



A Study on the Application of Sensitivity Analysis for Ship Hydrodynamic Coefficients

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Abstract

In the mathematical model with multiple input variables, the sensitivity analysis of the input variables is an important step to ensure the reliability of the mathematical model. In order to optimize the ship manoeuvring simulation, in particular the optimization of the trajectory ship, the sensitivity analysis should be performed in the mathematical model to select the group of the most sensitive hydrodynamic coefficients. In this paper, the author applied the sensitivity analysis method in mathematics model of ship manoeuvring programming in order to optimize the ship trajectory of Esso Bernicia 193000DWT tanker model.

Keywords: Sensitivity Analysis, Hydrodynamic Coefficients, Ship Manoeuvring Simulation, Ship Manoeuvring Optimization.

1. Introduction

According to the achieved result of K.T Tran et al. [1], to optimise the ship trajectory by minimizing the deviation of ship position and manoeuvring parameters, ship trajectories optimization problem turned into the problem of ship hydrodynamics coefficients optimization because of in such case the hydrodynamics coefficient become the variables of deviation functions (objective function).

Since the very large number of variables and the influences of these on the objective functions are not the same, it is necessary to analyse the variable sensitivity to identify the most sensitivity variables (corresponding the Step 2 in the flowchart of the ship trajectory optimization procedure, Fig.1). Otherwise, due to the complexity of the mathematic model in the computational program, reducing number of variables may increase the convergence of defining the variables' optimal values, and simultaneously reduces the amount of iteration steps to quickly reach the minimum values of objective functions, thus decreases the calculation time of that programme.

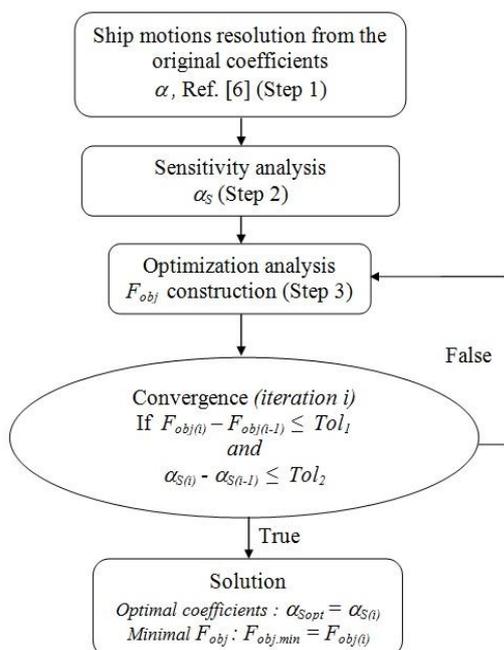


Fig.1: Flowchart of the ship manoeuvring optimization procedure

2. Methodology of Sensitivity Analysis [2, 3]

Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system (numerical or others) can be apportioned to different sources of uncertainty in its input. Due to the fact that uncertainty analyses are subject to change and quantity of the uncertainty factors (input variables). Ideally, the procedure of uncertainty and sensitivity analysis should be taken simultaneously (Fig. 2).

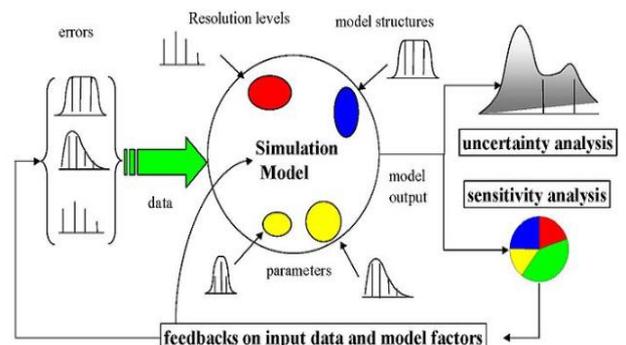


Fig.2: Scheme of sensitivity analysis model

There are several common sensitivity analysis methods:

- Local Methods
- Regression Analysis Methods
- Variance-Based Methods
- Emulator Methods
- HDMR (High-Dimensional Model Representations) Methods
- FAST (Fourier Amplitude Sensitivity Test) Methods
- Monte Carlo Methods.

