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Research paper

Anisotropic Seismic Wave Simulation via Pseudo-spectral and Pseudo-acoustic Approximations

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Abstract

Nowadays, the concept of seismic anisotropy is applied widely in seismic modelling, imaging and inversion. To appreciate the influences of seismic anisotropy, an anisotropic wave equation needs to be employed. Despite the fact that seismic anisotropy is innately an elastic phenomenon, the elastic anisotropic wave equation rarely employed in imaging methods because of its time-consuming computational operation. Pseudo-acoustic equations are proposed to reduce the computational cost, even though, they have a few drawbacks. Pseudo spectral approaches are appropriate alternatives for pseudo acoustic methods. In this paper, we aim to study different techniques applied to propagate seismic wave in anisotropic environment. Firstly, the theory and results of a pseudo acoustic wave propagator are demonstrated. Then, a spectral technique based on lowrank approximation for modelling pure acoustic waves is discussed, and we investigate and compare its accuracy and efficiency to the pseudo acoustic method.

Keywords: *pseudo-acoustic, pseudo-spectral, seismic anisotropy, wave modeling*.

1. Introduction

The importance of anisotropy in seismic imaging and processing is broadly recognised by geoscientists. The implementation of anisotropy in exploration became more essential when seismic data were acquired with wider offsets and azimuths. Anisotropy is defined as the variation of one or more properties of a medium with direction. In the case of seismic anisotropy, the velocity of seismic waves depends on the direction of propagation. The main sources of seismic anisotropy in the Earth are fracture networks, alignment of mineral grains and thin layering. Anisotropic techniques can be grouped into three categories: P wave imaging, S wave splitting and mode-converted PS-waves, and AVO analysis [1].

In P-wave scope, imaging algorithms have been mostly expanded for vertical transverse isotropy (VTI) and tilted transverse isotropy (TTI) [2]. Transverse isotropy occurs when thin bed series, perpendicular to the symmetry axis, are isotropic. When thin layers are horizontal, the medium is called VTI and when layers are tilted due to tectonic activity, it is TTI (Figures 1a and 1b). Cracks and fractures in thin bed interfaces can also be created due to strong tectonic stress, and they result in an azimuthal anisotropy [3]. In this condition, the transverse isotropic theory cannot justify the discrepancy of residual moveouts among common image gathers (CIGs) of different azimuths. Orthorhombic, as a more inclusive anisotropic model, can be employed not only to cope with azimuthal velocity variation, but also to obtain significant details of fracture networks (Fig. 1c).

The seismic wave equation performs as an engine for imaging and inversion algorithms. To appreciate the influences of seismic anisotropy in imaging, the isotropic wave equation must be replaced by the anisotropic one. Despite the fact that seismic anisotropy is innately an elastic phenomenon, the elastic anisotropic wave equation rarely employed in imaging methods because of its time-consuming computational operation. Pseudo-acoustic equations are proposed to reduce the computational cost[4].

Alkhalifah [5] indicated that one can derive a much simpler VTI dispersion relation by setting the vertical S wave velocity to zero. He subsequently suggested an acoustic VTI approximation by utilizing the dispersion relation, and it leaded to acceptable estimations to the elastic equation. Various pseudo-acoustic wave equations were later developed based on Alkhalifah's dispersion relation [6]. Using Hooke's law and the equations of motion, in which the vertical S wave velocity is also set to zero, is another approach to achieve a pseudo-acoustic approximation [7]. In both specified approaches, pseudo-acoustic TTI equations can be obtained by considering the tilt of the symmetry axis.

Despite the well performance of the pseudo-acoustic approximation in elliptical anisotropic and isotropic ($\varepsilon = \delta$, ε and δ are Thomsen's parameters) conditions, for anellipticity, where $\varepsilon \neq \delta$, the S wave velocity is only zero along the symmetry axis. The S wave velocity has a value in other directions and when the approximation is

