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***Research paper***


# Solving an improved heat transmission measuring equation using partial differential equations with variable coefficients

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**Abstract**

An application, for all of its importance in engineering or not, has a need to be less complex while covering a wide range of variables. In this paper, the Al-Tememe transformation was used to solve the linear partial differential equations with variable coefficients, and applied on an improved heat transmission measuring equation. It was found that the range of the heat transmission measuring equation variables had increased, including the time and temperature observation point variables.

***Keywords***: *Al-Tememe transformation, improved heat transmission measuring equation, linear partial differential equations with variable coefficients, partial differential equations with variable coefficients.*

## 1. Introduction

In mathematics, transformation comprises the conversion of one function into another that may or may not be in the same domain. The heat equation is a partial differential equation applied in a variety of scientific branches, which can describe a heat distribution function over a period of time [3] [4].
Many studies have proposed different methods to solve partial differential equations such as [5 - 9]. Some of them have found ways to incorporate the methods into the heat equation [5]. The Al-Tememe transformation tool, however, has not been performed as an application to solve the heat equation. It was not surprising to learn that Al-Tememe transformation had recently transformed the original differential equation into an elementary algebraic expression. Its concept can collectively add up and provide a significant benefit in various technological and scientific fields, including quantum physics and bio-medical signal processing [1] [2] [10].
This study explained the Al-Tememe transformation's role in solving the heat transmission measuring equation using variable coefficients.

## 2. Basic concepts

It is necessary to provide some relevant definitions and theorems in order to make this study's calculations clearer.

### 2.1. Definition [3]

Al-Tememe transformation (TT) for the function $f(x)$, where $(x > 1)$ is defined by the following integral:

$$T[f(x)] = \int_1^\infty x^{-s} f(x)dx = F(s)$$

By determining that integral is convergent and s is a positive constant, we can say that:

$$T[u(x,t)] = \int_1^\infty t^{-s} u(x,t)dt = v(x,s).$$

$u(x,t)$ is a function of x and t.

Remark (1): TT is characterized by the linear property, where A and B are constants, while the functions $u_1(x,t)$ and $u_2(x,t)$ are defined when $(t > 1)$.

$$T[Au_1(x,t) + Bu_2(x,t)] = AT[u_1(x,t)] + BT[u_2(x,t)]$$

### 2.2. Al-Tememe transformation for some functions, [3]

| Functions | $T[f(x)] = \int_1^\infty x^{-s} f(x)dx$ | Region of convergence |
|---|---|---|
| k, k is a constant | $\frac{k}{s-1}$ | $s > 1$ |
| $x^n, n \in R$ | $\frac{k}{s-(n+1)}$ | $s > n+1$ |
| $\ln x$ | $\frac{1}{(s-1)^2}$ | $s > 1$ |
| $\sin(a \ln x)$ | $\frac{a}{(s-1)^2+a^2}$ | $s > 1$ |
| $\cos(a \ln x)$ | $\frac{s-1}{(s-1)^2+a^2}$ | $s > 1$ |
| $x^n \ln x, n \in R$ | $\frac{1}{(s-(n+1))^2}$ | $s > n+1$ |
| $\sinh(a \ln x)$ | $\frac{a}{(s-1)^2-a^2}$ | $\|s-1\| > a$ |
| $\cosh(a \ln x)$ | $\frac{s-1}{(s-1)^2+a^2}$ | $\|s-1\| > a$ |
| $(\ln x)^n, n \in Z^+$ | $\frac{n!}{(s-1)^{n+1}}$ | $s > 1$ |

### 2.3. Al-Tememe transformation in solving linear partial differential equations with variable coefficients [3]

Studies have shown that one of the most important functions of TT is to solve the linear partial differential equations with variable

coefficients, through a transformation process from the linear partial differential equation to the linear partial differential equation with variable coefficients. It was found that by taking $T^{-1}$ (inverse Al-Tememe) to the solution of the ordinary differential equation, it is possible to find the solution for the linear partial differential equation with variable coefficients.
The special form of the second order linear partial differential equation is:

$$At^2u_{xx} + Btu_t + Ch_1(x)u_{xx} + Eh_2(x)u_x + Gh_3(x)u_{tx} + Pu = f(x,t)$$

Where: $A, B, C, E, G$ and $P$ are constants, and $f(x,t)$ is a function of $x$ and $t$ such that TT for $f(x,t)$ is known.