In this research, the authors have applied the Local Methods on: investigating potential change in system function variable Y on each input variable X_i at initial fixed points x^0 :

$$\left| \frac{\partial Y}{\partial X_i} \right|_{x^0}$$

Sensitivity may be then measured by the influence change in the system function variable value Y on variation of variable X_i around the initial fixed point x^0 .

3. Mathematical Model of Ship Manoeuvring Simulation and Optimization [4,5,6,7]

(i) The non-dimensional differential equations of surface ship motions (3 degrees of freedom, applied for model of Esso Bernicia 193000 DWT oil tanker model [10]) is identified by:

$$\dot{u} - vr = gX''$$

$$\dot{v} + ur = gY'' \tag{1}$$

$$(Lk_z'') \cdot 2\dot{r} = gLN''$$

where:

u and v are ship velocities in GX- axis (surge motion) and GY- axis (sway motion) respectively;

\dot{u} and \dot{v} are ship axial accelerations in GX- axis (surge motion) and GY-axis (sway motion), respectively;

r is turning rate about GN-axis (yaw motion);

\dot{r} is turning acceleration about GN-axis; g is the gravity acceleration;

L is the ship length between perpendiculars;

k_z'' is non-dimensional of radius gyration of initial moment about vertical axis N.

The right sides of the equations in the set are the summation of the projection of hydrodynamics forces along the longitudinal axis X, transverse axis Y and vertical axis N respectively. Those components can be identified by:

$$\begin{aligned} g.X'' &= X_u'' \dot{u} + \frac{1}{L} X_{|u|}'' |u| + \frac{1}{L} X_{vr}'' vr \\ &+ \frac{1}{L} X_{vv}'' v|v| + \frac{1}{L} X_{c|c|\delta\delta}'' c|c|\delta^2 \\ &+ \frac{1}{L} X_{c|c|\beta\delta}'' c|c|\beta\delta + gT'' (1 - td) \\ &+ \frac{1}{L} X_{\dot{u}\xi}'' \dot{u}\xi + \frac{1}{L} X_{|u|\xi}'' |u|\xi \\ &+ \frac{1}{L} X_{vr\xi}'' vr\xi + \frac{1}{L} X_{vv\xi\xi}'' v^2\xi^2 \end{aligned} \tag{2}$$

$$\begin{aligned} g.Y'' &= Y_v'' \dot{v} + \frac{1}{L} Y_{uv}'' uv + \frac{1}{L} Y_{|v|v}'' |v|v \\ &+ \frac{1}{L} Y_{c|c|\beta}'' c|c|\beta + \frac{1}{L} Y_{ur}'' ur \\ &+ \frac{1}{L} X_{c|c|\beta|\beta\delta}'' c|c|\beta|\beta\delta + Y_T gT \\ &+ \frac{1}{L} Y_{ur\xi}'' ur\xi + \frac{1}{L} Y_{uv\xi}'' uv\xi \\ &+ \frac{1}{L} Y_{v\xi}'' v\xi + \frac{1}{L} Y_{|v|v\xi}'' |v|v\xi \\ &+ \frac{1}{L} Y_{c|c|\beta|\beta\xi}'' c|c|\beta|\beta\xi \end{aligned} \tag{3}$$

$$\begin{aligned} g.L.N'' &= \frac{1}{L^2} N_r'' r + \frac{1}{L^2} N_{uv}'' uv \\ &+ \frac{1}{L} N_{|v|v}'' |v|v + \frac{1}{L^2} N_{c|c|\delta}'' c|c|\delta \\ &+ \frac{1}{L} N_{ur}'' ur + \frac{1}{L^2} X_{c|c|\beta|\beta\delta}'' c|c|\beta|\beta\delta \\ &+ \frac{1}{L} N_T gT + \frac{1}{L} N_{ur\xi}'' ur\xi + N_{r\xi}'' r\xi \\ &+ \frac{1}{L^2} N_{uv\xi}'' uv\xi + \frac{1}{L} N_{vr\xi}'' vr\xi \end{aligned} \tag{4}$$

$$+ \frac{1}{L^2} N_{c|c|\beta|\beta\xi}'' c|c|\beta|\beta\xi$$

where:

c is the flow velocity at rudder;

δ is the rudder angle;

t_d is the thrust deduction;

β is the drift angle;

ξ is the index evaluating the water depth effect respect to ship draft;

T'' is non-dimensional propeller thrust;

$X_u'', X_{|u|}'', \dots, Y_v'', Y_{uv}'', \dots, N_r'', N_{uv}'', \dots, N_{c|c|\beta|\beta\xi}''$ are ship hydrodynamics coefficients.

