

Solidness investigation of ward thickness on Ferro convection in an anisotropic permeable framework in the presence of flat liquid temperature

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Abstract

The force field dependent viscousness result of ferroconvection in associate aeolotropic disposed porous system has been studied. The layer of fluid is undertaken to be delimited by free partition. The proper manner of constitutional equation is obtained by liner hypothesis theory. The limit worth of the horizontal magnitude relation and vertical temperature changes, aeolotropic particle parameters, magnetic porosity of porous system, increasing issue of magnetic dependent viscousness and therefore the magnetization parameter are studied. This methodology is found to stabilize solely unchanged mode of instability.

Keywords: aeolotropic porous medium; Ferro convection, horizontal thermal gradient, porous medium, instability analysis

1. Introduction

A ferrofluid is a liquid it becomes strongly magnetized in the presence of magnetic field. It is not possibly easy to found in nature but it can be artificially produced in different process. Ferrofluids do not continue to exist magnetization because of the non-existence of an externally applied fields. Ferrofluids are colloidal dispersions of sub-domain magnetic particles that retain their liquid characteristics in the media in which it is possible to induce substantial magnetic forces, which results in unusual fluid mechanical properties in the presence of an applied field. Ferrofluids are used in liquids cooled loud speakers where in the voice coil of the loud speakers gets heated up and also sound quality is affected by unwanted resonance in an undamped system. Some of the effects of drugs and medical techniques on the biological system of ferrofluids are magnetic drug targeting, high fever, dissimilar improvement in the quantity of imaginary Resonance magnetic and division of magnetic of cells. Ferro fluids are used as dissimilarity factor for MRI and also can be used for diagnosis the Cancer Index. Diagnosis of such cases the ferrofluids are made up of iron oxide nanoparticles, for "Super paramagnetic Iron oxide Nanoparticle". ferrofluids area unit employed in liquid cooled loudspeakers, that use being nothing quite specified drops of ferrofluids to conduct heat far from speaker coils. It's conjointly accustomed create zero-leakage turn shaft seals employed in pc-disk drives. Finlayson studied a vertical regular flux deliberated the convective instability of one element ferrofluid heated from below [1]. Rudraiah have studied and analysed the role of basic take temperature changes on Marangoni convection in magnetic attraction fluid [2]. Vaidyanathan had studied the role of dependent flux consistency of ferroconvection in a very sparsely distributed porous medium [3]. Bharti studied the result of thermo solutal convection

in magnetic attraction fluid [4]. The result of flux dependent consistency and property of porous medium on Ferro convection exploitation Darcy model has been analyzed by Ramanathan and Surendra. P [5]. Sunil analyzed the consequence of Associate in nursing action of dirt particle in a very rotating magnetic attraction fluid heated and saluted from below [6]. Vaidyanathan analyzed the result of flux dependent consistency on ferroconvection in a very rotating medium [7]. They conjointly discuss the result of force on solet driven ferrothermo-haline convection in a very medium of distributed particle suspension [8]. Study of ferroconvection and ferrothermo-haline convection for a rotating fluid within the presence of porous medium has been analyzed by several researchers. Mayboudi have studied ferrofluids in 2-dimensional system [9]. Within the existence of porous medium, Divya deliberated the result of rotation on fluid that is heated and saluted from [10]. Hoeman and Gurganai determined the result of the temperature dependent consistency on Bernad convection in a very porous medium exploitation Non-Darcy model [11]. Vanishree and Siddheshwar dead a linear stability method in respect to mono-diffusive convection throughout the aeolotropic rotating porous medium with temperature dependent viscosity [12]. Holyer studied and analyzed the 2 totally different density of traditional fluids within the existence of horizontal temperature gradients [13]. The result of horizontal gradient on ferroconvection has been investigated by Vaidyanathan [14]. Sekar investigated the result of horizontal thermal gradient on double scattering ferroconvection in a very moderately divided aeolotropic porous system within the presence of horizontal gradient [15].

