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***Research paper***

# Fractional order based on genetic algorithm PID controller for controlling the speed of DC motors

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## Abstract

Buck boost converter is a good interface between photo voltaic (PV) and the load. It is utilized in applications that require an output voltage of a value above or below the input voltage level. The benefit of using this circuit is to supply a stable and controlled output voltage regardless of the input voltage level. This research presents an optimal design method for fractional-order proportional–integral-derivative (FOPID) controllers of buck boost converter for the purpose of acquiring a group of desired properties. FOPID controller, usually denoted by⟦PI^μ D⟧^λ, is a special PID controller type in which its derivative and integral orders are fractions between zero and one. Thus, FOPID controller has five variables instead of three in compare with the classical PID controller. In this work, the FOPID is designed to control the speed of Direct Current (DC) motor fed by a buck boost converter. Genetic algorithm will be implemented to set the variables related to the fractional order controller of PID using various type of fitness function such as MSE, ISE and ITSE. The obtained results indicate an enhancement in the steady and transient state performance of system, including the time of settling and rise as well as peak overshoot.

***Keywords***: *Buck Boost Converter; Genetic Algorithm; Fractional Order PID Controller; Speed Controlling on DC Motor.*

## 1. Introduction

Robust, efficient and reliable drives are in great demand for many industrial applications such as mobile robot, electric cars, home appliances, etc. DC motors have been considered as primary drives of these applications because of their precise control, simple design and cost effective manufacturing [1-3]. The development of DC motor drives is important to enhance the dynamic characteristics and response of the motor. Dc-Dc converter is the most efficient way to drive actuators of electromechanical systems. They supply a regulated dc output voltage even when they are exposed to load and input voltage variations. However, the control of output voltage of buck boost dc-dc converters is quite difficult because of their switching characteristics and nonlinearity [4-6]. Therefore, the linear traditional control techniques are not suitable for these converters. A suitable control strategy for dc-dc buck boost converters must deal with their intrinsic nonlinearity, input voltage variation and load fluctuations. Therefore, an advanced and robust control approach is required to overcome the complexities associated with these converters [7-9].

So far, the controllers of proportional-integral–derivative (PID) are considered as the most dominant type of feedback control in the industry. This is because of their design simplicity and capability in achieving decent transient and steady state responses [10]. However, PID controllers exhibits some drawbacks such as sensitivity to changes in system parameter, performance drop as the order of the system increases and weak performance with nonlinear systems [11], [12]. Lately, fractional order PID has drawn the attention of developers and researchers in the field of control system design [13], [14]. Utilizing the controllers of non-integer derivation and integration is an approach of enhancing the response of the traditional PID controllers. It has been concluded that equations of fractional-order differential can define the dynamic behavior of the system better than the integer-order counterparts [15]. They also have more degrees of freedom in tuning control parameters and show a better response characteristic with high order systems [16]. However, the designing procedure of fractional-order PID (FOPID) involves a certain amount of difficulties in compared with traditional PID controllers due to the fact that controllers of FOPID contain integral and derivative orders as additional tuning parameters [17].

Recently, a considerable amount of literature has been introduced to explain the tuning methods of FO-PID controller. These methods are usually classified as analytical, graphical and optimization approaches [18], [19]. In optimization-based method, the controller's parameters are calculated to obtain some predefined time domain specifications. The genetic algorithm is considered as one of the most commonly used optimization-based method for tuning FOPID controller. This tuning method is mostly depending on the estimation of the fractional order calculus first, then setting other variables similar to setting the controllers of integer-order methods [20].

## 2. System modeling

### 2.1. DC motor model

DC motor can be defined as a common plant component in industrial systems. It directly delivers a rotary movement responding to an input electric voltage (V). The electrical equivalent circuit of the DC motor armature includes a resistor (R), an induced back EMF (e) and a self-inductor (L). The DC motors' mathematical model is a well-known and was stated in detail in several studies and therefore, this research does not need to emphasize the assumptions and process of derivation. The final transfer function [21] of a brushed DC motor is stated as follow:

$$\mathrm{G1(s)} = \frac{\mathrm{w(s)}}{\mathrm{E(s)}} = \frac{\mathrm{K}}{[(\mathrm{R+Ls})(\mathrm{Js+B})+\mathrm{K}^2]} \tag{2.1}$$