The set of equations (1) can be solved by Euler numerical method using Matlab programming language. The solutions of the set (1) are the ship motion components including:

+ Ship positions and heading angle: $\eta = [x \ y \ \psi]^T$

+ Ship velocity components: $\gamma = [u \ v \ r]^T$

(ii) The objective function form in two typical of ship manoeuvring tests:

According to the ‘‘Standards for ship manoeuvrability – Resolution MSC 137(76)’’ proposed by International Maritime Organization IMO [8], there are seven tests: Turning Circle, Spiral, Pull-out, Zigzag, Stopping, Rudder, and Williamson.

In this research, the author has simulated and defined the objective functions [9] for analysing the sensitivity of the ship hydrodynamic coefficients in two typical ship manoeuvring tests: Turning Circle and Zigzag.

+ Turning Circle test:

$$F_{obj} = \left(\sum_{i=1}^n f_i(\alpha)^2 \right)^{1/2} = \left(\sum_{i=1}^n \Delta S_i^2 \right)^{1/2}$$

Where:

$f_i(\alpha) = \Delta S_i$ is the deviation function of ship position between simulation and experiment at the measured point (i), calculated by:

$$\Delta S_i^2 = \left(x_i^{cal} - x_i^{exp} \right)^2 + \left(y_i^{cal} - y_i^{exp} \right)^2$$

Subscripts *cal* and *exp* indicate the computed and experimental data respectively, (x_i, y_i) are the coordinates of the point i on the ship’s trajectory, and n is the number of pairs of points to be approximated.

$\alpha = [\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_M]^T$: is the matrix of M (M=35) of the influences of ship’s hydrodynamic coefficients on the deviation of ship position between simulation and experiment data;

+ Zigzag test:

$$F_{obj} = \left(\sum_{i=1}^n f_i(\alpha)^2 \right)^{1/2} = \left(\sum_{i=1}^n \Delta \psi_i^2 \right)^{1/2}$$

where:

$f_i(\alpha) = \Delta \psi_i$ is the deviation function of heading angle between simulation and experiment at the measured point, calculated by:

$$\Delta \psi_i^2 = \left(\psi_i^{cal} - \psi_i^{exp} \right)^2$$

Subscripts *cal* and *exp* indicate the computed and experimental data respectively, (i) is the point i on the ship’s yaw angle, and n is the number of pairs of points to be approximated.

$\alpha = [\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_M]^T$: is the matrix of M (M=35) the influences of ship hydrodynamic coefficients on the deviation of heading angle between simulation and experiment;

(iii) Objectives of optimization problem and the necessity of ship hydrodynamic coefficients sensitivity analysis:

Objective functions F_{obj} are the functions of variables α_i (hydrodynamic coefficient). Optimizing ship trajectory is essentially programming the computation of minimizing the values of those objective functions, thereby determining the optimum values of variables α_i .

4. Application of Sensitivity Analysis in Mathematical Model of Ship Manoeuvring Simulation and Optimization

In general, objective function F_{obj} can be identified as follows:

$$F_{obj} = F(\alpha_i, x, t)$$

where:

α_i are the ship hydrodynamic coefficients (35 coefficients);

x is the vector of ship motions (position, heading angle, surging velocity, sway velocity, ...);

t is computing time.

