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Research paper

Determination of Number of New Elements for Sequence of Fuzzy Topographic Topological Mapping

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Abstract

Problem involving neuro magnetic inverse problem can be solved with Fuzzy Topographic Topological Mapping (FTTM). FTTM is a model consists of four components and connected by three algorithms. FTTM version 1 and version 2 were designed to present 3D view of an unbounded single current and bounded multicurrent sources, respectively. Several definitions related to sequence of FTTM were introduced by Suhana and the feature, namely the cube of FTTM are developed. In this paper, sequence of FTTM namely $FTTM_n$ are discussed. Consequently, some theorems are proved to describe the number of the new elements produced from the $FTTM_n$. Besides that, the number of new elements produced from combination of sequence of $FTTM_n$ can be written as a combination from cubes of $FTTM_n$.

Keywords: *Fuzzy Topographic Topological Mapping; Sequence; Cubes of $FTTM_n$.*

1. Introduction

Fuzzy Topographic Topological Mapping (FTTM) is a model designed to solve neuro magnetic inverse problem. There are four components and connected by three algorithms in FTTM. Detail of FTTM can be found in [1]. In general, there are two version of FTTM namely FTTM version 1 and FTTM version 2. Each were designed to present 3D view of an unbounded single current and bounded multicurrent sources, respectively. In 2008, Suhana introduced some definitions on sequence of FTTM. In this paper, the vertices of FTTM are extended. The geometrical features for some finite vertices of FTTM; namely FK_n are investigated. One of the geometrical features produced from the combination of sequences of FTTM are cubes of $FTTM_n$ [1]. From cubes of FTTM, one can determine the number of new elements produced from the combination of FTTM.

In general term, FTTM is a 4-tuple of topological spaces homeomorphic to each other [2]. The representation for FTTM are given by the equation

$$FTTM = \{(M,B,F,T) : M \cong B \cong F \cong T\}. \quad (1)$$

Sequential elements of FTTM [1] are present exactly as shown in Figure 1.
FTTM version 1 was developed to present a 3-D view of an unbounded single current source [3, 4] in one angle observation (upper of a head model). There are three algorithms, which link between four components as shown in Figure 2. FTTM version 2 was developed to present a 3-D view of a bounded multi current source [5] in 4 angles of observation (upper, left, right and back of a head model). It contains three algorithms, where all four components of FTTM connected to the three algorithms (refer Figure 3).

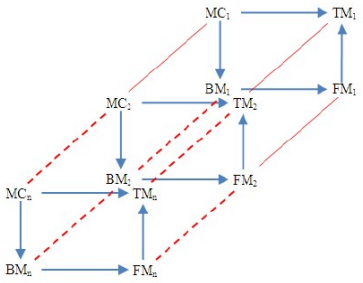


Fig. 1: Sequence of $FTTM_n$

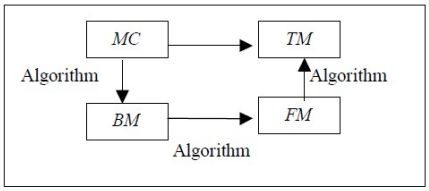


Fig. 2: FTTM version 1

1.1. Cube of $FTTM_n$

A cube is a combination of two, three or more FTTM in $FTTM_n$ (see Figure 4). A cube of FTTM is denoted by $FTTM_{k/n}$ where k is a combination of k number of terms in $FTTM_n$.

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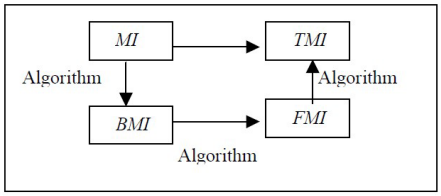


Fig. 3: FTTM version 2

Figure 4 shows some combination and the number of cube for some FTTM$_n$. In general, a cube of FTTM$_n$ is defined as follows.

Definition 1.1: *Sequence of FTTM$_{k/n}$, namely FTTM$_{k/1}$, FTTM$_{k/2}$, FTTMK$_{k/3}$,... is given recursively by equation*

$$FK_{k/n} = \frac{n(n-1)(n-2)(n-3)(n-4)\ldots(n-k-1)}{k!},\ n \geq 1,\ k \geq 2. \quad (2)$$

1.2. FTTM as Pascal's Triangle

Jamaian [6] found that there exists a relation on the sequences of FTTM and Pascal's Triangle. Jamaian showed that the sequence of FTTM$_{2/n}$, FTTM$_{3/n}$, and FTTM$_{4/n}$ are presented in the third, the fourth, and the fifth main diagonals of Pascal's Triangle respectively as shown in Figure 5.

2. Relation between Sequence of *FTTM$_n$* and cube of *FTTM$_n$*

The determination of numbers of elements produced from the extended FTTM$_n$ proved to be time consuming. For example, one has to list all 36 elements of FTTM$_4$ in order to find the number of elements. Alternatively, using the conjecture introduced by Jamaian which was later proved by Sayed and Ahmad, the number new elements of FTTM$_n$ is given by $n^4 - n$ [7]. In addition, the elements of FTTM$_n$ can be written as the linear combination of the sequence of cubes of FTTM$_n$.