2. Mathematical formulation

Let us consider an immeasurable area of incompressible viscous ferrofluid containing temperature and magnetic variation in horizontal and vertical direction. The co-ordinates are selected with z axis and calculating the vertically distances towards upwards and the values of x horizontally. Only the length and breadth derivation of the system is taken into assumed. The incompressible Boussinesq approximations density is assumed. A detailed examination of linear unchanged hypothesis analysis is used and vertical mode method is used. Brinkman model is used. Fluid dynamics basic incompressible continuity equation is given by.

$$\nabla \cdot \mathbf{q} = 0 \quad (1)$$

Let us consider the fluid as incompressible fluid and which having a various viscosity given by:

$$\mu = \mu_1 (1 + \delta \cdot \mathbf{B}) \quad (2)$$

Where μ_1 is taken as the viscosity of the fluid when the applied magnetic field is absent. The variation in the coefficient of magnetic field dependent viscosity has been taken to be isotropic $\delta_1 = \delta_2$. Hence the component wise μ can be written as:

$$\begin{aligned} \mu_x &= \mu_1 (1 + \delta B_1) \\ \mu_z &= \mu_1 (1 + \delta B_3) \\ \nu &= \frac{\mu}{\rho_0}, \quad \nu = \nu_1 (1 + \delta \cdot \mathbf{B}) \end{aligned} \quad (3)$$

Improved momentum equation for magnetic fluids is

$$\left(\frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) \mathbf{q} = \frac{\nabla p}{\rho_0} + \frac{\rho \mathbf{g}}{\rho_0} + \nu_1 \nabla^2 \mathbf{q} + \frac{\nabla \cdot (\mathbf{B} \mathbf{H})}{\rho_0} - \frac{\nu_1}{k} \mathbf{q} \quad (4)$$

Thermal diffusivity equation is as follows:

$$\left(\frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) T = K_1 \nabla^2 T \quad (5)$$

Where ability to attract a magnetic personality terms are vanished as their archive is assumed to be insignificantly very small. The porosity effect on thermal conductivity of a porous medium has not been considered, as the medium is assumed to be sparsely distributed.

The density equation of state for a single component fluid is:

$$\rho = \rho_0 [1 - \alpha_1 (T - T_0)] \quad (6)$$

Where $\mathbf{q}$, p , $\mathbf{g}$, $\mathbf{v}$, $\mathbf{B}$, $\mathbf{H}$, K_1 , α_1 , ρ , ρ_0 , T , S are velocity, pressure, acceleration due to gravity, kinematic viscosity, magnetic induction, magnetic field, thermal diffusivity, thermal expansion coefficient, density, density at reference temperature T_0 . Since the system has the highest growth rate for 2-dimensional disturbances, the role study involves towards the 2-dimensional instabilities due to small disturbance applied on basic state. The basic state equation can be written as:

$$\mathbf{Q}_b = 0, \quad T_b = T_0 + \bar{T}_x x + \bar{T}_z z \quad (7)$$

Where $\bar{T}_x$, $\bar{T}_z$ denote the horizontal and vertical temperature and changes in salinity gradients, respectively.

The continuous flow of fluid function ψ is defined as

$$\mathbf{Q}(u, v) = \left(-\frac{\partial \psi}{\partial z}, \frac{\partial \psi}{\partial x} \right) \quad (8)$$

The beginning state of magnetization and magnetic field are $[M_0(x), M_0(z)]$ and $[H_0(x), H_0(z)]$, respectively.