This above equation is considered as $G_1(s)$. After inserting the DC motor parameter values, listed in table (1), the final function of DC motor transfer is:

$$\frac{\mathrm{w(s)}}{\mathrm{Ea(s)}} = \frac{0.023}{0.005\mathrm{s}^2+0.01\mathrm{s}+0.000559} \tag{2.2}$$

**Table 1:** Parameter Values of DC Motor [6].

| Specification of DC motor | |
|---|---|
| L | 0.5 H |
| R | 1 Ω |
| K | 0.023 |
| J | $0.01\ \mathrm{Kg.m^2}$ |
| B | $0.00003\ \mathrm{N.m.S^2/rad}$ |

## 2.2. Buck boost converter model

It is a basic switching - mode converter circuit in which the voltage of the output could be higher or lower than the voltage of the input. The equivalent circuit of buck boost converter of is demonstrated in Figure (1). The mathematical model [22] of the open loop transfer function is derived to apply an appropriate design technique for obtaining the ideal values for the controller parameters which will meet the specifications of steady and transient state of the system of closed loops.

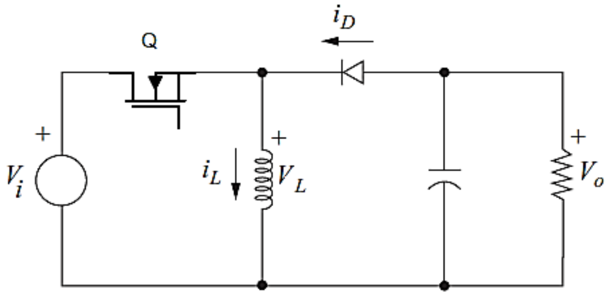

**Fig. 1:** The Equivalent Circuit of the Converter of Buck Boost.

During the on period of the circuit, the transistor Q is closed while the diode is reversed biased. The current flows input via the inductance causing the inductor current to raise and hence the energy of store. Concurrently, the capacitance C will supply the load R with the required power. The equivalent circuit of the on period is demonstrated in Figure (1).

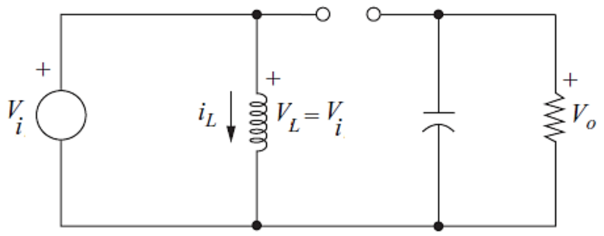

**Fig. 2:** The Buck Boost Converter and Switch ON.

$$\mathrm{V_L} = \mathrm{V_i} \rightarrow \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}} = \mathrm{V_i}$$

$$\frac{\mathrm{di}}{\mathrm{dt}} = \frac{\mathrm{V_i}}{\mathrm{L}} \tag{2.3}$$

$$\mathrm{I_C} = \mathrm{c}\,\frac{\mathrm{dvc}}{\mathrm{dt}} \rightarrow \frac{-\mathrm{V_o}}{\mathrm{R}} = \mathrm{c}\,\frac{\mathrm{dvc}}{\mathrm{dt}}$$

$$\frac{\mathrm{dvc}}{\mathrm{dt}} = -\frac{\mathrm{V_c}}{\mathrm{Rc}} \tag{2.4}$$

During the off period of the circuit, the Q transistor is turned off and the current that flowed via the L inductor, will be flowing via the C capacitor and the load R. Concurrently, the current of inductor will fall down to the Q transistor which is turned on again. Its equivalent circuit of the off period is shown in Figure (3) below:

