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*Research paper*

# On Some new Results of a Subclass of Meromorphic Univalent Functions with Negative Coefficients Defined by liu – Srivastava Linear Operator

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## Abstract

In this paper, we introduce and study a new subclass of meromorphic univalent functions with negative coefficients defined by Liu – Srivastava linear operator in the $U^* = \{z: z \in \mathbb{C} \text{ and } 0 < z < 1\} = U/\{0\}$. We obtain some properties like, coefficients inequalities, growth and distortion theorems, closure theorems, partial sums and convolution properties.

***Keywords:*** *Meromorphic univalent functions, coefficients inequalities, growth and distortion, partial sums, convolution properties, Liu – Srivastava linear operator.*

***Mathematics subject classification:*** 30

## 1. Introduction

Let $B$ denoted the class of meromorphic univalent functions of the form :

$$f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} a_k z^k \qquad (a_k \geq 0). \qquad (1)$$

Which are analytic in the punctured unite disc $U^* = \{z: z \in \mathbb{C} \text{ and } 0 < z < 1\} = U/\{0\}$.
Let $B^*$ be the subclass of the class $B$ containing of functions of the form:

$$f(z) = \frac{1}{z} - \sum_{k=1}^{\infty} a_k z^k \qquad (a_k \geq 0). \qquad (2)$$

Let $g(z) \in B^*$ be given by:

$$g(z) = \frac{1}{z} - \sum_{k=1}^{\infty} b_k z^k , \qquad (3)$$

then the Hadmamard product (or convolution ) of two power series

$$f(z) = \frac{1}{z} - \sum_{k=1}^{\infty} a_k z^k , \quad g(z) = \frac{1}{z} - \sum_{k=1}^{\infty} b_k z^k , \qquad (4)$$

in $B^*$ is defined by:

$$(f * g)(z) = f(z) * g(z) = \frac{1}{z} - \sum_{k=1}^{\infty} a_k b_k z^k . \qquad (5)$$

A function $f \in B^*$ is meromorphicstar like function of order $\mu$ $(0 \leq \mu < 1)$ if

$$Re\left\{1 + \frac{zf'(z)}{f(z)}\right\} > \mu \qquad (z \in U). \qquad (6)$$

A function $f \in B^*$ is meromorphic convex function of order $\mu$ $(0 \leq \mu < 1)$ if

$$Re\left\{1 + \frac{zf''(z)}{f'^{(z)}}\right\} > \mu \qquad (z \in U). \qquad (7)$$

Some other classes by W.G.Atshan and Kukarni[2] , N.E.Chen, H. Rmark, H.M.Srivastava and C.S.Yn [3], M.K.Aouf [1], M.Darns[6], Miller[8], Mogra[9] eta…
For complex parameters $\alpha_1, \alpha_2, \dots, \alpha_\tau$ and $\beta_1, \beta_2, \dots, \beta_m (\beta \neq 0, -1, \dots; j = 1,2, \dots, m)$,
the generalized hypergeometric function $\tau F_m(z)$ , is defined by :

$$\tau F_m(z) = \tau F_m(\alpha_1, \alpha_2, \dots, \alpha_\tau ;\ \beta_1, \beta_2, \dots, \beta_m; z) = \sum_{0}^{\infty} \frac{(\alpha_1)_n \dots (\alpha_\tau)_n}{(\beta_1)_n \dots (\beta_m)_n} \frac{z^n}{n!} , \qquad (8)$$

$$(\tau \leq m + 1;\ \tau, m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\};\ z \in U),$$

where $(\theta)_n$ is the pochhammer symbol defined by :

$$(\theta)_n = \frac{\Gamma(\theta + n)}{\Gamma(\theta)} = \begin{cases} 1 , & n = 0 ;\ \theta \in \mathbb{C} / \{0\} \\ \theta(\theta+1)(\theta+2)\dots(\theta+n-1) , & n \in \mathbb{N} ;\ \theta \in \mathbb{C} \end{cases} . \qquad (9)$$