The Maxwell's equation for not able to conducting fluids are:

$$\nabla \cdot \mathbf{B} = 0 \quad (9)$$

$$\nabla \times \mathbf{H} = 0$$

Further relation between $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ are given by

$$\mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H}) \quad (10)$$

Where μ_0 is the quantity of measuring the influence on the substance of magnetic flux. It is considered that the magnetization joined with the magnetic field and temperature gradient field so that, the magnetization $\mathbf{M}$ is written as:

$$\mathbf{M} = \mathbf{H}/H \, M(H, T) \quad (11)$$

The modified magnetic equation of present state is given by

$$M = M_0 + X(H - H_0) - K(T - T_0) \quad (12)$$

Where

$\chi = \left(\frac{\partial M}{\partial H} \right)_{H_0, T_0}$ is the magnetic susceptibility

$K = - \left(\frac{\partial M}{\partial T} \right)_{H_0, T_0}$ is the Pyromagnetic coefficient of magnetic fluid

$$H_b(x, z) = \left[H_0(x) + \frac{K \bar{T}_x x}{(1 + \chi)} \right] \hat{i} + \left[H_0(z) + \frac{K \bar{T}_z z}{(1 + \chi)} \right] \quad (13)$$

$$H_1 + M_1 = (1 + \chi) H_1 - K T$$

$$H_1 + M_3 = (1 + \chi) H_3 - K T$$

The continuous flow of fluid equations for the small changes ψ , temperature T and the magnetic potential ϕ are given by:

$$\begin{aligned} \rho \left(\frac{\partial}{\partial t} - \nu_1 \nabla^2 \right) \nabla^2 \psi + \frac{\nu_1}{k_2} \frac{\partial^2 \psi}{\partial x^2} + \frac{\nu_1}{k_1} \frac{\partial^2 \psi}{\partial z^2} &= g \alpha_1 \frac{\partial T}{\partial x} + \frac{\mu_0}{\rho_0} \frac{\partial^2 \phi}{\partial x \partial z} [K(\bar{T}_z - \bar{T}_x)] \\ - \frac{\mu_0 K^2}{\rho_0 (1 + \chi)} \left(\bar{T}_z \frac{\partial T}{\partial x} - \bar{T}_x \frac{\partial T}{\partial z} \right) + \nu_1 \delta \mu_0 (M_0 + H_0) \left(\nabla^2 - \frac{1}{k_1} \right) \frac{\partial^2 \psi}{\partial z^2} &+ \nu_1 \delta \mu_0 \\ (M_0 + H_0) \left(\nabla^2 - \frac{1}{k_2} \right) \frac{\partial^2 \psi}{\partial x^2} \end{aligned} \quad (14)$$

$$\left(\frac{\partial}{\partial t} - K_1 \nabla^2 \right) T = \frac{\partial \psi}{\partial z} \bar{T}_x - \frac{\partial \psi}{\partial x} \bar{T}_z \quad (15)$$

$$(1 + \chi) \nabla^2 \phi - K \left(\frac{\partial T}{\partial x} - \frac{\partial T}{\partial z} \right) = 0 \quad (16)$$

The permeability of the porous medium along the horizontal and the vertical directions are k_1 and k_2 are respectively.

3. Stability analysis

Now we can apply linear stability analysis and normal mode analysis method, the solution of the equation (14)-(15) are derived:

$$(\psi, T, \phi) = (\psi_0, T_0, \phi_0) \exp[i(a_x x + a_z z) + \sigma t] \quad (17)$$

The horizontal and vertical wave numbers are a_x and a_z . The growth rate of instability is the real part of σ and the frequency is the imaginary part of σ . Substitution of equation (17) in equation (14)-(15) yields the following three equations.

$$\begin{aligned} \left[a^2 (\sigma + \nu_1 a^2) + \nu_1 X a^4 + \left(\frac{\nu_1}{k_2} a_x^2 + \frac{\nu_1}{k_1} a_z^2 \right) + X \left(\frac{\nu_1}{k_2} a_x^2 + \frac{\nu_1}{k_1} a_z^2 \right) \right] \psi &- \left[\frac{\mu_0 K^2}{\rho_0 (1 + \chi)} [(a_x \bar{T}_z - a_z \bar{T}_x) - g \alpha_1 a_x] \right] i T \\ + \frac{\mu_0}{\rho_0} K a_x a_z (\bar{T}_z - \bar{T}_x) \phi &= 0 \end{aligned} \quad (18)$$

