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Research paper

Mathematical Model of the Controlled Object when Regulating the Working Volume of the Manipulator Pump

Bulat F. Bairamov;Artur A. Fardeev;Albert R. Fardeev

Kazan Federal University
**Corresponding author E-mail: oafardeeva@mail.ru*

Abstract

This paper describes the constructed mathematical model of the controlled object of the automated system for controlling the displacement volume of the manipulator pump. The change in pump displacement volume is performed in order to save energy by minimizing the fluid drain through the overflow valve. The use of the considered manipulator is intended in the process of automated supply of blanks to the working area of a press used in forging and hot stamping processes. The reactive forces acting on the moving parts of the mobility modules from the fixed parts of the manipulator are determined. Conversion of the useful power of the pump into the useful power of hydraulic motors is performed. This takes into account the losses using efficiency. An equation describing the mathematical model of the controlled object is obtained. This equation is represented in relative variables. The derivation of this equation and the determination of the reactive forces are made on the basis of the assumption that all transport degrees of freedom are translational. This limitation is caused by the peculiarities of the working area of the press used in technological processes of forging and hot forming of blanks. The working zone is a deep horizontal tunnel, the movement of the working object in which is possible only with translational degrees of freedom.

Keywords: *mathematical model, mobility modules, automated control system, pump displacement volume*

1. Introduction

The movement of the workpiece from one machine to another in the course of technological processes of forging and hot stamping takes place either along a conveyor belt or in a container using a loader.

Manual but amenable to automation in this technological process are the following steps:

1) feeding of the workpiece from the receiving tray to the upsetting area of the stamp of the stamping press;
2) shifting the workpiece in the preparatory impression;
3) shifting the workpiece to the finishing impression;
4) removal of forgings onto the conveyor;
5) feeding of the workpiece from the receiving tray to the stamp of the trimming press;
6) removal of forgings, slugs and burrs in appropriate containers.

These stages can be automated with the help of two manipulators, since they are associated with two presses. Both manipulators will perform similar work, so you can limit yourself to developing a single manipulator.

The use of manipulators to automate these steps is provided by the fact that it is impossible to do this with the help of other means (for example, a conveyor), because, in addition to horizontal and vertical movements of blanks and forgings, it is necessary to rotate them relative to one or another axis of coordinates. In addition, it is necessary to remove forgings from impressions.

The speed of a manipulator exceeds the speed of human operations. Therefore, in addition to an increase in labor productivity upon the considered replacement of a person by a manipulator, an increase in the efficiency of using equipment (presses) will occur.

By virtue of the above, the use of manipulators instead of manual labor in the process under consideration is possible and beneficial.

In this case, the manipulator is the controlled object. There is a control not only over movement the working object, but also the displacement volume common to all degrees of pump mobility depending on the speed of movement of the moving parts of the manipulator. The control of the pump displacement volume is made to reduce energy costs without the need to provide the maximum speed to the working object.

2. Methods

We define the reactive forces acting on the moving parts of the modules of mobility from the side of the fixed parts. Let the manipulator have only translational degree of freedom. Reaction forces, inertia forces and external loads acting on the moving parts of the modules of mobility in this case are shown in Figure. Reactive moments are not shown in the figure, since the problem of their definition is not set. This is due to the fact that in this case no torques are used to control the manipulator.

To determine the reactions we use the d'Alembert principle, i.e. we will make the equilibrium equations for the moving parts of each of the modules of mobility taking into account the inertia forces.

We make the equilibrium equations for the moving parts of the first mobility module:

$$\sum_{k=1}^{n} F_{kx} = 0;\ R_{11} - \left(m_1^{|} + m_0\right)a_1 = 0;$$

$$\sum_{k=1}^{n} F_{ky} = 0;\ R_{12} - \left(m_1^{|} + m_0\right)a_2 = 0; \qquad (1)$$

$$\sum_{k=1}^{n} F_{kz} = 0;$$

$$R_{13} - \left(m_1^{|} + m_0\right)g - \left(m_1^{|} + m_0\right)a_3 = 0.$$

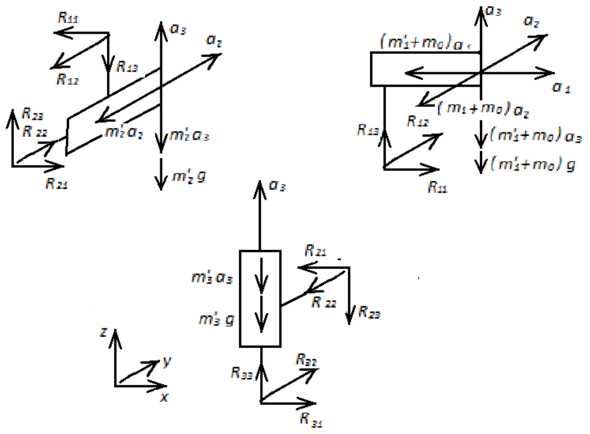

Figure: Forces applied to the moving parts of the mobility modules of the manipulator