Corresponding to function $\tau F_m(\alpha_1,\alpha_2,\dots,\alpha_\tau\,;\,\beta_1,\beta_2,\dots,\beta_m;z)$, defined by:

$$Q(\alpha_1,\alpha_2,\dots,\alpha_\tau\,;\,\beta_1,\beta_2,\dots,\beta_m;z) = z^{-1}\tau F_m(\alpha_1,\alpha_2,\dots,\alpha_\tau\,;\,\beta_1,\beta_2,\dots,\beta_m;z). \tag{10}$$

Liu and Srivastava [10] consideral a linear operator

$$H(\alpha_1,\alpha_2,\dots,\alpha_\tau\,;\,\beta_1,\beta_2,\dots,\beta_m): B^* \to B^*,$$

$$H(\alpha_1,\alpha_2,\dots,\alpha_\tau\,;\,\beta_1,\beta_2,\dots,\beta_m)f(z) = H(\alpha_1,\alpha_2,\dots,\alpha_\tau\,;\,\beta_1,\beta_2,\dots,\beta_m;z) * f(z)$$

$$= z^{-1} - \sum_{k=1}^{\infty} \left| \frac{(\alpha_1)_{n+1}\dots(\alpha_\tau)_{n+1}}{(\beta_1)_{n+1}\dots(\beta_m)_{n+1}} \right| \frac{z^k}{(n+1)!}, \tag{11}$$

where, $\alpha_i > 0, (i = 1,\dots,\tau),; \beta_j > 0, (j = 1,\dots,m), \tau \le m + 1\,; m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

For notional simplicity, we use a shorter notations $H[\alpha_1]$ for

$$\Gamma_n(\alpha_1) = \sum_{n=1}^{\infty} \left| \frac{(\alpha_1)_{n+1}\dots(\alpha_\tau)_{n+1}}{(\beta_1)_{n+1}\dots(\beta_m)_{n+1}} \right| \frac{1}{(n+1)!}, \tag{12}$$

unless otherwise stated in the squel. We note that the linear operator $H_m^\tau[\alpha_1]$ was earlier defined for multivalent functions by Dziok and Srivastava [5] and was investigated by Liu and Srivastava [7]. Motivated by Ravichandaranetal. [11] and Atshan and Kularni[2] making use of the operator $H[\alpha_1]$, now defined a new subclass $B^*(v,c,\delta)$ of $B^*$.

**Definition 1:**for$0 \le v < 1, \frac{1}{2} \le c \le 1, 0 < \delta \le 1$, we let $B^*(v,c,\delta)$ by the subclass of $B^*$ consisting of functions of the form (2) and satisfying the condition :

$$\left| 1 - \frac{z^3[H[\alpha_1]f(z)]''}{(2vc-1) + z^2[H[\alpha_1]f(z)]'} \right| < \delta. \tag{13}$$

In the following theorem, we obtain necessary and sufficient condition for a function $f$ to be in the subclass $B^*(v,c,\delta)$.

**Theorem 1:** Let $f \in B^*$ be given by (2). Then $f \in B^*(v,c,\delta)$ if and only if

$$k(k+\delta)\sum_{k=1}^{\infty} \Gamma_n(\alpha_1) a_k z^{k+1} \le 2[2 + vc(\delta - 1)], \tag{14}$$

where $0 \le v < 1, \frac{1}{2} \le c \le 1, 0 < \delta \le 1, n \in \mathbb{N}_0, z \in U^*$.