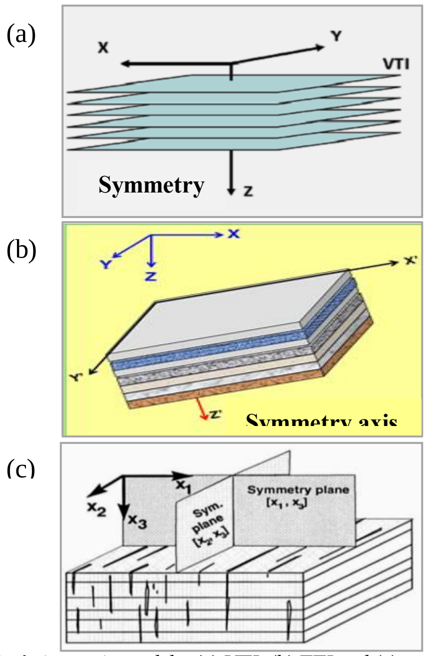

Fig. 1: Anisotropic models: (a) VTI, (b) TTI and (c) orthorhombic.

applied for imaging, Sv wave components add noise into the images [8]. Another difficulty for employing the pseudo-acoustic wave equations is that they are only valid when $\varepsilon \geq \delta$, however, subsurface formations may not practically comply with this condition. Moreover, the S-wave presence in the result of pseudo-acoustic TTI approximation leads to numerical instability, which introduces noticeable errors into dip and azimuth angles [9].

Various methods have been suggested to solve the problems involved in the pseudo-acoustic approximations. One technique to eliminate S-wave artifacts is to locate the source in either an isotropic or elliptical anisotropic environment [10]. However, P waves can still convert to Sv waves due to interfaces with sharp impedance contrasts, while waves propagate in the subsurface. One can eliminate the converted S waves either by adding finite Sv wave velocities along the axis of symmetry (which also fixes the instability issue [11]) or by applying a filter at each output time step [12]. An efficient approach to remove the Sv wave mode completely is to derive pure acoustic wave equations.

Several pure P-wave approximations have been developed for VTI, TTI and orthorhombic media. Klíe and Toro [13] modified Alkhalifah's acoustic wave equation. Their proposed equation does not create any S wave artifacts, and is applicable for any anisotropic strength. Du et al. [6] derived P and Sv wave equations for TTI media based on the weak anisotropy approximation to apply in a reverse time migration. The equations were written in the time-wavenumber domain, and the pseudo-spectral method was used for numerical solutions. Liu et al. [14] further developed new VTI pure P wave propagators from the optimised separable approximation. Decoupling the P - Sv dispersion relation and deriving separate equations for P and Sv waves resulted in unconditionally stable and free S wave solutions for P wave equations. Given that shear and mode-converted waves carry valuable information about the anisotropy of the Earth, in these methods, shear wave equations can be implemented for anisotropic S-wave modelling.

Chu et al. [15] developed a pure acoustic VTI wave equation, which can be efficiently solved by implementing the pseudo-spectral approach. They started from the pure P wave dispersion relation and showed that it could be rewritten in a summation form. The pseudo-analytical method is another way to approximate the pseudo-differential operator using interpolation of several constant parameter modelling responses, similar to the phase shift plus interpolation algorithm for one-way wave propagation. The lowrank approximation, the rapid expansion and the Fourier finite difference (FFD) methods are improved versions of the pseudo-analytical technique that was introduced by Fomel et al. [16]. Moreover, Kang and Cheng [17] introduced new coupled second-order systems called pseudo-pure-mode wave equations. They separated P wave and S wave data by applying a filtering step to correct projection deviations resulting from the variation between polarisation direction and its isotropic references [18].

In this paper, we aim to study different techniques applied to simulate seismic wave propagation in anisotropic media. In section 2, the theory and results of a pseudo acoustic wave propagator are demonstrated. Next, a spectral technique for modelling pure acoustic waves proposed by Fomel et al. [16] is discussed, and we investigate and compare its accuracy and efficiency to the pseudo acoustic method.

2. Methodology

2.1. Pseudo-acoustic approximation for TTI wave modelling

The wave equation introduced by Alkhalifah [19], was called an acoustic approximation, which was later changed to pseudo-acoustic due to its features to produce both P and Sv waves. Following Alkhalifah's study [19], various alternatives to the coupled equations have been proposed. We demonstrate the theory and numerical examples of the TTI pseudo-acoustic wave equation suggested by Fletcher et al. [4].