Here $m_1^{|}$ - mass of moving parts of the first mobility module; m_0 - **the** mass of the working object together with the seizure; a_1, a_2, a_3 - projections of the acceleration vector of the manipulation object points and points of moving parts of the first mobility module along the axis x, y, z respectively; R_{11}, R_{12}, R_{13} reactions applied to the moving parts of the first mobility module from the fixed parts of this module.

We express the reactions R_{11}, R_{12}, R_{13} from the equations (1) :

$$R_{11} = \left(m_1^{|} + m_0\right)a_1;$$

$$R_{12} = \left(m_1^{|} + m_0\right)a_2; \qquad (2)$$

$$R_{13} = \left(m_1^{|} + m_0\right)g + \left(m_1^{|} + m_0\right)a_3.$$

We make a replacement in equations (2):

$$m_1 = m_1^{|} + m_0,$$

Where m_1 - **the** mass of the moving parts of the manipulator (together with the working object) having the ability to move along the axis x.Then equations (2) will take the form:

$$R_{11} = m_1 a_1;$$

$$R_{12} = m_1 a_2; \qquad (3)$$

$$R_{13} = m_1\left(a_3 + g\right).$$

Here (for points of the moving part of the first mobility module and the working object) a_1 by the absolute value is equal to the relative acceleration modulus created by the first mobility module; a_2, a_3 are equal by the absolute values to the modules of the transporting accelerations created by the second and third modules of mobility, respectively. Coriolis acceleration does not occur, because transporting rotation is missing.

We make the equilibrium equations for the moving parts of the second mobility module.

$$\sum_{k=1}^{n} F_{kx} = 0;\ R_{21} - R_{11} = 0;$$

$$\sum_{k=1}^{n} F_{ky} = 0;\ R_{22} - R_{12} - m_2^{|} a_2 = 0; \qquad (4)$$

$$\sum_{k=1}^{n} F_{kz} = 0;\ R_{23} - R_{13} - m_2^{|} a_3 - m_2^{|} g.$$

Here $m_2^{|}$ - mass of moving parts of the second mobility module; a_2, a_3 - projection of the acceleration vector for points of moving parts of the second mobility module along the axis y, z respectively (the projection of this vector on the axis x equals zero); R_{21}, R_{22}, R_{23} **are** reactions applied to the moving parts of the second mobility module from the fixed parts of this module.

We express the reactions R_{21}, R_{22}, R_{23} from equations (4) in view of (3):

$$R_{21} = m_1 a_1;$$

$$R_{22} = \left(m_1 + m_2^{|}\right)a_2; \qquad (5)$$

$$R_{23} = m_1\left(a_3 + g\right) + m_2^{|}\left(a_3 + g\right).$$

In equations (5) we will make the replacement:

$$m_2 = m_1 + m_2^{|},$$

Where m_2 **is the** mass of the moving parts of the manipulator (together with the working object) having the ability to move along the axis y.Then equations (5) take the form:

$$R_{21} = m_1 a_1;$$

$$R_{22} = m_2 a_2; \qquad (6)$$

$$R_{23} = m_2\left(a_3 + g\right).$$

Here (for points of the moving part of the second mobility module) a_2 by the absolute value is equal to the relative acceleration modulus created by the second mobility module; a_3 by the absolute value is equal to the modulus of the transporting acceleration created by the third mobility module. Coriolis acceleration does not occur, because transporting rotation is missing. Therefore, the velocity projection of any point of the

movable part of the second mobility module on the axis x equals to zero.
We make the equilibrium equations for the moving parts of the third mobility module.

$$\sum_{k=1}^{n} F_{kx} = 0;\ R_{31} - R_{21} = 0;$$

$$\sum_{k=1}^{n} F_{ky} = 0;\ R_{32} - R_{22} = 0; \tag{7}$$

$$\sum_{k=1}^{n} F_{kz} = 0;\ R_{33} - R_{23} - m_3^{|} a_3 - m_3^{|} g.$$

Here $m_3^{|}$ **is** the mass of moving parts of the third mobility module; a_3 **is** the acceleration vector projection of the moving parts' points of the third mobility module on the axis z(projections of this vector on the axis x and y are equal to zero); R_{31}, R_{32}, R_{33} **are** reactions applied to the moving parts of the third mobility module from the fixed parts of this module.