The result (14) is sharp for the function

$$f(z) = \frac{1}{z} - \frac{2[2 + vc(\delta - 1)]}{k(k+\delta)\Gamma_n(\alpha_1)} z^{k+1}, (k = 1,2,\dots).$$

**Proof :**Assume that the inequality (14) hold true and $|z| = 1$. Then, we have

$$\left| 1 - \frac{z^3[H[\alpha_1]f(z)]''}{(2vc-1) + z^2[H[\alpha_1]f(z)]'} \right| < \delta$$

$$= \left|(2vc-1) + z^2[H[\alpha_1]f(z)]' - z^3[H[\alpha_1]f(z)]''\right| - \delta\left|(2vc-1) + z^2[H[\alpha_1]f(z)]'\right|$$

$$\le k(k+\delta)\sum_{k=1}^{\infty} \Gamma_n(\alpha_1)\, a_k z^{k+1} - \left[2\left(2 + vc(\delta - 1)\right)\right] \le 0,$$

by hypothesis, hence, by the principle of maximum modulus, $f \in B^*(v,c,\delta)$.

Conversely, suppose that $f$ defined by (2) is in the class $B^*(v,c,\delta)$ .

Hence

$$\left| \frac{(2vc-1) + z^2[H[\alpha_1]f(z)]' - z^3[H[\alpha_1]f(z)]''}{(2vc-1) + z^2[H[\alpha_1]f(z)]'} \right| < \delta, \quad z \in U$$

we get

$$\left| \frac{(2vc-2) + \sum_{k=1}^{\infty} k^2\Gamma_n(\alpha_1) a_k z^{k+1}}{2vc - \sum_{k=1}^{\infty} k\Gamma_n(\alpha_1) a_k z^{k+1}} \right| < \delta.$$

Since $Re(z) < |z| forall\ z$, we have

$$Re\left\{ \frac{(2vc-2) + \sum_{k=1}^{\infty} k^2\Gamma_n(\alpha_1) a_k z^{k+1}}{2vc - \sum_{k=1}^{\infty} k\Gamma_n(\alpha_1) a_k z^{k+1}} \right\} < \delta. \tag{15}$$

We can choose the value of $z$ on real axis, so that $z^3[H[\alpha_1]f(z)]''$is real .

Let $z \to 1-$, through real values, so we can write (15) as

$$k(k+\delta)\sum_{k=1}^{\infty} \Gamma_n(\alpha_1) a_k z^{k+1} \le 2[2 + vc(\delta - 1)],$$

This completes the proof of theorem (1).

**Corollary 1:**Let the function $f(z)$ defined by (2) be in the class $B^*(v,c,\delta)$. Then

$$a_k \le \frac{2[2 + vc(\delta - 1)]}{k(k+\delta)\Gamma_n(\alpha_1)} (k \ge 1),$$

with equality for function

$$f(z) = \frac{1}{z} - \frac{2[2 + vc(\delta - 1)]}{k(k+\delta)\Gamma_n(\alpha_1)} z^k. \tag{16}$$

In the next theorems, we prove the growth and distortion theorems for the function $f(z) \in B^*(v,c,\delta)$.

**Theorem 2:**Let the function $f(z) \in B^*(v,c,\delta)$, then for $0 < |z| = r < 1$, we have

$$\frac{1}{r} - \frac{2[2 + vc(\delta - 1)]}{(1+\delta)} r \le |f(z)| \le \frac{1}{r} + \frac{2[2 + vc(\delta - 1)]}{(1+\delta)} r, \tag{17}$$

for the function :

$$f(z) = \frac{1}{z} - \frac{2[2 + vc(\delta - 1)]}{(1+\delta)} z. \tag{18}$$

**Proof:**It is easy to see from Theorem 1 , that

$$(1+\delta)\sum_{k=1}^{\infty} a_k \le \sum_{k=1}^{\infty} k(k+\delta) a_k \le 2[2 + vc(\delta - 1)].$$

Then

$$\sum_{k=1}^{\infty} a_k \le \frac{2[2 + vc(\delta - 1)]}{(1+\delta)}. \tag{19}$$

Making use of (19), we have

$$|f(z)| \ge \frac{1}{|z|} - |z|\sum_{k=1}^{\infty} a_k \ge \frac{1}{r} - \frac{2[2 + vc(\delta - 1)]}{(1+\delta)} r, \tag{20}$$

and

$$|f(z)| \le \frac{1}{|z|} + |z|\sum_{k=1}^{\infty} a_k \ge \frac{1}{r} + \frac{2[2 + vc(\delta - 1)]}{(1+\delta)} r, \tag{21}$$

Which proves the assertion (17). The proof is completed.

