International Journal of Engineering & Technology, 7 (4.31) (2018) 487-490

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

A Review on Z-Numbers

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Abstract

Reliable information is very important to make accurate and right decision. The concept of Z-number can be used to describe the real-life information into computer-understandable that is relates to the issue of reliability information. This concept has a potential for many application such as Engineering, Management, Education, Mathematics, and also in many other fields. In this paper, applications and methodologies of the Z-numbers are reviewed systematically since 2011 until 2018 from several online databases such as ScienceDirect, Springer, Emerald, Wiley, and Taylor & Francis. The studied articles are grouped into four categories: the concepts papers, new approach papers, develop methods, and application papers.

Keywords: *Decision Making; Multi Criteria Decision Making; Multi-Attribute Decision Making; Z-information; Z-numbers*

1. Introduction

Rational decisions can be made if the information is incomplete, uncertain, and sometimes imprecise. In the existing decision theories, decision based on relevant information can be generalized into three levels of valuations; crisp, interval and fuzzy number valuation [1]. Von Neumann and Morgenstern in [2] proposed an expected utility theory, and Savage in [3] proposed the subjective expected utility theory. Based on [4], these approaches in [2] and [3] require known values for all the objective or subjective probabilities. However, it is hard to determine the values of needed information in this real world [4].

One of very important part in decision analysis is to know the reliability of the numbers we are dealing with [5]. We call this uncertainty as a pervasive phenomenon. Human do not has problem to make rational decisions based on incomplete, imprecise or uncertain information. In 2011, Zadeh [6] has introduced Z-numbers to deal with reliable information.

Many decision has been made based on uncertain in information. Human probably can easily make a decision even the information is uncertain, or incomplete. However, it is hard and challenging to formalize this capability. Based on Zadeh, Z-numbers relates to reliability of the information. For example, component of Z is (X, Y), where Z= (X, Y). In this case, X is fuzzy restriction, and Y is a reliability of component X [6]. This new concept will be used in the process of uncertain information widely, since more close to describe the knowledge of human being [7]. Based on [8], in term of restraint and reliability, Z-number is more capable to describe the human knowledge compared to the classical fuzzy number.

Since 2011, attention for research in Z-numbers is increasing. Figure 1 shows the number of publications on Z-numbers since 2011.

In 2011, only one paper has been published, i.e. by Zadeh [6], where the first time Z-numbers been introduced. The number of publication keep increased, where in 2017, the number of publication is about 27 papers. The diagram also shows that in recent years this subject is a hot topic and it has received a great deal of attention from practitioners and researchers and also, there is plenty of works for them to doing on it.

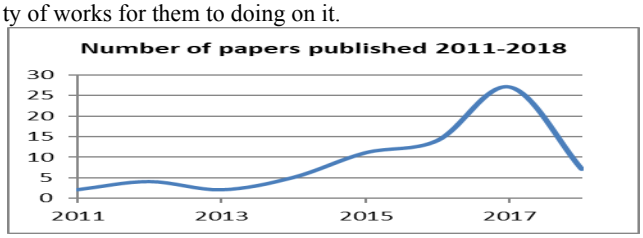

Fig. 1. The bibliometric number of papers on Z-Numbers since 2011 until 2018

The intention of this paper is to review the previous work on Z-numbers that have not been discussed by the previous researchers, between 2011 and 2017. To achieve this intention, peer-reviewed journals from 2011 until 2018 have been extracted from online databases such as ScienceDirect, Springer, Emerald, Wiley, and Taylor & Francis and as well as a limited number of papers from conferences. We also discuss the research gaps and future research agenda that be done to fill these gaps. We hope this review will provide a single platform for academicians to understand the available literature on the topic of Z-number. Several question we try to answer from these original researches as follows:

- What they have been done to develop their research in Z-numbers?
- How this research field has been evolved in the past 7 years?
- What is the main research area of the Z-number?