Sensitivities of the hydrodynamic coefficients are expressed by the influences of each factor with respect to the change of objective function F_{obj} . These changes may be exposed through the relative gradient of F_{obj} at each α_i , as follows:

$$\frac{\partial F_{obj}}{\partial \alpha_i} = \frac{F_{obj}(\alpha_i + \varepsilon \alpha_i) - F_{obj}(\alpha_i - \varepsilon \alpha_i)}{2\varepsilon \alpha_i}$$

where:

ε is a small chosen tolerance coefficient for evaluating how the relative gradient of F_{obj} response to the small variation of α_i (ship hydrodynamic coefficients), these changes are shown in Fig. 3:

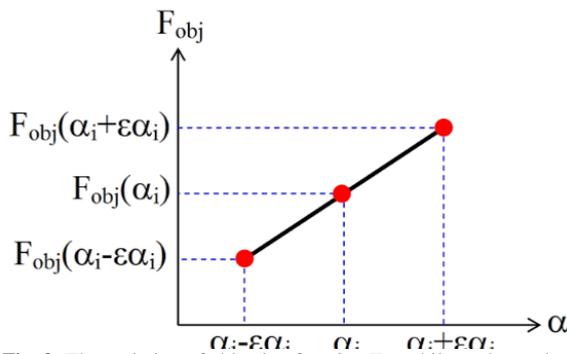


Fig. 3: The variation of objective function F_{obj} while α_i changed

In this study, we proposed: $\varepsilon = 1\%$

5. Numerical Results.

The author has written a sensitivity analysis program using Matlab to evaluate the influence of input variables (hydrodynamic parameters) on the variation of objective function F_{obj} in each test: Turning Circle and Zigzag [1,2,3].

The important input factors consist of:

The number of hydrodynamic parameters taken into analysing sensitivity:

The total number of hydrodynamic parameters is 35 (Table 1).

Table 1: The original hydrodynamic parameters of Esso Bernicia 193000 dwt tanker model [4,10].

ID No.	Coefficient	Reference value
1	$X_{\dot{u}}''$	-0.05
2	X_{vr}''	1.020
3	Y_v''	-0.020
4	$Y_{ c c \beta \beta\delta}''$	-2.16
5	Y_T''	0.04
6	N_T''	-0.02
7	$N_{\dot{r}}''$	-0.0728
8	$Y_{ v v}''$	-2.4

ID No.	Coefficient	Reference value
9	$N_{ v r}''$	-0.3
10	$X_{ v v}''$	0.3
11	Y_{uv}''	-1.205
12	N_{uv}''	-0.451
13	$X_{\dot{u}\xi}''$	-0.05
14	$Y_{\dot{v}\xi}''$	-0.378
15	$Y_{ur\xi}''$	0.182
16	$N_{ur\xi}''$	-0.047
17	$X_{vr\xi}''$	0.378
18	$Y_{ v v\xi}''$	-1.5
19	$N_{vr\xi}''$	-0.12
20	$Y_{ c c \delta}''$	0.208
21	$Y_{uv\xi}''$	0
22	$N_{uv\xi}''$	-0.241
23	$X_{ c c \beta \delta}''$	0.152
24	$N_{ c c \delta}''$	-0.098
25	$X_{vv\xi\xi}''$	0.0125
26	$Y_{ c c \beta \beta\delta}''$	-2.16
27	$N_{ c c \beta \beta\delta}''$	0.688
28	$Y_{ c c \beta \beta\delta\xi}''$	-0.191
29	$N_{ c c \beta \beta \delta \xi}''$	0.344
30	Y_{ur}''	0.248
31	N_{ur}''	-0.207
32	$X_{u u}''$	-0.0377
33	$N_{\dot{r}\xi}''$	-0.0045
34	$X_{ u u\xi}''$	-0.0061
35	$X_{ c c \delta\delta}''$	-0.093

The range of varying objective function are used as variables filtering boundary condition (Tol):

$$\frac{\partial F_{obj}}{\partial \alpha_i} > |Tol|$$

In our study, we proposed: $Tol = \pm 0.1$.

The selection of most influential hydrodynamic parameters (highest sensitivity) is based on the following criteria: In the same range of small variation in the vicinity of coefficients α_i , the most sensitive coefficients are the ones made the gradient of the objective function values change the most (i.e. the graph of F_{obj} is steepest). The group of most sensitive factors are identified depending on the filtering of the variation of the relative gradient of F_{obj} .

