		4	5	6
$X_i = \max\{X_{iA}, X_{iB}\}$	Case 1	r=1	10.65	8.52	7.10
		r=k	88.78	97.31	104.41
	Case 2	r=1	87.45	69.96	58.30
		r=k	728.76	798.72	857.02
	Case 3	r=1	50.02	40.02	33.35
		r=k	416.83	456.85	490.19
$X_i = X_{iA} + X_{iB}$	Case 1	r=1	7.08	5.66	4.72
		r=k	59.01	64.67	69.39
	Case 2	r=1	45.68	36.55	30.46
		r=k	380.69	417.24	447.7
	Case 3	r=1	28.50	22.80	19.00
		r=k	237.52	260.32	279.32

Table 2: Effect of 'k' on the performance measures for Model- 2

	k		4	5	6
$X_i = \max\{X_{iA}, X_{iB}\}$	Case 1	r=1	1.40	1.11	0.93
		r=k	11.63	12.74	13.68
	Case 2	r=1	3.93	3.14	2.62
		r=k	32.72	35.86	38.47
	Case 3	r=1	1.67	1.33	1.11
		r=k	13.89	15.22	16.34
$X_i = X_{iA} + X_{iB}$	Case 1	r=1	1.15	0.92	0.76
		r=k	9.55	10.47	11.24
	Case 2	r=1	2.71	2.17	1.81
		r=k	22.63	24.79	26.60
	Case 3	r=1	1.34	1.07	0.90
		r=k	11.20	12.27	13.17

Table 3: Effect of 'k' on the performance measures for Model- 3

	k		4	5	6
$X_i = \max\{X_{iA}, X_{iB}\}$	Case 1	r=1	18.33	14.67	12.22
		r=k	152.77	167.43	179.65
	Case 2	r=1	312.06	249.65	208.04
		r=k	2600.51	2850.16	3058.20
	Case 3	r=1	113.21	90.56	75.47
		r=k	943.38	1033.94	1109.42
$X_i = X_{iA} + X_{iB}$	Case 1	r=1	11.88	9.50	7.92
		r=k	98.96	108.46	116.38
	Case 2	r=1	154.64	123.71	103.09
		r=k	1288.68	1412.39	1515.49
	Case 3	r=1	62.20	49.76	41.47
		r=k	518.34	568.10	609.56

4. Findings

It is observed that, as the number of decisions increases, E(S) decreases for minimum and it increase for maximum order statistics respectively.

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