In order to answer the identified research questions, firstly, we defined search arguments, keywords and the databases mentioned above to identify all articles related to the Z-number or Z-information from 2011 to 2018. Secondly, we focused on examining the study elements and results. We examined development process and application of the Z-number was being used to solve practical real-world problems, and we found that they have been developed for a number of different purposes. In the next step, after article's reading by focusing on introduction and results of each paper based on type of study (tools and approaches and also

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techniques) we found that they can be categorized into four categories. The studies that work solely on the theoretical concepts of Z-number is categorized as Concept approach; the papers that offered a new method to create a desired results are categorized as the Novell approach; studies that developed the previous methods put into the Developed approach; and the papers that have used Z-number by appliance is categorized as Applied approach.
This paper has four sections: Section I presents the introduction, Section II, Literature reviews of the Z-number, and we conclude our review in Section III.

2. Literature Review on Z-number

In this area of study, several number of approaches and techniques have been introduced. The articles under review can be divided into several categories. The first category is the articles that expand alone the concept of Z-number merely the point of view of theoretical. The second category is the articles that offer a new approach to make advances especially in order to create a desired result. The third category is the articles that develop the previous methods, and the last category is the articles that show the application.

2.1. Z-number Concept Approach

This sub-section will review any papers that discuss on the theoretical concept of Z-numbers. The first time Z-number concept introduced by Zadeh is in 2011 [6]. In [6], the Z-number is used to formalize the human capability in making a rational decision in uncertainty and imprecision situation. There are several questions been touched in [6] but have no answer such as one of important problem is ranking the Z-numbers, however no suggestion how to solve it.
After Z-number been introduced in 2011, in 2012, Yager in [8] has suggested that the information of uncertain variable V can be provided from Z-numbers and it is called as Z-valuation. The probability of V is A is equal to B can be interpreted by the Z-valuation. Multiple Z-valuations were manipulated and combined, and the relationship between Z-numbers and linguistic summaries has been shown in [8]. At the end of [8], the author showed a representation of Z-valuations in terms of Dempster–Shafer belief structures that made use of type-2 fuzzy set.
The Z-number theories that introduced by Zadeh [6] was not mature and all researchers use classical fuzzy number for their applications. Therefore, Kang [5] has proposed a method that transforms Z-numbers to classical fuzzy numbers. After presenting it, many applications could be done under the description method with Z-number directly.
In 2013, for the first time, Z-numbers been used for uncertainty modeling of load values in [9]. A standard classification of uncertainty handling methods about energy system has been proposed by Soroudi and Amraee [9]. Several methodologies has been assessed and found out that each method is suitable for a specific type of uncertainty [9].