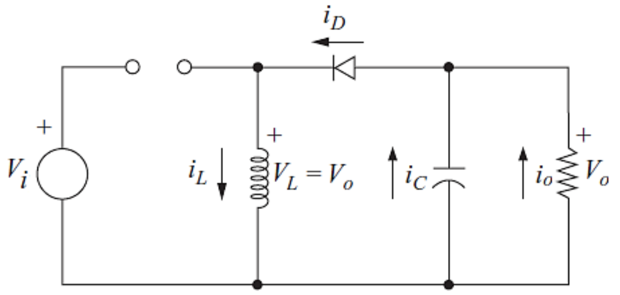

**Fig. 3:** The Buck Boost Converter and Switch OFF.

$$V_L = V_c \rightarrow L\frac{di}{dt} = V_c$$

$$\frac{di}{dt} = \frac{V_c}{L} \tag{2.5}$$

$$I_C = -\,(i_L + I_o)$$

$$c\,\frac{dvc}{dt} = -\,(i_L + \frac{V_c}{R}) \rightarrow \frac{dvc}{dt} = \frac{-\,i_L}{c} - \frac{V_c}{Rc} \tag{2.6}$$

Combining the two periods together yield

$$\frac{di}{dt} = \frac{V_i}{L}\,d + \frac{V_c}{L}\,(1-d) \tag{2.7}$$

$$\frac{dvc}{dt} = -\frac{V_c}{Rc}\,d + \left(\frac{-\,i_L}{c} - \frac{V_c}{Rc}\right)(1-d) = -(1-d)\,\frac{i_L}{c} - \frac{V_c}{Rc} \tag{2.8}$$

$$\dot{X} = A\,x(t) + B\,u(t)$$

$$\dot{X}_1 = \frac{di}{dt}\,, X_1 = i_L$$

$$\dot{X}_2 = \frac{dvc}{dt}\,, X_2 = V_c\,, u(t) = V_i$$

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dvc}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1-d}{L} \\ \frac{-(1-d)}{c} & \frac{-1}{Rc} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} d/L \\ 0 \end{bmatrix} V_i \tag{2.9}$$

Taking Laplace transform to investigate the function of transfer of the output to the input:

$$G(s) = C[SI - A]^{-1}B + D \tag{2.10}$$

$$[SI - A]^{-1} = \frac{\begin{bmatrix} S+\frac{1}{Rc} & \frac{1-d}{L} \\ \frac{d-1}{c} & S \end{bmatrix}}{S^2+\frac{S}{Rc}-\frac{(-d^2+2d-1)}{Lc}} \tag{2.11}$$

$$C[SI - A]^{-1} = \frac{[0\ 1]\ x \begin{bmatrix} S+\frac{1}{Rc} & \frac{1-d}{L} \\ \frac{d-1}{c} & S \end{bmatrix}}{S^2+\frac{1}{Rc}S+\frac{(d^2-2d+1)}{Lc}} = \frac{\begin{bmatrix} \frac{d-1}{c} & S \end{bmatrix}}{S^2+\frac{1}{Rc}S+\frac{(1-d)^2}{Lc}} \tag{2.12}$$

$$C[SI - A]^{-1}B = \frac{\begin{bmatrix} \frac{d-1}{c} & S \end{bmatrix} x \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix}}{S^2+\frac{1}{Rc}S+\frac{(1-d)^2}{Lc}} = \frac{\frac{d^2-d}{Lc}}{S^2+\frac{1}{Rc}S+\frac{(1-d)^2}{Lc}} \tag{2.13}$$

$$\frac{V_o}{V_i} = \frac{\frac{d^2-d}{Lc}}{S^2+\frac{1}{Rc}S+\frac{(1-d)^2}{Lc}} \tag{2.14}$$

This above equation is marked as $G_2(s)$. After inserting the parameter values of buck boost circuit, listed in table (2), the final function of transfer of the converter of buck boost becomes:

$$G(s) = \frac{-1.299 * exp(5)}{s^2 + 23.19\,s + 2.16 * exp(5)} \qquad (2.15)$$

**Table 2:** Parameter Values of Buck Boost Converter

| Specification of buck boost converter | |
|---|---|
| R | 5.1 Ω |
| L | 2.2 e-4 H |
| c | 8200 e-6 F |
| Vi | 20 V |
| Vo | $-12\,V$ |
| d | 0.385 |

The combined transfer function of DC motor and buck boost converter given below.

$$G(s) = G_1(s) * G_2(s)$$

$$G(s) = \frac{-4.434}{0.005s^4 + 0.131s^3 + 1.951s^2 + 4.822\,s + 0.1792} \qquad (2.16)$$

## 3. 3. Controller of fractional order

The controller of fractional Order PID, referred by $PI^{\lambda}D^{\mu}$, has been first introduced by Igor Podlubny in the year of 1997 [23]. This controller depending on fractional calculus comprises two additional parameters, which are (λ, and μ), compared to typical PID controllers. μ and λ are the order of the differentiator and integrator respectively. These two more variables tend to increase the flexibility and robustness of the controller. The general architecture of fractional order for the controller of PID is demonstrated in Figure (4) below.