2.1.1. Theory

Alkhalifah's dispersion relation with assumption of *Vsz* equal to zero in the exact TTI dispersion relation, is given by:

$$\omega^4 = \left[v_{px}^2\left(\hat{k}_x^2 + \hat{k}_y^2\right) + v_{pz}^2 \hat{k}_z^2\right]\omega^2 + v_{pz}^2\left(v_{pn}^2 - v_{px}^2\right)\left(\hat{k}_x^2 + \hat{k}_y^2\right)\hat{k}_z^2 \quad (1)$$

where $\hat{k}$ is wavenumber and the circumflex presents its direction in a rotated coordinate system aligned with the symmetry axis, ω is angular frequency, v_{px} is P wave horizontal velocity given by $v_{px} = v_{pz}\sqrt{1+2\varepsilon}$, v_{pz} is P wave vertical velocity, v_{pn} is the P wave normal moveout (NMO) velocity which is defined by $v_{pn} = v_{pz}\sqrt{1+2\delta}$, and v_{sz} is Sv wave vertical velocity.

The rotated wavenumbers can be defined as:

$$\hat{k}_x = k_x cos\theta cos\varphi + k_y cos\theta sin\varphi + k_z sin\theta$$

$$\hat{k}_y = -k_x sin\varphi + k_y cos\varphi$$

$$\hat{k}_z = -k_x sin\theta cos\varphi - k_y sin\theta sin\varphi + k_z cos\theta, \quad (2)$$

where θ is the angle between the wavefront normal and the vertical axis and φ is the tilt angle, i.e. the angle between the symmetry axis and the vertical axis. By replacing (2) in (1), the new equation can be written as:

$$\omega^4 = \left(v_{px}^2 f_2 + v_{pz}^2 f_1\right)\omega^2 + v_{pz}^2\left(v_{pn}^2 - v_{px}^2\right) f_1 \cdot f_2 \quad (3)$$

Where

$$f_1 = k_x^2 sin^2\theta cos^2\varphi + k_y^2 sin^2\theta sin^2\varphi + k_z^2 cos^2\theta + k_x k_y sin^2\theta sin2\varphi + k_y k_z sin2\theta sin\varphi + k_x k_z sin2\theta cos\varphi \quad (4)$$

And

$$f_2 = k_x^2 + k_y^2 + k_z^2 - f_1. \quad (5)$$

Applying an inverse Fourier transform on equations (4) and (5) generates the following differential operators:

$$H_1 = sin^2\theta cos^2\varphi \frac{\partial^2}{\partial x^2} + sin^2\theta sin^2\varphi \frac{\partial^2}{\partial y^2} + cos^2\theta \frac{\partial^2}{\partial z^2} + sin^2\theta sin2\varphi \frac{\partial^2}{\partial x \partial y} + sin2\theta sin\varphi \frac{\partial^2}{\partial y \partial z} + sin2\theta cos\varphi \frac{\partial^2}{\partial x \partial z} \quad (6)$$

and

$$H_2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - H_1. \quad (7)$$

Solving the fourth order partial differential equation (PDE) (3) in time is cumbersome. Moreover, mixed space and time derivatives need more computation than single spatial derivatives. Hence, equivalent coupled lower order equations are desired. Fletcher et al. [4] proposed a coupled system of second order PDEs in time which is applicable when $\varepsilon - \delta \geq 0$. If both sides of equation (3) are multiplied with the wavefield $p(\omega, k_x, k_y, k_z)$, and using the auxiliary function

$$q(\omega, k_x, k_y, k_z) = \frac{\omega^2 + (v_{pn}^2 - v_{px}^2) f_2}{\omega^2} p(\omega, k_x, k_y, k_z), \quad (8)$$

the new wave equations are defined by:

$$\frac{\partial^2 p}{\partial t^2} = v_{px}^2 H_2 p + v_{pz}^2 H_1 q$$

$$\frac{\partial^2 q}{\partial t^2} = v_{pn}^2 H_2 p + v_{pz}^2 H_1 q \ . \quad (9)$$

The aforementioned pseudo acoustic equations are derived for 3D media, but one can modify them as a 2D propagator by removing one of the spatial variables.

2.2. Modelling pure P wave in anisotropic media using lowrank approximation

The phase shift extrapolation technique provides P wavefields without shear wave artifacts since the P and S wave solutions are given by different parts of the wavenumber spectrum [20]. We describe a spectral method that uses the TTI dispersion relation, mentioned in the previous section, and a mixed domain acoustic wave extrapolator. The lowrank approximation is used to conduct the mixed domain operator.