2.2. Z-number Novell Approach

Any papers that proposed a new method will be reviewed in this sub-section. Azadeh et al. in [10] uses Z-numbers to propose a new Analytical Hierarchy Process (AHP) method. The objective of this work is to solve problem in linguistic decision-making and also develop a model to find the criteria's in evaluating the best universities. The Z-number and numerical example has been used to illustrate the proposed procedure [10].
In 2014, a technique to measure the progress of any project that under uncertain condition has been proposed by Salari et al. [11]. In [11], a Z-number has been applied efficiently and in fact, this work also presented a forecasting of the project completion time and cost [11].
A new approach to solve a decision making in a real economic problem using Z-value information has been presented in [1]. Zeinalova in [1] had pointed out that relevant information is missing in most of existing decision theories. Experimental investigation in [1] showed that making decision based on Z-information is more realistic and adequate to decision issues in real-life. The results of suggested approach only proved on the investment problem that it could be applied to decision making in other various areas where decision relevant information is imperfect.
Aliev and Memmedova in [12] have suggested a new approach for modeling of the effect of the Pilates exercises on students' motivation, attention, anxiety, and educational achievement. The uncertainty of data related with cognitive measuring of psychological parameters and their partial reliability have been promoted. To embrace incomplete and lack of matching rules, the fuzzy rule has been used for modeling the considered relationship and then, Z-interpolation method by Zadeh [1] to approximate the reasoning.
TOPSIS method using Z-numbers has been proposed by Yaakob and Gegov in [13]. The ability of fuzzy rule-based has been extended in [13] to make more flexible in multi criteria decision-making by considering expert experience and knowledge. Meanwhile, spearman rho correlation is used to validate the ranking.
The efficiency of decision-making units (DMUs) can be measured by effective mathematical technique called Data envelopment analysis (DEA) [14]. Sadi-Nezhad and Sotoudeh-Anvari [14] uses Z-numbers to propose the DEA model.In [14], the inputs and outputs have been formulated to achieve the solution and Z-numbers was converted to into classical fuzzy numbers, which is the Z-DEA model is transformed into a traditional fuzzy DEA model, based on fuzzy expectation. The author claimed that their framework is flexible and can be applied to evaluate different DMUs in fuzzy environment.
A method called Analytic Hierarchy Process Z-number Data Envelopment Analysis (AHPFDEA) is an extension of Z-numbers had been proposed in [15], where the objective is to rank the risk priority of 20 bridge structures. Work in [15] have utilized two ways of incorporating the reliability components in [5] and [10].
Differential evolution optimization and Z-number arithmetic has been used in Z-number based Linear Programming (Z-LP) model [16]. A comparative analysis has been conducted and validity of the approach is shown [16].
In [17], the uncertain information with Z-number and Dempster-Shafer (DS) theory has been combined to enhance the reliability of the fault detection. At the end of [17], the author claimed that the complementary sensors' information under different fault features can be adopted.
Aliyevs in [18] has used fuzzy uncertainty to characterized decision relevant information. The value and weight from these characterizations is then derived from experts. The applicability and efficiency of the proposed method is demonstrated through numerical example on MADM Web services selection problem [18].
The arithmetic of Z-numbers and distance between Z-numbers has been used to find best supplier [19]. This method is quite similar with in [18], however, the method in [18] only use concept of negative ideal solution and the Z-numbers were defined by discrete. The values of the Z-numbers information are not converted to fuzzy information in [18].
Kang et al. in [20] has developed a method that can be applied on uncertain environment for multi-criteria decision making (MCDM) application. In this work [20], several experiments have been run to demonstrate the effectiveness of ordering Z-numbers application and MCDM.

2.3. Z-Number Developed Approach

Any studies that develop the previous methods will be reviewed in this subsection. Aliev and Zeinalova [21] in 2014 showed how Z-numbers can be used to make decisions by considering reliability. Authors investigated two approaches, i.e. reducing the Z-numbers to classical fuzzy numbers and direct computation with Z-

numbers. Numerical examples have been used to validate the proposed work [21].
Zeinalova in [22] pointed out that relevant information is missing in the reliability of decision. A problem of decision making under Z-valued information was considered and Choquet integral was used to construct a utility function in Z-number environment. To validate the approach, it has been applied to solve the real world business problem.
In 2016, Zeinalova and Mammadova [23] applied Z-information to make decision on oil extraction problem. A decision making frameworks has been developed where the Z-information is calculated based on direct arithmetic Z-numbers.
Peng and Wang focuses on multi-criteria group decision making (MCGDM) problems, where the weight information is incompletely known [24]. A reliable tool called Hesitant Uncertain Linguistic Z-Numbers (HULZNs) has been developed to depict complex and uncertain decision-making information [24]. The VIKOR model is combined with power aggregation operators was developed.
Also, a more confident Z-number model with DEA models by Azadeh and Kokabi [25] is proposed in 2016. Based on [25], the inputs and outputs are imprecise and vague when data are crisp. Most of these data are expert-based and therefore reliability is very important. In this study [25], a crisp linear programming model can be obtained by transforming the Z-number DEA model into possible linear programming and then apply an alternative a-cut approach. In order to tackle the uncertainty, interaction between projects and reliability, the proposed model has been applied portfolio selection problem.