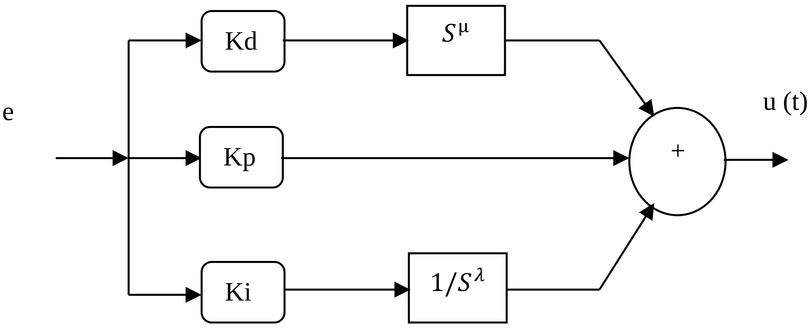

**Fig. 4:** Fractional Order for Controller of PID.

The equation of integral-differential for identifying the control action of a fractional order of PID controller is specified as

$$u(t) = kp\ e(t) + ki\ D^{-\lambda}\ e(t) + kd\ D^{-\mu}\ e(t) \qquad (3.1)$$

Where e(t) is the tracking system error signal, u(t) is the signal of control.

By using of Laplace transformation for this equation with nullify of basic conditions, the function of transfer for fractional order PID controller can be expressed as

$$G_c(s) = kp + \frac{ki}{s^{\lambda}} + kd\ s^{\mu}\ (\lambda\,, \mu\ > 0) \qquad (3.2)$$

## 4. Genetic algorithm optimization

Genetic Algorithms (GA) are a stochastic global search engine that simulates the natural process development. GA begins with an initial inhabitants comprising a chromosomes number where each one represents a possible problem's solution which its performance is assessed by the fitness role. Essentially, GA includes three primary steps: Mutation, Crossover and Selection. The implementation of the setting process via GAs starts with defining the representation of chromosome which is formed by five values as shown in Figure (5) that correspond to the gains to be adjusted to attain a suitable behavior. The gains Kd, Kp, Ki, μ and λ, are real numbers and the individual distinguish to be assessed [24].

| $K_p$ | $K_i$ | $K_d$ | λ | μ |
|---|---|---|---|---|

**Fig. 5:** Chromosome Definition.

The fitness function is the measurement of the chromosome's quality. The three objective functions applied for this particular problem are the Integral Time Square Error (ITSE), Integral Square Error (ISE) and Mean Square Error (ITSE) which have the following forms:

$$ITSE = \int_0^T t(e(t))^2 dt = \int_0^T t(r(t) - y(t))^2 dt \qquad (3.3)$$

$$ISE = \int_0^T (e(t))^2 dt = \int_0^T (r(t) - y(t))^2 dt \qquad (3.4)$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(e(t))^2 \qquad (3.5)$$

Where r is variable of reference, y is output controlling, e is error controlling.

The specification of the Genetic Algorithm applied for this particular system is given in table 2.

**Table 3:** Parameters of GA

| GA Parameter | Value/Method |
|---|---|
| Population Size | 50 |
| Max No. of Generations | 100 |
| Fitness Function | ITSE, ISE, MSE |
| Selection Method | Normalized Geometric Selection |
| Crossover Method | Scattering |
| Mutation Method | Uniform Mutation |

The GA architecture flow chart is depicted in the Figure (6) below.