**Theorem 3:**Let the function $f(z) \in B^*(v,c,\delta)$, then for $0 < |z| = r < 1$, we have

$$\frac{1}{r^2}-\frac{2[2+vc(\delta-1)]}{(1+\delta)}\le|f'(z)|\le\frac{1}{r^2}+\frac{2[2+vc(\delta-1)]}{(1+\delta)}\quad, \qquad (22)$$

With equality for the function $f(z)$ given by (18).

**Proof:** From Theorem 1 and (18), we have

$$\sum_{k=1}^{\infty}ka_k\le\frac{2[2+vc(\delta-1)]}{(1+\delta)}\,. \qquad (23)$$

The remaining part of the proof is similar of Theorem 2.

In the next theorems, we obtain the closure theorems.

Let the functions$f_j(j=1,2,\dots,m)$ be defined by :

$$f_j(z)=\frac{1}{z}-\sum_{k=1}^{\infty}a_{k,j}z^k,\qquad (a_{k,j}\ge 0,k\ge 1)\,. \qquad (24)$$

**Theorem 4:** Let the functions$f_j(z)$ defined by (240 be in the class $B^*(v,c,\delta) for\ every\ j=1,2,\dots,m$. Then the function $h(z)$ defined by :

$$h(z)=\frac{1}{z}-\sum_{k=1}^{\infty}c_kz^k\,,\qquad (c_k\ge 0,k\in\mathbb{N},k\ge 1)$$

Also belongs to the class $B^*(v,c,\delta)$, where

$$c_k=\frac{1}{m}\sum_{j=1}^{m}a_{k,j}.$$

**Proof :** Since $f_j(z)\in B^*(v,c,\delta)$, therefore from Theorem , we obtain

$$\sum_{k=1}^{\infty}k(k+\delta)\Gamma_n(\alpha_1)a_{k,j}z^{k+1}\le 2[2+vc(\delta-1)], \qquad (25)$$

Then

$$\sum_{k=1}^{\infty}k(k+\delta)\Gamma_n(\alpha_1)c_kz^{k+1}=\sum_{k=1}^{\infty}k(k+\delta)\Gamma_n(\alpha_1)[\frac{1}{m}\sum_{j=1}^{m}a_{k,j}]\,z^{k+1}\le 2[2+vc(\delta-1)].$$

Hence $h(z)\in B^*(v,c,\delta)$. This complete the proof.

**Theorem 5:** Let the functions$f_j(z)$ defined by (24) be in the class $B^*(v,c,\delta)$, for every $j=1,2,\dots,m$. Then the function $h(z)$ defined by:

$$h(z)=\sum_{j=1}^{m}d_jf_j\,(z)\quad and\quad \sum_{j=1}^{m}d_j=1,d_j\ge 0.$$

Is also in the class $B^*(v,c,\delta)$.