**2.3.1. Theorem, [3]**

1) $T[tu_t(x,t)] = -u(x,1) + (s-1)v(x,s)$ .
2) $T[t^2u_{tt}(x,t)] = -u_t(x,1) - (s-2)u(x,1) + (s-2)(s-1)v(x,s)$ .
3) $T[h(x)u_x(x,t)] = h(x)\frac{d}{dx}v(x,s)$ .
4) $T[h(x)u_{xx}(x,t)] = h(x)\frac{d^2}{dx^2}v(x,s)$ .
5) $T[h(x)tu_{tx}(x,t)] = h(x)\frac{d}{dx}[-u(x,1) + (s-1)v(x,s)]$ .

# 3. Solving an improved heat transmission measuring equation using Al_Tememe transform

Let's consider the problem:

$$tu_t = x^2u_{xx} \tag{3.1}$$

With the boundary equations:

$$u(1,t) = 0\ , u(10,t) = 0\ and\ u(x,1) = 3sin(2\pi\ ln\ x)$$

Where

$$0 < x < 10\ , t > 0$$

This partial differential equation represents an improved heat transmission measuring equation.
In order to solve the equation (3.1), TT is taken for both sides:

$$T[x^2u_{xx}] - T[tu_t] = T(0)$$

$$D^2v - [-u(x,1) + (s-1)v(x,s)] = 0$$

By substituting the values of the boundary conditions:

$$D^2v + 3sin(2\pi\ ln\ x) - (s-1)v(x,s) = 0$$

$$D^2v - (s-1)v = -3sin(2\pi\ ln\ x)$$

By considering the homogeneous solution:

$$D^2v - (s-1)v = 0\ , m - (s-1) = 0$$

Then

$$m = \pm\sqrt{s-1}$$

So

$$V_a = C_1e^{\sqrt{s-1}\,x} + C_2e^{-\sqrt{s-1}\,x}$$

By considering the non-homogeneous solution:

$$D^2v - (s-1)v = 3sin(2\pi\ ln\ x)\ , V_p = \frac{-3sin(2\pi\ ln\ x)}{D^2-(s-1)}$$

$$V_p = \frac{-3sin(2\pi\ ln\ x)}{-(2\pi)^2-(s-1)} = \frac{3\ sin(2\pi\ ln\ x)}{4\pi^2+(s-1)}$$

The general solution to equation (3.1) is:

$$u = C_1e^{\sqrt{s-1}\,x} + C_2e^{-\sqrt{s-1}\,x} + \frac{3\ sin(2\pi\ ln\ x)}{4\pi^2+(s-1)}$$

By taking TT to the boundary conditions, it is possible to determine the values of $C_1$ and $C_2$, which are $C_1 = 0$ and $C_2 = 0$ :
Since $u(1,t) = 0$, then:

$$C_1e^{\sqrt{s-1}} + C_2e^{-\sqrt{s-1}} = 0 \tag{3.2}$$

And since $u(10,t) = 0$, then:

$$C_1e^{10\sqrt{s-1}} + C_2e^{-10\sqrt{s-1}} = 0 \tag{3.3}$$

From equations (3.2) and (3.3)

$$C_1 = C_2 = 0$$

The solution equation (3.1) is:

$$u = \frac{3\ sin(2\pi\ ln\ x)}{4\pi^2+(s-1)}$$

# 4. Conclusions

The improved heat transmission measuring equation $tu_t = x^2u_{xx}$ can be solved using the Laplace transformation. However, it is not an easy solution due to lack of ready-made theorems. In Al-Tememe Transform there are already calculated ready-made theorems that can simplify the solution for the equation. With the use of Al-Tememe transformation with variable coefficients, the range of the heat equation is increased, that's include increasing the range of time by multiplying by $t$ and increasing the range of temperature observation point by multiplying $x^2$ by $u_{xx}$.

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