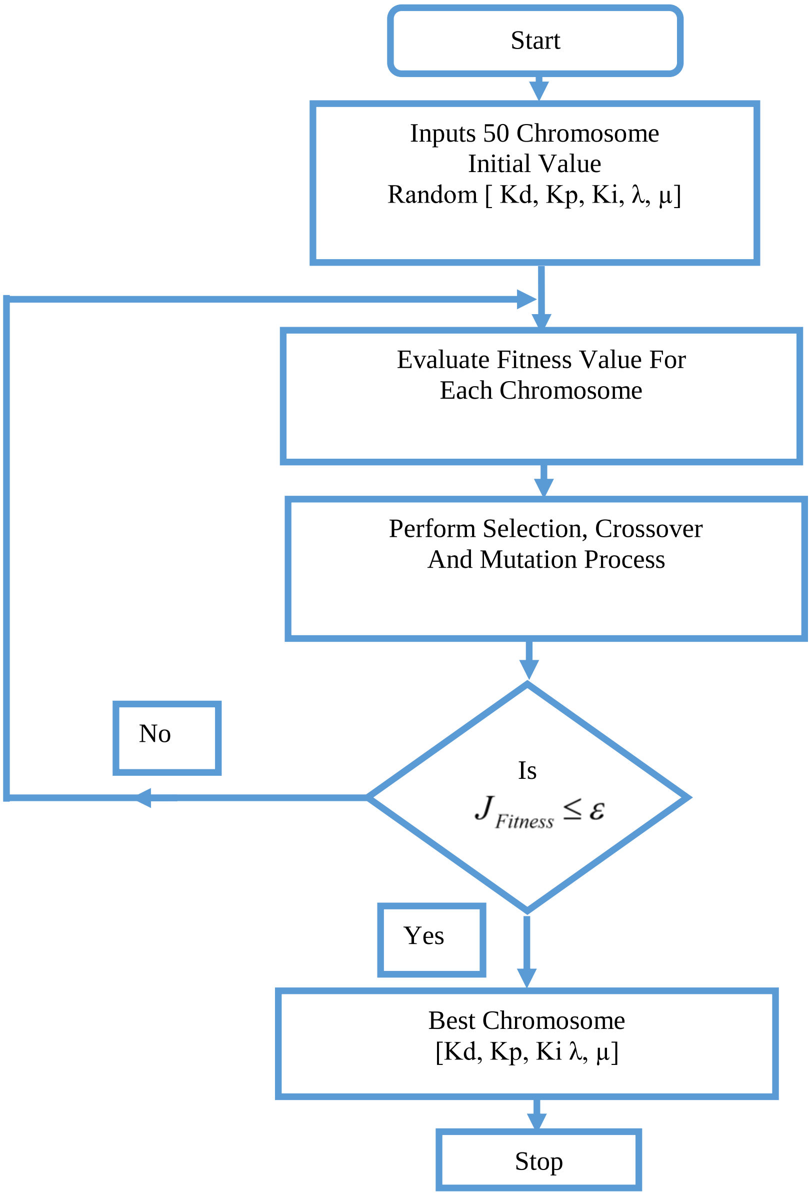

**Fig. 6:** GA Flowchart for Calculating Cost Matrices.

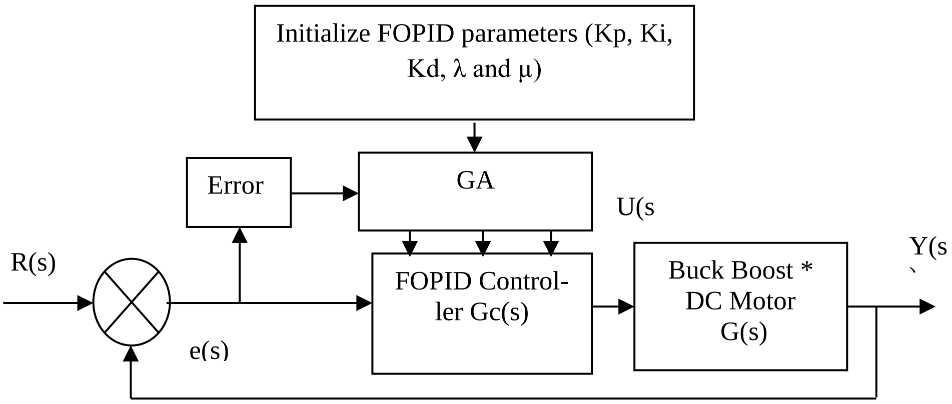

**Fig. 7:** GA-FOPID Controller.

The schematic diagram of FOPID tuned by GA for DC motor speed control drive by a converter of buck boost is shown in Figure (7). The reference speed is fed as an input and associated with actual motor speed. The generated error is given to the controller that gives the control signal as an input to the PWM module of the buck boost converter to drive the DC motor.