**Proof:**By definition of $h(z)$, we have

$$h(z)=\sum_{j=1}^{m}d_j\frac{1}{z}-\sum_{k=1}^{\infty}[\sum_{j=1}^{m}d_j\,a_{k,j}]z^{k+1},$$

since $f_j(z)$ are in the class $B^*(v,c,\delta)$,

then

$$\sum_{k=1}^{\infty}k(k+\delta)\Gamma_n(\alpha_1)a_{k,j}z^{k+1}\le 2[2+vc(\delta-1)],$$

for every $j=1,2,\dots,m$. Hence we can see that

$$\sum_{k=1}^{\infty}k(k+\delta)\Gamma_n(\alpha_1)[\sum_{j=1}^{m}d_j\,a_{k,j}]z^{k+1}=\sum_{j=1}^{m}d_j\,[\sum_{k=1}^{\infty}k(k+\delta)\Gamma_n(\alpha_1)a_{k,j}]z^{k+1}\le 2[2+vc(\delta-1)]\sum_{j=1}^{m}d_j=2[2+vc(\delta-1)].$$

Thus $h(z)\in B^*(v,c,\delta)$. This complete the proof.

In the following theorem, we obtain the partial sums for the class$B^*(v,c,\delta)$.

Some of these property partial sums was studies by Liu and Song [6].

**Theorem 6:**Let $f_1(z)=\frac{1}{z}\quad and\ \ f_\iota(z)=\frac{1}{z}-\sum_{k=1}^{\iota-1}a_kz^k$, suppose also that $\sum_{k=1}^{\infty}d_ka_k\le 1$,

$$(d_k=\frac{k(k+\delta)\Gamma_n(\alpha_1)}{2[2+vc(\delta-1)]}). \qquad (26)$$

Then, we have

$$Re\left\{\frac{f(z)}{f_\iota(z)}\right\}>1-\frac{1}{d_\iota}, \qquad (27)$$

and

$$Re\left\{\frac{f(z)}{f_\iota(z)}\right\}>\frac{d_\iota}{1+d_\iota}. \qquad (28)$$

Each of the bounds in (27) and (28) is best possible for $k\in\mathbb{N}$.

**Proof:** For the coefficients $d_k$ given by (26), it is not difficult to verify that

$$d_{k+1}>d_k>1,\quad k=1,2,\dots.$$

Therefore by using hypothesis (26), we have

$$\sum_{k=1}^{\iota-1}a_k+d_\iota\sum_{k=1}^{\infty}a_k\le\sum_{k=1}^{\infty}d_\iota a_k\le 1. \qquad (29)$$

By setting

$$g_1(z)=d_\iota(\frac{f(z)}{f_\iota(z)}-\left(1-\frac{1}{d_\iota}\right))=1-\frac{1-d_\iota\sum_{k=1}^{\infty}a_kz^{k+1}}{1-\sum_{k=1}^{\infty}a_kz^{k+1}}, \qquad (30)$$

and applying (29), we find that

$$\left|\frac{g_\iota(z)-1}{g_1(z)+1}\right|\le\frac{d_\iota\sum_{k=\iota}^{\infty}a_k}{2-2\sum_{k=1}^{\iota-1}a_k-d_\iota\sum_{k=\iota}^{\infty}a_k}\le 1, \qquad (31)$$

which readily yields the assertion (27). If we take

$$f(z)=\frac{1}{z}-\frac{z^\iota}{d_\iota}, \qquad (32)$$

than

$$\frac{f(z)}{f_\iota(z)}=1-\frac{z^\iota}{d_\iota}\to 1-\frac{1}{d_\iota}(z\to 1-),$$

which shows that the bound in (27), is the best possible for each $k\in\mathbb{N}$.

Similarly, if we put

$$g_2(z)=(1+d_\iota)\left[\frac{f_\iota(z)}{f(z)}-\frac{d_\iota}{1+d_\iota}\right]=1+\frac{(1+d_\iota)(\sum_{k=\iota}^{\infty}a_k\,z^{k+1})}{1-\sum_{k=\iota}^{\infty}a_k\,z^{k+1}}, \qquad (33)$$

and make use of (29), we have

$$\left|\frac{g_2(z)-1}{g_2(z)+1}\right|\le\frac{(1+d_\iota)(\sum_{k=\iota}^{\infty}a_k)}{2-2\sum_{k=1}^{\infty}a_k+(1+d_\iota)(\sum_{k=\iota}^{\infty}a_k)}, \qquad (34)$$

which leads us to the assertion (28). The bound in (28)is sharp for each $k\in\mathbb{N}$, with function given by (32). The proof of the theorem is complete.