The result of sensitivity analysis can be defined as follows:

(1) Turning Circle test:

The variation of partial derivative of F_{obj} for each α_i is shown in Fig.4. The most sensitive parameters corresponding to Tol (Tol = ± 0.1) consist of 10 variables:

$$N''_T, Y''_{ur\xi}, N''_{ur\xi}, Y''_{|c|c\delta}, N''_{vv\xi}, X''_{|c|\beta\delta}, N''_{|c|c\delta}, N''_{ur}, X''_{|u|u\xi}, X''_{|c|c\delta}$$

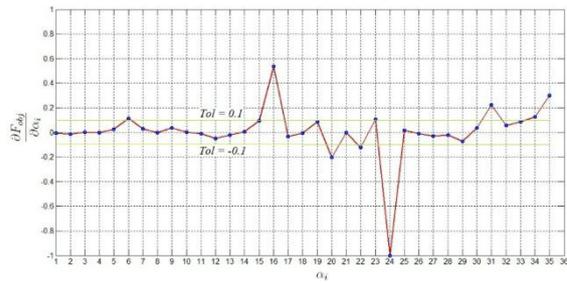


Fig. 4: Sensitivity analysis results of Turning Circle test

(2) Zigzag test:

The variation of partial derivative of F_{obj} for each α_i is shown in Fig.5. The most sensitive parameters corresponding to Tol (Tol = ± 0.1) consist of 10 variables:

$$Y''_T, N''_T, N''_f, Y''_{ur\xi}, N''_{ur\xi}, N''_{uv\xi}, N''_{|c|c\delta}, X''_{|u|u}, N''_{r\xi}, X''_{|u|u\xi}$$

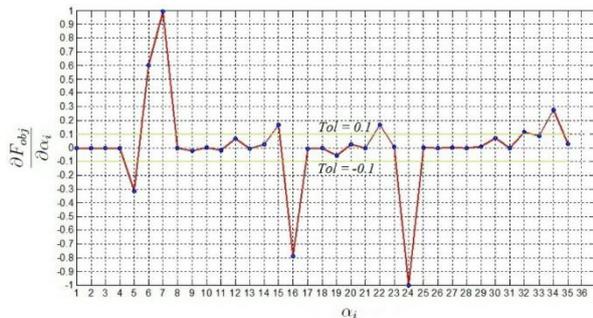


Fig. 5: Sensitivity analysis results of Zigzag test

(3) Combination of the sensitivity analysis results of Turning Circle and Zigzag tests: It can be observed from the results of analysing and selecting the most sensitive hydrodynamic coefficients for each test that: there are 5 common sensitive coefficients and 5 independent sensitive coefficients for each test. Thus, in total the combination has 15 most sensitive coefficients for both tests:

No.	The most sensitive hydrodynamic coefficients	No.	The most sensitive hydrodynamic coefficients
1	N''_T	9	$X''_{ u u\xi}$
2	$Y''_{ur\xi}$	10	$X''_{ c c\delta}$
3	$N''_{ur\xi}$	11	Y''_T
4	$Y''_{ c c\delta}$	12	N''_f
5	$N''_{vv\xi}$	13	$N''_{uv\xi}$
6	$X''_{ c \beta\delta}$	14	$X''_{ u u}$
7	$N''_{ c c\delta}$	15	$N''_{r\xi}$
8	N''_{ur}		

After selecting the set of the most sensitive hydrodynamic coefficients of ship, the procedure of ship trajectory simulation and optimization will be carried out, the deviation between experimental data and simulation data will be closed. Thus, it may also identify the optimal value of these most sensitive hydrodynamic coefficients corresponding the optimal ship trajectory.

6. Conclusion

The authors have presented the overview of methodology, algorithm, and application of sensitivity analysis method. This article has summarized the approaches of the analyzing sensitive of 35 hydrodynamic parameters. Therefore, 15 most sensitive parameters of Esso Bernicia 193000 dwt full-scale tanker model was determined to cater the numerical procedure of ship manoeuvring simulation and optimization. The identification of these 15 ship's hydrodynamic coefficients helps reducing the number of identified coefficients in the next research steps of ship manoeuvring optimization. As a result, the most influential hydrodynamic coefficients on the derivation of the simulation trajectories was identified. This paper is useful reference for further study of ship trajectory simulation and optimization, methodology of ship's hydrodynamic coefficient identification [5,6,7], as well as the simulation of the ship-to-ship interaction [12], ship-to-bank/bottom interaction [11], ship collision prediction, etc.

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