We express the reactions R_{31}, R_{32}, R_{33} from equations (7) in view of (6):

$$R_{31} = m_1 a_1;$$

$$R_{32} = m_2 a_2; \tag{8}$$

$$R_{33} = m_2(a_3 + g) + m_3^{|}(a_3 + g).$$

We will make the replacement in equations (8):

$$m_3 = m_2 + m_3^{|},$$

Where m_3 **is the** mass of the moving parts of the manipulator (together with the working object) having the ability to move along the axis z. Then equations (8) will take the form:

$$R_{31} = m_1 a_1;$$

$$R_{32} = m_2 a_2; \tag{9}$$

$$R_{33} = m_3(a_3 + g).$$

Here (for points of the moving part of the third mobility module) a_3 by the absolute value is equal to the acceleration modulus created by the third mobility module. There is no transporting acceleration, since there is no transporting movement. Movement occurs along the axis z, therefore, the projections of the acceleration of any point of the second mobility module's moving part on the axis x and y are zero.

From the simultaneous equations (3), (6), (9), we consider equations with the participation of R_{11}, R_{22}, R_{33}:

$$R_{11} = m_1 a_1;$$

$$R_{22} = m_2 a_2; \tag{10}$$

$$R_{33} = m_3(a_3 + g).$$

The rest of the reactions will not be necessary in the preparation of a mathematical model for the controlled object of the automated system for regulating the displacement volume of the manipulator pump.

3. Results and discussion

Useful pump power $(N_{ПН})$ minus power lost in the valve (N_K) is passed through the pipeline to the hydraulic motor. At the same time there is a loss of some power. The power received in the hydraulic motor is called the power consumed by the hydraulic motor $(N_Г)$. This process is described by the formula [1]:

$$(N_{ПН} - N_K)\eta_{ТР} = N_Г, \tag{11}$$

Where $\eta_{ТР}$ is a hydraulic efficiency of hydraulic drive, taking into account the total hydraulic pressure loss in the pipeline [2].
Further power $N_Г$ is transmitted through hydraulic motors to the load in the form of the useful power of the hydraulic drive $N_{ПГ}$. In this case, losses also occur (volumetric - due to leaks and mechanical - due to friction force), which are taken into account by the introduction of the efficiency of the hydraulic motor $\eta_Г$ [3]:

$$N_{ПГ} = N_Г \eta_Г. \tag{12}$$

Multiplying both sides of (11) by $\eta_Г$ and taking into account (12), we get:

$$(N_{ПН} - N_K)\eta_{ТР}\eta_Г = N_{ПГ} \tag{13}$$

We determine the values $N_{ПН}$, N_K, $N_{ПГ}$ **[4]**:

$$N_{ПН} = p_{нас} Q_{нас}$$

$$N_K = p_K Q_K = p_{нас} Q_K \tag{14}$$

$$N_{ПГ} = R_{11}\upsilon_1 + R_{22}\upsilon_2 + R_{33}\upsilon_3,$$

Where $p_{нас}$, $Q_{нас}$ are pump pressure and flow; p_K, Q_K - pressure and flow of the overflow valve; R_{11}, R_{22}, R_{33} - projections onto the corresponding axis of reactions applied to the corresponding plungers from the load side [5]; u_1, u_2, u_3 - projections of the speeds of the corresponding plungers on the corresponding axis.
Here $p_K = p_{нас}$, because the valve is in close proximity to the pump.
Substituting equations (10) into (14), and then the resulting equations in (13) and taking into account that $a_i = \frac{d\upsilon_i}{dt}$ $(i = \overline{1,3})$, we get:

$$p_{нас}(Q_{нас}-Q_K)\eta_{TP}\eta_{Г}=m_1\frac{d\upsilon_1}{dt}\upsilon_1+m_2\frac{d\upsilon_2}{dt}\upsilon_2+m_3\left(\frac{d\upsilon_3}{dt}+g\right)\upsilon_3. \quad (15)$$

Let's express Q_K through the pump pressure. The throughput capacity of the valve gap which is lowered to the height of the opening σ_1 **is** determined by the formula

$$Q_K=\mu_щ A_щ\sqrt{\left(\frac{2}{\rho}\right)p_{нас}} \quad (16)$$

Where $\mu_щ$ is the experimental coefficient depending on the number Re; $A_щ$ - the gap area; ρ - the density of the liquid. Since the number Re depends on σ_1 and $p_{нас}$ which change within small limits, we will consider $\mu_щ$ as constant. $A_щ$ is determined by the formula

$$A_щ=\pi d_K\sigma_1\sin\beta, \quad (17)$$

Where d_K - working diameter of the valve; β is the angle between the valve cone generator and a vertical line.
It can be shown that

$$\sigma_1=\frac{p_{нас}A_k\psi}{C}-z_0. \quad (18)$$

Substituting (18) and (17) in (16) we get:

$$Q_K=\mu_щ\pi d_K\left(\frac{p_{нас}A_k\psi}{C}-z_0\right)\sin\beta\sqrt{\left(\frac{2}{\rho}\right)p_{нас}}$$