2.4. Z-Number Applied Approach

This sub-section will review any studies that used Z-number by appliance. In 2014, Kokabi et al. [26] has proposed approach to help the decision makers about the future behaviour of the trusted agent. There are two agents had been considered in this work, i.e. past agent and future trust values agent. The Z-numbers and ANN has been used as an algorithm of all rational scenarios of past trust behaviours. One of the limitation is that can predict trust values for a short time [26].
The Z-number based regression model was constructed by Sadikoglu et al. [27]. The objective of this work is to determine the psychological effect on educational achievement. Fuzziness and partial reliability was used to characterize the observed data and then will be described by Z-numbers. The validity of this approach is measured by using Jaccard similarity to justify the adequacy. The researcher has applied the Newton method to obtain optimal value of the Jaccard similarity criterion [27].
Stefanini in [29] has proposed generalized difference (g-difference) to overcome the Hukuhara difference (H-difference) of discrete Z-numbers [30]. In [28], Qiu et al. has proposed g-difference of continuous Z-numbers. Two concept has been proposed to solve fuzzy optimization problem, which are g-derivative and the convexity of Z-number function [28].

2.5. Summary

As mentioned before, all papers are published from 2011 to 2018 and have been extracted from online databases such as ScienceDirect, Springer, Emerald, Wiley, and Taylor & Francis. Table 1 shows the summary of reviewed paper based on author(s), year, type of study, approaches, and methods or techniques.

3. Conclusion

A systematically review of Z-number is provided by authors which classified papers based on Z-information to four approach. In this paper, papers were collected from well-known and credible publication, published since 2011. Based on our review, they can be categorized into four main approaches; includes of concepts, novel, developed, and applied approaches. In Concept approach, we reviewed all works that related to discuss on the theories of Z-numbers. This included the first time Z-numbers been introduced in decision-making.
Any works that proposed a new method related to Z-numbers and create a new desired result was grouped as Novel approach. Meanwhile, any works that develop the previous methods was categorized as Develop approach. Lastly, the Applied approach reviewed any works that uses Z-numbers to solve the real problems.

Acknowledgement

This work is funded by Universiti Putra Malaysia under Geran Putra (GP/2018/9596100).

Table 1: Summary of Reviewed Papers about Z-numbers (2011-2018)

Authors	Year	Approach type	Objective, Tools and approaches	Methods & Techniques
Zadeh, L.A.	2011	Concept Approach	Introducing the concept of a Z-number and the way to computer Z-numbers.	CWW with Z-number
Yager et al.	2012	Concept Approach	Using type-2 fuzzy sets to represent the structure of Z-valuation and Dempster-Shafer.	Z-num with fuzzy type 2
Kanga et al.	2012	Novel Approach	Using fuzzy to develop a method to transform Z-number to classical fuzzy number.	Fuzzy Set
Alireza and Turaj	2013	Concept Approach	A novel concept of Z-numbers is introduced, where the possibility of utilization Z-numbers was demonstrated to model uncertainty of load values	MCS and PEM with Z-number
Azadeha et al.	2013	Novel Approach	Propose a new AHP method for the first time based on Z-number.	AHP & Fuzzy set
Zeinalova	2014	Novel Approach	A problem of make decision represented and suggested the new approach to solve a real economic problem in various areas.	Z-information in MCDM
Salari, M.	2014	Novel Approach	A novel concept of a Z-number to measure project progress under uncertain condition.	EVM with Z-number
Kokabi et al.	2014	Applied Approach	Artificial Neural Networks and Z-numbers were used in a trusted entity to predict the future trust value.	ANN with Z-numbers
Aliev and Zeinalova	2014	Developed Approach	Using Z-numbers and Choquet integral to solve decision making issue.	Z-number and Choquet Integral
Zeinalova	2014	Developed Approach	Suggest a framework to solve a real world issue in business by using Choquet integral with Z-informations.	Z-number and Choquet Integral
Aliev and Memmedova	2015	Novel Approach	Used an inference techniques for approximate reasoning based on *Z*-interpolation method	*Z*-number with Rules
Yaakob and Gegov	2015	Novel Approach	Enhanced fuzzy rule based approach in MCDM by using Z-number on TOPSIS	Z-number and TOPSIS