2.2.1. Theory

For acoustic wave extrapolation, we start with the following acoustic wave equation [21]:

$$\frac{\partial^2 p}{\partial t^2} = v(\boldsymbol{x})^2 \nabla^2 p \quad (10)$$

where $p(x,t)$ is the seismic pressure wavefield and $v(x)$ is the wave velocity. Assuming that $v(x) \equiv v_0$ for a homogeneous medium and after applying a Fourier transform in space on equation (10) leads to:

$$\frac{d^2 \hat{p}}{dt^2} = -v_0^2 |\boldsymbol{k}|^2 \int_{-\infty}^{+\infty} p(\boldsymbol{x}, t) e^{i\boldsymbol{k}.\boldsymbol{x}} d\boldsymbol{x} \quad (11)$$

and its analytical solution results in the well-known second order time marching scheme [22]:

$$p(x, t+\Delta t) + p(x, t-\Delta t) = 2\int_{-\infty}^{+\infty} \hat{p}\,(k,t) \cos(|k| v_0 \Delta t) e^{-ik.x} dk. \quad (12)$$

Equation (12) yields a precise and effective solution by using the fast Fourier transform (FFT) when the medium has a constant velocity. In the case of a variable velocity medium, v_0 can be replaced with $v(x)$, however, FFT cannot be applied directly because of a space-wavenumber mixed domain expression in the integral operation:

$$W(\boldsymbol{x}, \boldsymbol{k}) = \cos(|\boldsymbol{k}| v(\boldsymbol{x}) \Delta t) \quad (13)$$

and the mixed domain matrix increases the computational cost from $O(N_x log N_x)$ to $O(N_x^2)$ where N_x is the total size of the 3D space grid. Fomel et al. [16] proposed a numerical approach, known as lowrank approximation, to overcome the mixed domain problem. In the lowrank decomposition, equation (13) can be rewritten as:

$$W(\boldsymbol{x}, \boldsymbol{k}) \approx \sum_{m=1}^{M} \sum_{n=1}^{N} W(\boldsymbol{x}, \boldsymbol{k}_m) a_{mn} W(\boldsymbol{x}_n, \boldsymbol{k}), \quad (14)$$

where $W(\boldsymbol{x}, \boldsymbol{k}_m)$ is a submatrix of $W(\boldsymbol{x}, \boldsymbol{k})$ that includes columns related to $\boldsymbol{k}_m$, $W(\boldsymbol{x}_n, \boldsymbol{k})$ is the submatrix involving rows associated with $\boldsymbol{x}_n$, and a_{mn} are the middle matrix coefficients. Substituting equation (14) into equation (13) gives:

$$p(\boldsymbol{x}, t+\Delta t) + p(\boldsymbol{x}, t-\Delta t) \approx 2\sum_{m=1}^{M} W(\boldsymbol{x}, \boldsymbol{k}_m) \left(\sum_{n=1}^{N} a_{mn} \left(\int e^{-i x \boldsymbol{k}} W(\boldsymbol{x}_n, \boldsymbol{k}) \hat{p}(\boldsymbol{k}, t) d\boldsymbol{k} \right) \right) \quad (15)$$

which provides faster computation for $p(\boldsymbol{x}, t+\Delta t)$ by applying N inverse FFTs. The above decomposition decreases the cost to $O(N\, N_x log N_x)$, in which N is a small number, and determines the rank of the decomposition. The rank of the approximation (M and N) and correspondingly the number of the FFTs may increase if the time step size Δt increases.

To incorporate anisotropy into the lowrank approximation, we need to use the anisotropic formula for $v(\boldsymbol{x})|\boldsymbol{k}|$ which can be obtained by considering $\omega = \boldsymbol{k}\boldsymbol{v}_p$. Anisotropic velocity takes the following form[23]:

$$v_p(\boldsymbol{x},\hat{\boldsymbol{k}},\eta) = \sqrt{\frac{1}{2}(v_{px}^2\hat{k}_x^2 + v_{pz}^2\hat{k}_z^2 + \frac{1}{2}\sqrt{(v_{px}^2\hat{k}_x^2 + v_{pz}^2\hat{k}_z^2)^2 - \frac{8\eta}{1+2\eta}v_{px}^2 v_{pz}^2\hat{k}_x^2\hat{k}_z^2}} \quad (16)$$

where $\hat{k}_x$ and $\hat{k}_z$ are defined by equation (2) for TTI media, and $\eta = \frac{\varepsilon-\delta}{1+2\delta}$. The aforementioned expressions can be used to extrapolate P wave in a TTI environment.