$$(a_x \bar{T}_z - a_z \bar{T}_x) i \psi - (\sigma + K_1 a^2) T = 0 \quad (19)$$

$$K i (a_x + a_z) T + (1 + \chi) a^2 \phi = 0 \quad (20)$$

The wave number $a^2 = a_x^2 + a_z^2$

Remove the terms of ψ , T and ϕ from the above three equation (18)-(20) leads to the characteristic equation given by:

$$- \left[a^2(\sigma + v_1 a^2) + v_1 X a^4 + \left(\frac{v_1}{k_2} a_x^2 + \frac{v_1}{k_1} a_z^2 \right) \right] \times (\sigma + K_1 a^2)(1 + \chi) a^2 + \left[g \alpha_t a_x - \frac{\mu_0 K^2}{\rho_0(1+\chi)} [(a_x \bar{T}_z - a_z \bar{T}_x)] \right] \times (1 + \chi) a^2 (a_z \bar{T}_x - a_x \bar{T}_z) + \frac{\mu_0}{\rho_0} K^2 a_x a_z [(\bar{T}_z - \bar{T}_x)(a_z \bar{T}_x - a_x \bar{T}_z)] (a_x + a_z) = 0 \quad (21)$$

$$\text{where } X = \delta^* M_3, \delta^* = \delta \mu_0 H_0 (1 + \chi) M_3 = \frac{(1 + \frac{M_0}{H_0})}{(1 + \chi)}$$

It is importance to indicate that in the disappear of the porous system, equation (8) reduces to the characteristic equation by Vaidyanathan and in the disappears of magnetic field, it reduces to the characteristic equation as given by Holyer [22].

4. Stationary instability

To evaluate the rules for the inception of unchanged instability, the increase rate must be equal to zero. Applied $\sigma = 0$ in equation (22), we obtained the critical Rayleigh number R_{sc} is :

$$R_{sc} = \frac{A^2(1 + \delta^* M_3) \left(1 + \frac{1}{\frac{k_2 a}{a_z}} \right) + \left(1 + \frac{\varepsilon}{\frac{k_2 a}{a_z}} \right)}{A(A-G) \left[M_1 \left(1 + \frac{(G-1)(A+1)}{A^2+1} - \frac{G}{A} \right) - 1 \right]} \quad (22)$$

where $A = \frac{a_x}{a_z}$, $G = \frac{\bar{T}_x}{\bar{T}_z}$, $\varepsilon = \frac{k_2^*}{k_1^*}$, $a' = ad$, $k_2^* = \frac{k_2}{a^2}$, $M_1 = \frac{\mu_0 K^2 \bar{T}_z}{\rho_0(1+\chi)g\alpha_1}$ and $R_{sc} = \frac{g\alpha_1 \bar{T}_z}{v k_1 a^4}$. Here k_1^* and k_2^* are the permeability of the porous medium and is the anisotropic parameter. The magnetic thermal Rayleigh number is given by:

$$N_c = R_{sc} M_1 = \frac{A^2(1 + \delta^* M_3) \left(1 + \frac{1}{\frac{k_2 a}{a_z}} \right) + \left(1 + \frac{\varepsilon}{\frac{k_2 a}{a_z}} \right)}{A(A-G) \left(1 + \frac{(G-1)(A+1)}{A^2+1} - \frac{G}{A} \right)} \quad (23)$$