Sadi-Nezhad and Sotoudeh-Anvari	2015	Novel Approach	Introduce a new DEA model with compound to Z-number to measure the efficiency decision-making for handling real problems	Z-number with DEA model
Sahrom and Dom	2015	Novel Approach	Presented a hybrid of AHP-DEA method using Z-number	Z-number and Hybrid AHP-DEA
Aliev et al.	2015	Novel Approach	A model of Z-number based LP (Z-LP) had been suggested to fit real-world problems with the framework of LP.	LP with Z-number
Zeinalovaa and Mammadovab	2016	Developed Approach	A framework based on Z-numbers was proposed to solve the decision problem by studying on oil extraction issues as a case base	Z-number and Choquet Integral
Jiang et al.	2016	Novel Approach	Enhancing reliability of the fault detection by combining Z-numbers and Dempster-Shafer evidence theory.	Z-number and D-S theory
Peng and Wang	2016	Developed Approach	Introduced a reliable tool to solve hesitant uncertain linguistic Z-numbers problems which integrated linguistic variables by means of VIKOR and ERP	VIKOR, Z-number and ERP
Aliev et al.	2016	Applied Approach	Human-like fundamental approach for ranking of Z-numbers which is based on two main ideas	Z-number with FPO
Agakishiyev	2016	Novel Approach	Solving supplier selection problems under fuzzy and partially reliable information to choose the best supplier through Z-numbers arithmetic.	Z-number with negative ideal
Sadikoglua et al.	٢٠١٦	Applied Approach	A new regression model using Z-numbers was proposed to describe the attachment of educational achievement on motivation, attention and anxiety parameters.	Z-Regression and Z-number
Azadeh and Kokabi	2016	Developed Approach	Proposed Z-number DEA model is transformed into possible linear programming.	DEA with Z-numbers
Aliyev	2016	Novel Approach	Solving MADM problem based on ideal solutions of positive and negative ideal solutions, and also distance between Z-vectors and Z-information processing.	new approach for MADM
Kang et al.	2017	Novel Approach	Measuring effect of Z-numbers through Z-numbers utility.	MCDM with Z-number
Qiu et al.	2017	Applied Approach	General-difference of continuous Z-numbers was introduced and also, a new partial order to sort the Z-numbers is defined by means of generalized centroids	Z-number with g-difference

References

[1] Zeinalova, L.M., (2014), "Expected Utility Based Decision Making Under Z-information", Intelligent Automation and Soft Computing, 20 (3), 419–431. http://dx.doi.org/10.1080/10798587.2014.901650

[2] Von Neumann, J., & Morgenstern, O., (1947), "Theory of games and economic behaviour", Princeton University Press.

[3] Savage, L.J., (1954), "The foundations of statistics", New York: Wiley.

[4] Montero, J., & Ruan, D., (2010), "Modeling Uncertainty", Information Sciences, 180(6), 799–802.

[5] Kang B., Wei D., Li Y., & Deng Y., (2012), "A Method of Converting Z-number to Classical Fuzzy Number", Journal of Information & Computational Science, 9(3), 703-709.

[6] Zadeh, L.A., (2011), "A note on Z-numbers", Information Sciences, 181(14), 2923-2932.

[7] Deng, Y., & Chan, F.T.S., (2011), "A new fuzzy demister MCDM method and its application in supplier selection", Expert Systems with Applications, 38, 9854-986

[8] Yager, R., (2012), "On Z-Valuations Using Zadeh's Z-Numbers", International Journal Of Intelligent Systems, vol. 27, 259–278.

[9] Soroudi, A., & Amraee, T., (2013), "Decision making under uncertainty in energy systems: State of the art", Elsevier, Renewable and Sustainable Energy Reviews, in 2013, 28 : 376-384

[10] Azadeh, A., Saberib, M., Zandi Atashbara, N., Chang, E., & Pazhoheshfar, P., (2013), "Z-AHP: A Z-number Extension of Fuzzy Analytical Hierarchy Process. Digital Ecosystems and Technologies (DEST)," in 7th IEEE International Conference on 2013, DOI: 10.1109/DEST.2013.6611344

[11] Salari, M., & Wang, J., (2014), "A novel earned value management model using Z-number", International Journal of Applied Decision Sciences. DOI: 10.1504/IJADS.2014.058037

[12] Aliev, R., & Memmedova, K., (2015), "Application of *Z*-Number Based Modeling in Psychological Research", Computational Intelligence and Neuroscience, vol. 2015. http://dx.doi.org/10.1155/2015/760403

[13] Yaakob, A.M., & Gegov, A., (2015), "Fuzzy Rule Based Approach with Z-Numbers for Selection of Alternatives using TOPSIS", IEEE International Conference on Fuzzy Systems, Istanbul, Turkey, DOI: 10.1109/FUZZ-IEEE.2015.7337862

[14] Sadi-Nezhad, S., & Sotoudeh-Anvari1, A., (2016), "A new Data Envelopment Analysis under uncertain environment with respect to fuzziness and an estimation of reliability", OPSEARCH March 2016, vol. 53(1), 103–115 .