3. Results and discussion

3.1. Pseudo-acoustic approximation

2D wave modelling is performed in homogenous TI media by utilizing the finite difference approach. Figure 2 displays wavefields for different TI conditions at time t=0.8 s. The source dominant frequency is 10 Hz. The wavefield in an elliptical VTI medium, where ε and δ are equal to 0.25 (Fig. 2a), is effectively free of Sv waves, and only P waves are propagated.

In figures 2b and 2c, the tilt angles are $\varphi = 0°$ and $\varphi = 45°$, and the anisotropic parameters are defined as $\varepsilon = 0.22$ and $\delta = 0.12$, respectively. It is obvious that a diamond shaped Sv wavefield, which is known as an artefact Sv wave, appears along with the P wavefield. Another instance is a comparison of the isotropic and anisotropic shot gathers that are generated by applying the pseudo-acoustic algorithm on the Marmousi model (Fig. 3). Despite the isotropic gathers, the anisotropic ones involve Sv waves that appear as noise and need to be filtered. To eliminate noise in modelling, we can locate the source in an isotropic layer where Sv disappears.

3.2. lowrank approximation

The first example is wave modelling in an isotropic model with constant velocity by applying the lowrank approximation and finite difference. A Ricker wavelet with dominant frequency of 25 Hz is utilised for the source located in the middle of the model. A similar grid size and time step are used in both simulations. Figure 4 illustrates the difference of wavefields produced by the fourth order finite difference method and the fourth order lowrank approximation. It is obvious that the finite difference snapshot includes numerical dispersions, while the lowrank result is dispersion-free.

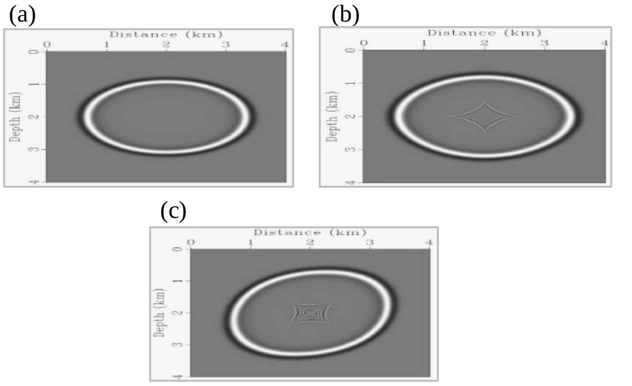

Fig. 2: Pseudo-acoustic wavefields in a medium with Vpz=3km/s and anisotropic condition (a) $\varepsilon = 0.25$ and $\delta = 0.25$, (b) $\varepsilon = 0.22$ and $\delta = 0.12$, and (c) $\varepsilon = 0.22, \delta = 0.12$ and $\varphi = 45°$.

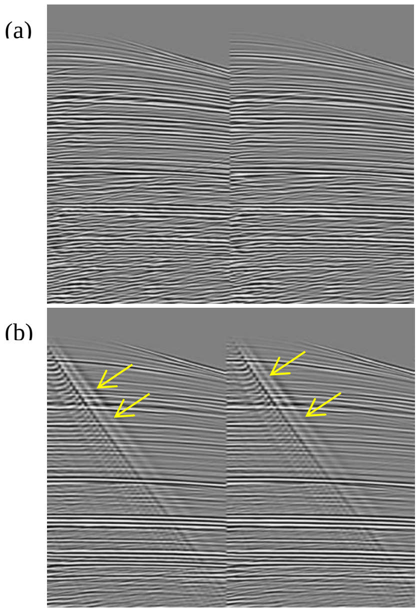

Fig. 3: (a) Isotropic and (b) VTI shot gathers created by pseudo-acoustic wave equation.