5. Oscillatory instability

For Oscillatory instability, put $\sigma = i \sigma_1$. Then, the characteristic equation (22) becomes:

$$- \left[-a^2 \sigma_1^2 \left\{ v_1 a^4 + v_1 X a^4 + \left(\frac{v_1}{k_2} a_x^2 + \frac{v_1}{k_1} a_z^2 \right) \right\} \right] \{ (1 + X) + k_1 a^4 \} + K_1 a^2 \left(v_1 a^4 + \left(\frac{v_1}{k_2} a_x^2 + \frac{v_1}{k_1} a_z^2 \right) (1 + X) \right) + \left[g \alpha_1 a_x - \frac{\mu_0 K^2}{\rho_0(1+\chi)} (a_x \bar{T}_z - a_z \bar{T}_x) \right] + \chi a^2 (a_z \bar{T}_x - a_x \bar{T}_z) + \frac{\mu_0}{\rho_0} K^2 a_x a_z [(\bar{T}_z - \bar{T}_x)(a_z \bar{T}_x - a_x \bar{T}_z)] (a_x + a_z) = 0 \quad (24)$$

By equating the imaginary part of equation (24), we get:

$$\sigma_1 \left[v_1 a^4 + v_1 X a^4 + \left(\frac{v_1}{k_2} a_x^2 + \frac{v_1}{k_1} a_z^2 \right) (1 + X) + k_1 a^4 \right] = 0 \quad (25)$$

It is found that $\sigma_1 = 0$, since

$$\left[v_1 a^4 + v_1 X a^4 + \left(\frac{v_1}{k_2} a_x^2 + \frac{v_1}{k_1} a_z^2 \right) (1 + X) + k_1 a^4 \right] \neq 0 \quad (26)$$

Hence the not possible of the event of the oscillatory instability is proved.

Differentiating the equation (6) with respect to δ^* , ε , M_3 , $k_2^* G$, A and we get:

$$\frac{dN_c}{d\delta^*} = \frac{A^2 M_3 \left(1 + \frac{1}{\frac{k_2 a}{a_z}} \right)}{A(A-G) \left(1 + \frac{(G-1)(A+1)}{A^2+1} - \frac{G}{A} \right)} > 0 \quad (27)$$

$$\frac{dN_c}{d\varepsilon} = \frac{\left(\frac{1}{\frac{k_2 a}{a_z}} \right)}{A(A-G) \left(1 + \frac{(G-1)(A+1)}{A^2+1} - \frac{G}{A} \right)} > 0 \quad (28)$$

$$\frac{dN_c}{dM_3} = \frac{A^2 \delta^* \left(1 + \frac{1}{\frac{k_2 a}{a_z}} \right)}{A(A-G) \left(1 + \frac{(G-1)(A+1)}{A^2+1} - \frac{G}{A} \right)} > 0 \quad (29)$$

$$\frac{dN_c}{dk_2^*} = \frac{\left(-\frac{1}{\frac{k_2 a}{a_z}} \right) [1 + A^2(1 + \delta^* M_3)]}{A(A-G) \left(1 + \frac{(G-1)(A+1)}{A^2+1} - \frac{G}{A} \right)} < 0$$

$$\frac{dN_c}{dG} < 0 \text{ and } \frac{dN_c}{dA} > 0 \quad (30)$$

This shows the stabilizing nature of δ^* , ε , M_3 , A and destabilizing nature of k_2^* and G .

6. Result and discussion

The reliant consistency attractive field impact on the Ferro convection exposed to temperature change and attractive zone in an anisotropic permeable framework has been considered, utilizing direct speculation hypothesis. The dependability examination is along a vertical and flat directions as it were. The attractive warm Rayleigh number N_c has been assessed for a few proportions of temperature inclinations and wave numbers, anisotropic parameters ε , the porousness of the permeable framework k and the attractive ward consistency coefficient δ . As far as possible presence estimations of G , A , k , ε and δ are accepted to decide the method of unsteadiness. G should take esteems from - 1 through 0 to +1 and the estimations of an are allowed fluctuate from - 4 to +4. As per examine the impact of inversion temperature change t_s M_1 is taken to be 1000. The anisotropic parameters has been given qualities running from 0.3 to 0.9 and the porousness of the permeable framework k is allowed to shift from 0.1 to 0.7 and the estimations of coefficient of attractive ward thickness are accepted from 0.001 to 0.007. The framework is found to balance out through stationary method of unsteadiness as it were. It is additionally observed that oscillatory unsteadiness can't showed up.