.
Let's denote:

$$K_{\mu\psi}=\frac{\mu_щ\pi d_K A_k\psi}{C}\sin\beta\sqrt{\left(\frac{2}{\rho}\right)};$$

$$K_{\mu 0}=\mu_щ\pi d_K z_0\sin\beta\sqrt{\left(\frac{2}{\rho}\right)},$$

then:

$$Q_K=K_{\mu\psi}(p_{нас})^{\frac{3}{2}}-K_{\mu 0}(p_{нас})^{\frac{1}{2}}. \quad (19)$$

We express $Q_{нас}$ through the displacement volume [6]:

$$Q_{нас}=V_{он}n_н, \quad (20)$$

Where $V_{он}$ **is the** pump displacement volume, $n_н$ **- the** frequency of rotation of the pump shaft in *rev / sec* .
Substituting (19) and (20) in (15) we get:

$$p_{нас}\left(V_{он}n_н-K_{\mu\psi}(p_{нас})^{\frac{3}{2}}+K_{\mu 0}(p_{нас})^{\frac{1}{2}}\right)\eta_{TP}\eta_{Г}=$$

$$=m_1\frac{d\upsilon_1}{dt}\upsilon_1+m_2\frac{d\upsilon_2}{dt}\upsilon_2+m_3\left(\frac{d\upsilon_3}{dt}+g\right)\upsilon_3$$

or

$$p_{нас}V_{он}n_н\eta_{TP}\eta_{Г}-K_{\mu\psi}\eta_{TP}\eta_{Г}(p_{нас})^{\frac{5}{2}}+K_{\mu 0}\eta_{TP}\eta_{Г}(p_{нас})^{\frac{3}{2}}=$$

$$=m_1\frac{d\upsilon_1}{dt}\upsilon_1+m_2\frac{d\upsilon_2}{dt}\upsilon_2+m_3\left(\frac{d\upsilon_3}{dt}+g\right)\upsilon_3. \quad (21)$$

4. Summary

Let's pass to relative variables [7]:

$$u_p=\frac{p_{нас}-p_{нас0}}{p_{нас0}}; \qquad u_V=\frac{V_{он}-V_{он0}}{V_{он0}};$$

$$u_{\upsilon 1}=\frac{\upsilon_1-\upsilon_{10}}{\upsilon_{10}};$$

$$u_{\upsilon 2}=\frac{\upsilon_2-\upsilon_{20}}{\upsilon_{20}};\ u_{\upsilon 3}=\frac{\upsilon_3-\upsilon_{30}}{\upsilon_{30}}, \quad (22)$$

Where $p_{нас0}$, $V_{он0}$, υ_{10}, υ_{20}, υ_{30} are pressure, displacement volume and velocity projections of the corresponding plungers on the respective axes in nominal mode.
We express from (22) $p_{нас0}$, $V_{он0}$, υ_{10}, υ_{20}, υ_{30} and substitute in (21) [8]:

$$(1+u_p)p_{нас0}(1+u_V)V_{он0}n_н\eta_{TP}\eta_{Г}-K_{\mu\psi}\eta_{TP}\eta_{Г}\left((1+u_p)p_{нас0}\right)^{\frac{5}{2}}+$$

$$+K_{\mu 0}\eta_{TP}\eta_{Г}\left((1+u_p)p_{нас0}\right)^{\frac{3}{2}}=$$

$$=m_1\frac{d(1+u_{\upsilon 1})\upsilon_{10}}{dt}(1+u_{\upsilon 1})\upsilon_{10}+m_2\frac{d(1+u_{\upsilon 2})\upsilon_{20}}{dt}(1+u_{\upsilon 2})\upsilon_{20}+$$

$$+m_3\left(\frac{d(1+u_{\upsilon 3})\upsilon_{30}}{dt}+g\right)(1+u_{\upsilon 3})\upsilon_{30}.$$