Type \ n	1	2	3	4	5	6
Segment $F2_n$						
$\binom{n}{2}$	0	1	3	6	10	15
Triangular $F3_n$						
$\binom{n}{3}$	0	0	1	4	10	20
Square $F4_n$						
$\binom{n}{4}$	0	0	0	1	5	15
Pentagonal $F5_n$						
$\binom{n}{5}$	0	0	0	0	1	6

Fig. 4: Cubes of FK$_n$

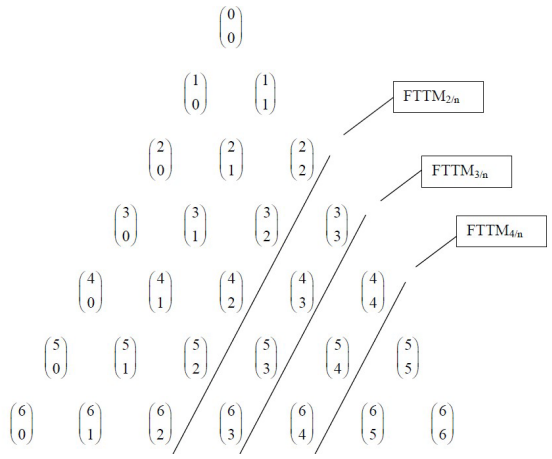


Fig. 5: FTTM in Pascal's Triangle

A sequence for FTTM$_n$ can be written as a combination of the cubes of FTTM$_n$. Therefore, a new theorem on elements for sequence of FTTM$_n$ is deduced. Generally, elements of FTTM$_n$ are given by

$$FK_n = C_1\binom{n}{2} + C_2\binom{n}{3} + C_3\binom{n}{4} + C_4\binom{n}{5} + \ldots + C_p\binom{n}{k}; k = K. \quad (3)$$

with k being the number of components i.e. the vertices of FTTM$_n$, and $C_1, C_2, C_3, C_4, \ldots, C_p$ are the coefficients for each combination. Finding the coefficient $C_1, C_2, C_3, C_4, \ldots, C_p$ is not trivial.

Remark: Since FK_n contains $FK_{2/n}$, $FK_{3/n}$, $FK_{4/n}$,..., $FK_{k/n}$, then it is obvious that there could only be a cube with a combination of k in FK_n. For combination of $K + 1$ FK in FK_n, i.e $FK_{K+1/n}$, this implies that there are $K + 1$ components in FK such that $\{T_1, T_2, T_3, T_4, \ldots, T_k, T_{k+1}\}$. It leads to contradiction since FK only has K components which is $\{T_1, T_2, T_3, T_4, \ldots, T_K\}$. Therefore a cube of FK can be formed from a combination of 2,3,4,...,k in FK_n; i.e $Fk_{2/n}, Fk_{3/n}, FTTM_{4/n}, FK_{K/n}$.
From the remark, FTTM was presented as a combination of cubes namely

$$FTTM_n = C_1\binom{n}{2} + C_2\binom{n}{3} + C_3\binom{n}{4} = n^4 - n. \quad (4)$$

3. Results and Discussion

The coefficients for FTTM$_n$ can be found as given in the following theorem.

Theorem 3.1 *The number of elements produced from sequence of FTTM$_n$ is*

$$FTTM_n = 14\binom{n}{2} + 36\binom{n}{3} + 24\binom{n}{4} = n^4 - n. \quad (5)$$

Proof of Theorem 3.1
$F4_n$ can be written as a system of simultaneous equations given in the following.

$$\begin{aligned} x_1\binom{n}{2}+x_2\binom{n}{3}+x_3\binom{n}{4} &= n^4-n \\ x_1\binom{n}{2}+x_2\binom{n}{3} &= n^4-n \\ x_1\binom{n}{2} &= n^4-n \end{aligned} \tag{6}$$

or

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & x_1 & x_2 \\ 0 & 0 & x_1 \end{bmatrix} \begin{bmatrix} \binom{n}{2} \\ \binom{n}{3} \\ \binom{n}{4} \end{bmatrix} = \begin{bmatrix} n^4-n \\ n^4-n \\ n^4-n \end{bmatrix}. \tag{7}$$

For $n = 2$, in (7) leads to

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & x_1 & x_2 \\ 0 & 0 & x_1 \end{bmatrix} \begin{bmatrix} \binom{2}{2} \\ \binom{2}{3} \\ \binom{2}{4} \end{bmatrix} = \begin{bmatrix} 14 \\ 14 \\ 14 \end{bmatrix} \tag{8}$$

which imply $x_1 = 14$. Next, for $n = 3$, in (7) will yield

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & x_1 & x_2 \\ 0 & 0 & x_1 \end{bmatrix} \begin{bmatrix} \binom{3}{2} \\ \binom{3}{3} \\ \binom{3}{4} \end{bmatrix} = \begin{bmatrix} 78 \\ 78 \\ 78 \end{bmatrix}. \tag{9}$$

Solving the above equation will resulting $x_2 = 36$. Similarly, letting $n = 4$ will give $x_3 = 24$. Therefore,

$$FTTM_n = 14\binom{n}{2}+36\binom{n}{3}+24\binom{n}{4}.$$

4. Conclusion

Homeomorphism between components of $FTTM_n$ gives many new elements. One can list all the new elements of $FTTM_n$. However, it proved to be time consuming. Alternatively, using the conjecture introduced by Jamaian which was later proved by Sayed and Ahmad, the number of new elements of $FTTM_n$ is given by n4 − n. In addition, the elements of $FTTM_n$ can be written as the linear combination of the sequence of cubes of $FTTM_n$. The method to find the coefficient for the sequence of cubes of $FTTM_n$ is discussed in this paper. An alternative method to find the coefficient for $FTTM_n$ is also presented.

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