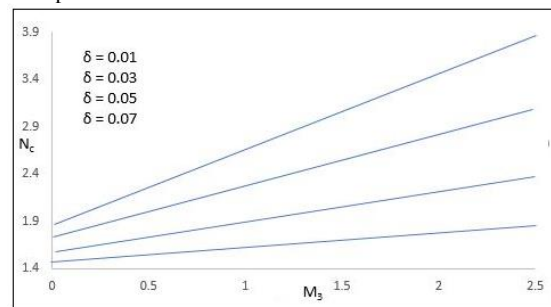


Fig. 1: Variation of the attractive warm Rayleigh number N_c ver-sus M_3 for various estimations of δ .

It is see from Fig.1 that as the estimations of M_3 increments from 5 to 25, the estimation of N_c tend to expand causing adjustment. It is likewise seen that the coefficient of attractive field subordinate thickness δ is expanded, the attractive warm Rayleigh number N_c additionally increas-es, this would suggest that attractive field subordinate consistency stabilizes through thickness variety as for the attractive field.

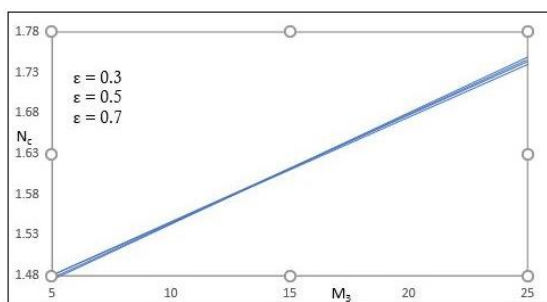


Fig. 2: Variation of N_c versus M_3 for various estimations of ε

Figure 2: Shows the variety of N_c versus M_3 for various estimations of the anisotropic parameter ε . As the estimations of M_3 tend to increment from 5 to 25, N_c esteems are found to increment and the diagram displays an adjustment. Higher estimations of M_3 balance out the framework more.

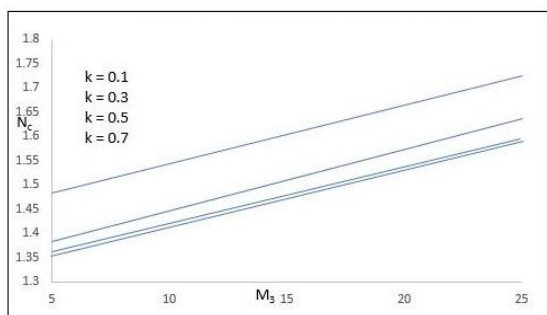


Fig. 3: The variation of N_c versus M_3 for various estimations of k .

In figurev3, it demonstrates that as the estimation of k increments from 0.1 to 0.7 the estimation of N_c has a tendency to diminishing causing a destabilization. This is on the grounds that in pore measure makes the stream of the liquid less demanding causing in-steadiness to set in before.

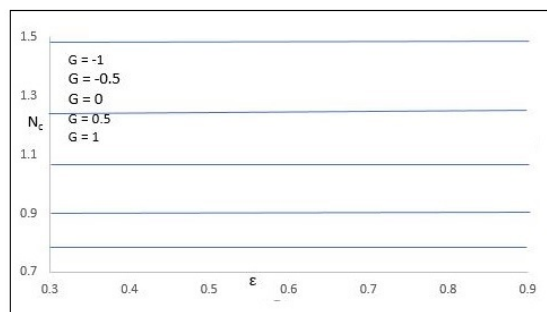


Fig. 4: The variation of N_c versus ε for various estimations of G

In figure 4 it demonstrates the variety of N_c versus ε for various estimations of G . As ε increments from 0.3 to 0.9, there is balancing out however it is just least qualities.

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