We convert the left and right sides of this equation neglecting small quantities relative variables in a higher degree of unity and their creations. In addition, we expand in the Maclaurin series the values $\left((1+u_p)p_{нас0}\right)^{\frac{5}{2}}$ and $\left((1+u_p)p_{нас0}\right)^{\frac{3}{2}}$, and we will take into account only the first two terms in these expansions. Then:

$$\left(1+u_p+u_V\right)p_{нас0}V_{он0}n_н\eta_{TP}\eta_Г-K_{\mu\psi}\eta_{TP}\eta_Г\left(1+\frac{5}{2}u_p\right)p_{нас0}^{\frac{5}{2}}+$$

$$+K_{\mu0}\eta_{TP}\eta_Г\left(1+\frac{3}{2}u_p\right)p_{нас0}^{\frac{3}{2}}= \quad =m_1\upsilon_{10}^2\frac{du_{\upsilon1}}{dt}\left(1+u_{\upsilon1}\right)+m_2\upsilon_{20}^2\frac{du_{\upsilon2}}{dt}\left(1+u_{\upsilon2}\right)+$$

$$+m_3\upsilon_{30}^2\left(\frac{du_{\upsilon3}}{dt}+\frac{g}{\upsilon_{30}}\right)\left(1+u_{\upsilon3}\right)$$

or

$$\left(1+u_p+u_V\right)p_{нас0}V_{он0}n_н\eta_{TP}\eta_Г-K_{\mu\psi}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{5}{2}}-\frac{5}{2}u_pK_{\mu\psi}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{5}{2}}+$$

$$+K_{\mu0}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{3}{2}}+\frac{3}{2}u_pK_{\mu0}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{3}{2}}=$$

$$=m_1\upsilon_{10}^2\frac{du_{\upsilon1}}{dt}\left(1+u_{\upsilon1}\right)+m_2\upsilon_{20}^2\frac{du_{\upsilon2}}{dt}\left(1+u_{\upsilon2}\right)+$$

$$+m_3\upsilon_{30}^2\left(\frac{du_{\upsilon3}}{dt}+\frac{g}{\upsilon_{30}}\right)\left(1+u_{\upsilon3}\right). \quad (23)$$

Let’s denote:

$$N_0=p_{нас0}V_{он0}n_н\eta_{TP}\eta_Г;\; K_{\mu1}=K_{\mu0}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{3}{2}}-K_{\mu\psi}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{5}{2}};$$

$$K_{\mu2}=\frac{3}{2}K_{\mu0}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{3}{2}}-\frac{5}{2}K_{\mu\psi}\eta_{TP}\eta_Г\,p_{нас0}^{\frac{5}{2}}.$$

Then equation (23) takes the form:

$$\left(1+u_p+u_V\right)N_0+K_{\mu1}+K_{\mu2}u_p=m_1\upsilon_{10}^2\frac{du_{\upsilon1}}{dt}\left(1+u_{\upsilon1}\right)+m_2\upsilon_{20}^2\frac{du_{\upsilon2}}{dt}\left(1+u_{\upsilon2}\right)+$$

$$+m_3\upsilon_{30}^2\left(\frac{du_{\upsilon3}}{dt}+\frac{g}{\upsilon_{30}}\right)\left(1+u_{\upsilon3}\right).$$

Let’s denote:

$$K_\mu=N_0+K_{\mu1};\; K_{\mu p}=N_0+K_{\mu2},$$

then

$$K_\mu+N_0u_V+K_{\mu p}u_p=m_1\upsilon_{10}^2\frac{du_{\upsilon1}}{dt}\left(1+u_{\upsilon1}\right)+m_2\upsilon_{20}^2\frac{du_{\upsilon2}}{dt}\left(1+u_{\upsilon2}\right)+$$

$$+m_3\upsilon_{30}^2\left(\frac{du_{\upsilon3}}{dt}+\frac{g}{\upsilon_{30}}\right)\left(1+u_{\upsilon3}\right) \quad (24)$$

Equation (24) describes the operation of the manipulator pump used in the process of hot stamping of the blanks, while controlling the displacement volume.

5. Conclusions

The mathematical model of the manipulator operation is compiled. The equation describing the operation of hydraulic cylinders is presented in relative variables and comprises a relative pressure and the displacement volume of the pump change. When constructing a mathematical model of the manipulator, the reactions were determined from the side of the fixed parts to the moving parts of each mobility module. These reactions were determined using the D'Alembert method. The required reactions can be determined by drawing up the Lagrange equations of the second kind. Further, the chain of conversion of the useful power produced by the pump into the useful power of hydraulic motors was traced, and losses due to the introduction of efficiency factors were taken into account. The resulting equation describing the operation of the manipulator was transformed into an equation in relative variables.

This equation will be necessary in the study [9] of the stability of the automated system for controlling the displacement volume of the manipulator pump in various modes [10].

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