## 5. Simulation and results

Initially, the objective function sub-routine is programmed and retrieved by the main genetic algorithm program. The GA involves using a double vector population with an initial range of [-10, 10]. The Crossover is implemented on 80% of the population and the Elite is applied on 5 % of the whole generation while 0.2 fraction of the population is migrated in both directions towards the previous and next sub-generations. The best generation that gives the optimal controller Gains (Kp, Kd, Ki, λ and μ) using different objective functions (MSE, ISE and ITSE) is shown in Figures 8, 9, 10 respectively.

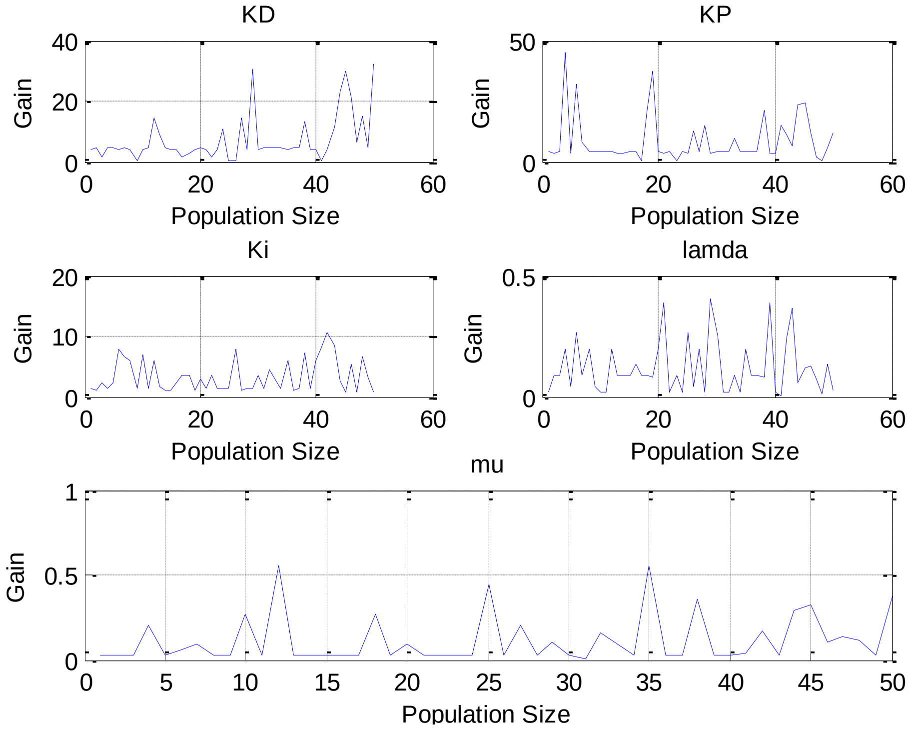

**Fig. 8:** The Integral Time Square Error (ITSE).

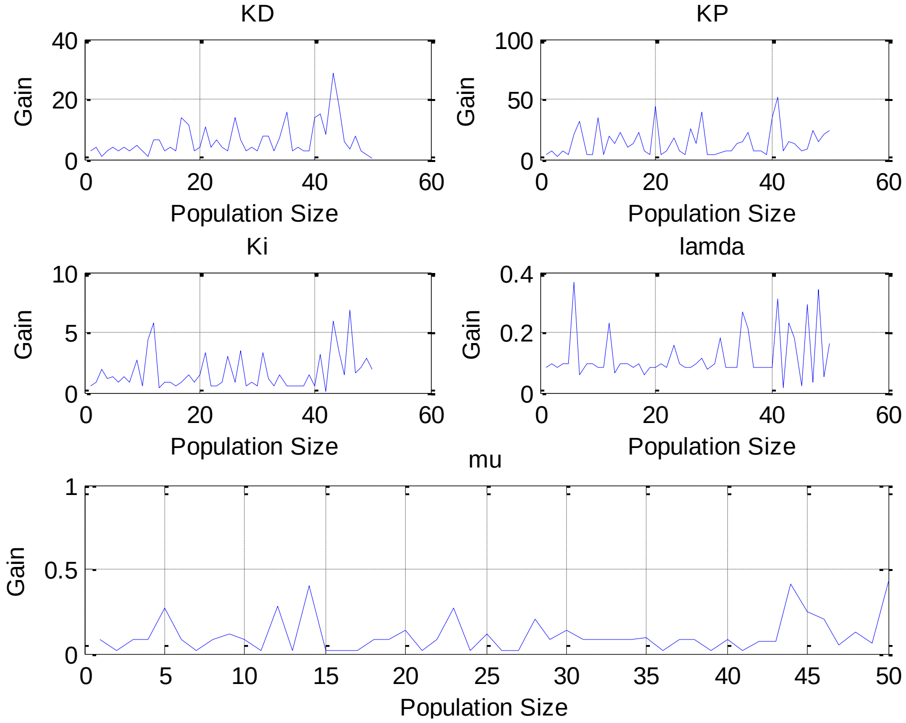

**Fig. 9:** The Integral Square Error (ISE).

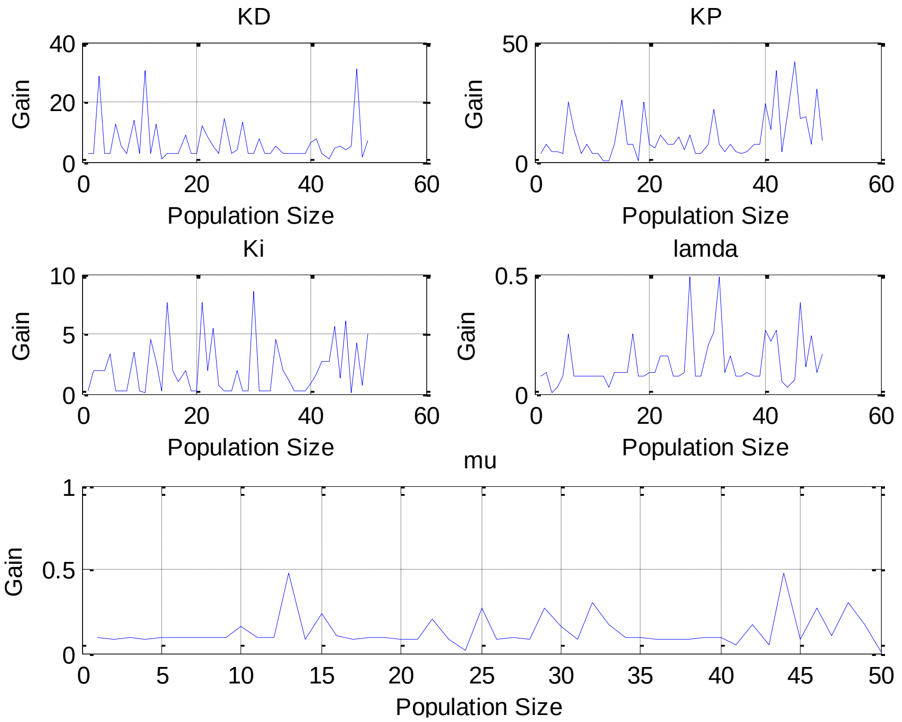

**Fig. 10:** The Mean Square Error (MSE).

The response of the system with GA-FOPID controller applying the MSE, ISE and ITSE objective functions is shown in Figures (11, 12, 13) respectively.