[15] Ashikin Sahrom, N.A., & Mohd Dom, R., (2015), "A Z-number Extension of the Hybrid Analytic Hierarchy Process – Fuzzy Data Envelopment Analysis for Risk Assessment," International Conference on Research and Education in Mathematics (ICREM7), DOI: 10.1109/ICREM.2015.7357019

[16] Aliev, R.A., Alizadeh A. V., Huseynov, O. H., & Jabbarova K. I., (2016), "Z-Number-Based Linear Programming," International Journal of Intelligent System, vol. 30, 563-589.

[17] Jiang, W., Xie, C., Zhuang, M., Shou, Y., & Tang, Y., (2016), "Sensor Data Fusion with Z-Numbers and Its Application in Fault Diagnosis," *Sensors*, *16*(9), 1509; doi:10.3390/s16091509

[18] Aliyev, R., (2016), "Multi-attribute decision making based on z-valuation," 12th International Conference on Application of Fuzzy Systems and Soft Computing, Vienna, Austria.

[19] Agakishiyev, E., (2016), "Supplier selection problem under Z-information", Procedia Computer Science 102, 418 – 425.

[20] Kang, B., Deng, Y., & Sadiq, R., (2018), "Total utility of Z-number," Applied Intelligence, vol. 48(3), 703–729.

[21] Aliev, R. A., & Zeinalova, L. M., (2014), "Decision Making Under Z-Information", Human-Centric Decision-Making Models for Social Sciences, 233-252.

[22] Zeinalova, L. M., (2014), "Choquet aggregation base decision making under Z-information", ICTACT Journal on Soft Computing, vol. 04(4), 819-824.

[23] Zeinalova, L. M., & Mammadova, M.A., (2016), "Decision making on oil extraction under z-information", Procedia Computer Science, vol. 102 (2016), 168-175.

[24] Peng, H.G., & Wang, J.Q., (2017), "Hesitant Uncertain Linguistic Z-Numbers and Their Application in Multi-criteria Group Decision-Making Problems," International Journal of Fuzzy Systems, vol. 19(5), 1300–1316.

[25] Azadeh, A., & Kokabi, R., (2016), "Z-number DEA: A new possibilistic DEA in the context of Z-numbers," Advanced Engineering Informatics 30 (3), 604–617.

[26] Kokabi, R., Azadeh, A., Saberi, M, & Hussain, F.K., (2014), "Trust Prediction Using Z-numbers and Artificial Neural Networks", IEEE Fuzzy Systems Conference, China.

[27] Sadikoglu, F., Huseynov, O., & Memmedova, K., (2016), "Z-Regression analysis in psychological and educational researches", Procedia Computer Science, 102 (2016), 385 – 389.

[28] Dong Qiu, D., Xing, Y., & Dong, R., (2018), "On Ranking of Continuous Z-Numbers with Generalized Centroids and Optimization Problems Based on *Z*-Numbers," International Journal of Intelligent Systems, vol. 33(1), 3-14.

[29] Stefanini L., (2010), "A generalization of Hukuhara difference and division for interval and fuzzy arithmetic", Fuzzy Set System, vol. 161(11), 1564–1584.

[30] Huseynov OH, Aliev RA, Aliyev RR, Alizadeh AA. The arithmetic of Z-numbers: theory

[31] and applications. Singapore: World Scientific; 2015.

[30] Huseynov, O.H., Aliev, R.A., Aliyev, R.R., & Alizadeh, A.A., (2015), "The arithmetic of Z-numbers: theory and applications", Singapore: World Scientific Publishing