Next, we conduct modelling in an anisotropic model with VTI and TTI conditions. The source frequency is 20 Hz, $\eta = 0.15$ and $\varphi = 45°$ for the TTI medium. The finite difference wavefields for both VTI and TTI consist of dispersion artifacts as well as an Sv wave component, however, the lowrank approximation, as a spectral method, does not lead to dispersion and Sv wave; its accuracy is remarkable (Figure 5). Using the exact dispersion relation in the lowrank approach causes no coupling of Sv waves and P waves in the anisotropic results. In these examples, we considered the time step for the finite difference and lowrank methods to be equal, but when it increases, in some situations the finite difference algorithm becomes unstable, while the lowrank algorithm is still stable and accurate. The reason for this behaviour is that increasing Δt to a specific value increases the rank N, and higher rank leads to a more accurate wave extrapolation, although with some extra computational time. The lowrank approximation is focused principally on the phase of wave, and does not provide precise amplitude. Thus, it is suitable for seismic imaging such as reverse time migration but not for seismic modelling which needs to consider the amplitude changes caused by elastic factors, namely, variable density. For obtaining true amplitude P waves, one may use effective mode separation and decomposition [18].

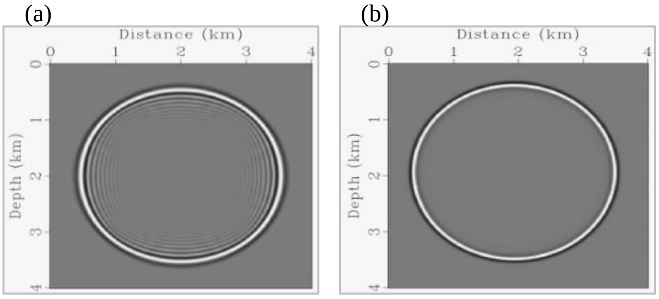

Fig. 4: Isotropic wavefields generated by (a) fourth order finite difference, and (b) fourth order lowrank approximation.

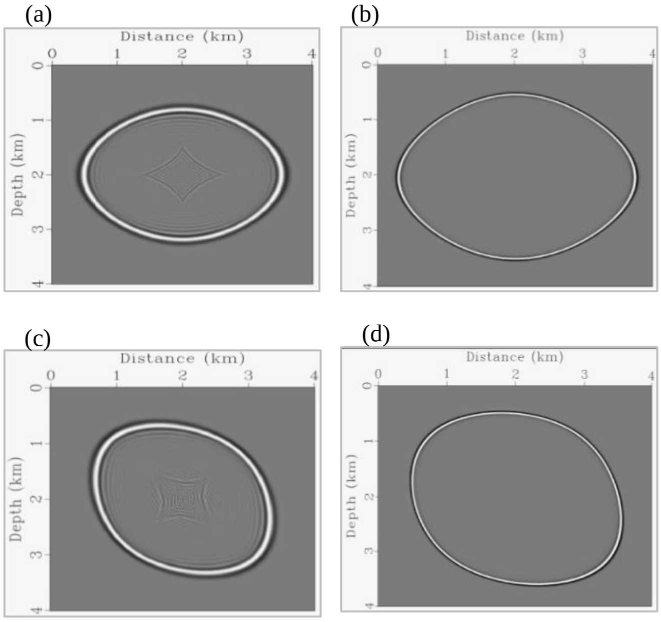

Fig. 5: Comparison of finite difference and lowrank approximation wavefields in anisotropic media: (a) VTI finite difference snapshot, (b) VTI lowrank approximation snapshot, (c) TTI finite difference snapshot with $\varphi = 45°$, and (d) TTI lowrank approximation snapshot with $\varphi = 45°$.

4. Conclusions

We studied the key concepts and state-of-the-art techniques for seismic wave simulation in anisotropic media. The pseudo-acoustic approximation produce perfect results in isotropic and elliptical anisotropy, but in anelliptic circumstances, Sv wave components, considered as noise, are introduced into the P wavefield. Another drawback of pseudo acoustic approximations is that they are only accurate if ε is equal or greater thanδ, however, subsurface structures may not adhere to this condition. Applying a filter or locating the source in isotropic or elliptical anisotropic medium can eliminate the Sv artifacts. To completely remove the Sv wave mode, starting with the pure P wave dispersion relation, and employing the spectral method leads to pure acoustic wavefields. We also described the lowrank approximation which is a spectral technique, resulting in VTI and TTI acoustic wavefields without any artifacts. The numerical examples indicated that this method is quite accurate and is free of dispersions and noise.

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