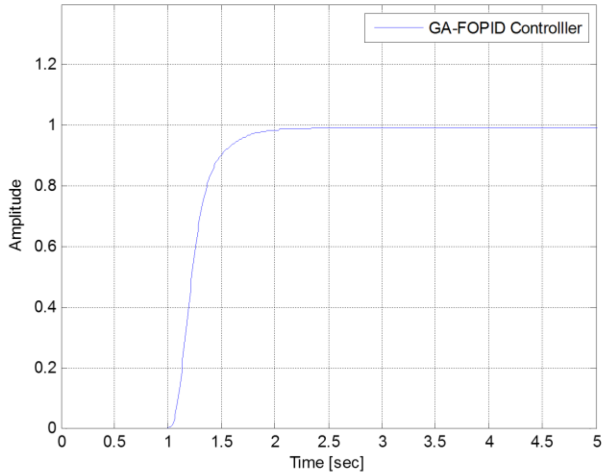

**Fig. 11:** GA-FOPID Close Loop Response with MSE Fitness Function.

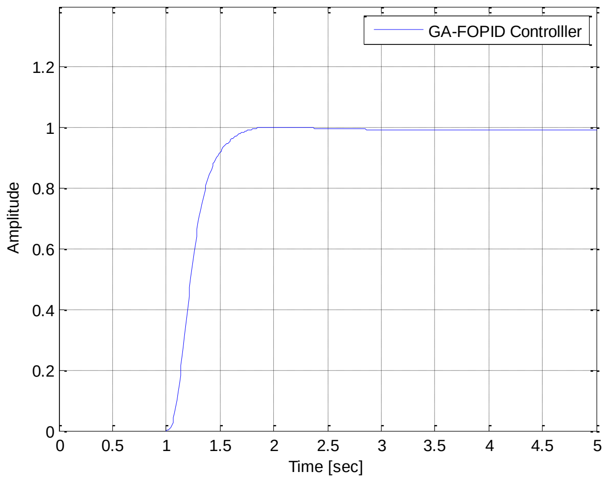

**Fig. 12:** GA-FOPID Close Loop Response with ISE Fitness Function.

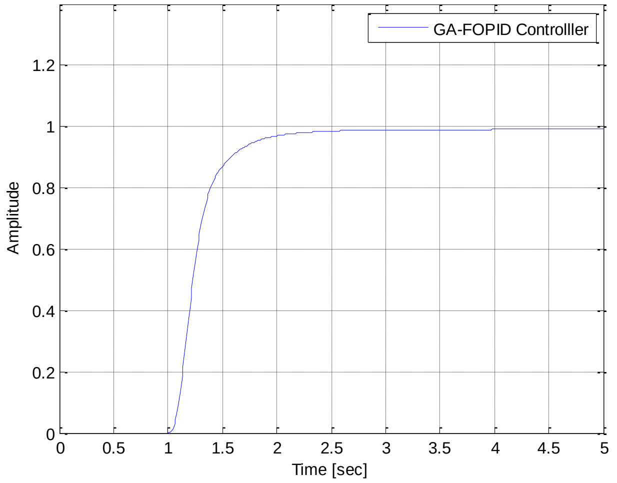

**Fig. 13:** GA-FOPID Close Loop Response with ITSE Fitness Function.

The gain values (Kd, Kp, Ki, μ and λ) of the best chromosome that produce the optimum performance of the system as well as several transient parameters like the rise time, settling time and peak overshot value are shown in table (4).

**Table 4:** The Performance Optimum Values

| GA_FOPID Controller Parameter | Fitness Function | | | Population Size |
|---|---|---|---|---|
| | MSE | ISE | ITSE | |
| Kd | 3.77668 | 2.58026 | 3.01735 | 50 |
| Kp | 4.03155 | 4.27675 | 3.65952 | |
| Ki | 1.35127 | 0.556677 | 0.262995 | |
| **λ** | 0.0165466 | 0.0835046 | 0.0739594 | |
| μ | 0.0288636 | 0.0787119 | 0.0882462 | |
| Tr (sec) | 0.398 | 0.368 | 0.477 | |
| Ts (sec) | 0.885 | 0.685 | 1.23 | |
| Mp % | 0 | 0 | 0 | |

## 6. Conclusion

This paper studied the scheme for feedback speed control of motor with DC current driven by a buck boost converter using a fractional order for the controller of PID. Buck boost converter is a nonlinear and time-variant system. Based on the assumption of low-frequency and small ripple condition, a mathematic converter of buck-boot model is derived using state-space averaging method. Then, Genetic Algorithms (GAs) were used to search for an ideal value for the parameters of (FOPID) controller. The optimization performance target is chosen to be the MSE, ISE and ITSE fitness functions. A comparison between the three fitness functions showed that the controller optimized by ISE provides the best performance in term of the time of rise and settling as well as peak overshoot. The FOPID controller also provided more robustness against load change due to the two extra degree of freedom added by $\boldsymbol{\lambda}$ and μ.

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