In the next theorems, we introduce the convolution properties of the functions belong to the class$B^*(v,c,\delta)$.

**Theorem 7:** Let $f(z)\ and\ g(z)\ \in B^*(v,c,\delta)$. Then $f(z)*g(z)\in B^*(\varphi,c,\delta)$ for

$$f(z)=\frac{1}{z}-\sum_{k=1}^{\infty}a_kz^k\quad,\quad g(z)=\frac{1}{z}-\sum_{k=1}^{\infty}b_kz^k,$$

where

$$\varphi\le\frac{2}{kc(k+\delta)(\delta-1)}.$$

**Proof:**Since $f(z),g(z)\in B^*(v,c,\delta)$, then we have

$$\sum_{k=1}^{\infty}\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_n(\alpha_1)}a_k\le 1\,, \qquad (35)$$

and

$$\sum_{k=1}^{\infty}\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_n(\alpha_1)}b_k\le 1. \qquad (36)$$

By the Cauchy Schwarz inequality, we have

$$\sum_{k=1}^{\infty}\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}\sqrt{a_k b_k}\leq 1. \qquad (37)$$

Thus it is sufficient to show that

$$\frac{2[2+\varphi c(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}a_k b_k\leq\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}\sqrt{a_k b_k}, \qquad (38)$$

or equivalently, that

$$\sqrt{a_k b_k}\leq\frac{2+\varphi c(\delta-1)}{2+vc(\delta-1)}. \qquad (39)$$

Connecting (38), it is sufficient to prove that

$$\frac{k(k+\delta)}{2[2+vc(\delta-1)]}\leq\frac{2+\varphi c(\delta-1)}{2+vc(\delta-1)}. \qquad (40)$$

It follows from (40), that

$$\varphi\leq\frac{2}{kc(k+\delta)(\delta-1)}. \qquad (41)$$

Which evidently completes the proof of Theorem (7).

**Theorem 8:** Let $f(z) and\ g(z)\in B^*(v,c,\delta)$. Then

$$h(z)=\frac{1}{z}-\sum_{k=1}^{\infty}({a_k}^2+{b_k}^2)z^k,$$

belong to the class $B^*(\rho,c,\delta)$,

where

$$\rho\geq\frac{k(k+1)-4[2[2+vc(\delta-1)]\Gamma_{\mathrm{n}}(\alpha_1)]^2}{[2[2+vc(\delta-1)]\Gamma_{\mathrm{n}}(\alpha_1)]^2 2c(\delta-1)}. \qquad (42)$$

**Proof:** Since $f(z), g(z)\in B^*(v,c,\delta)$, so by Theorem 1, yields

$$\sum_{k=1}^{\infty}\left[\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}a_k\right]^2\leq 1,$$

and

$$\sum_{k=1}^{\infty}\left[\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}b_k\right]^2\leq 1,$$

we obtain, from the last two inequalities

$$\sum_{k=1}^{\infty}\frac{1}{2}\left[\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}\right]^2\left({a_k}^2+{b_k}^2\right)\leq 1, \qquad (43)$$

but $h(z)\in B^*(\rho,c,\delta)$, if and only if

$$\frac{2[2+\rho c(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}\leq\left[\frac{2[2+vc(\delta-1)]}{k(k+\delta)\Gamma_{\mathrm{n}}(\alpha_1)}\right]^2.$$

Simplifying, we set

$$\rho\geq\frac{k(k+1)-4[2[2+vc(\delta-1)]\Gamma_{\mathrm{n}}(\alpha_1)]^2}{[2[2+vc(\delta-1)]\Gamma_{\mathrm{n}}(\alpha_1)]^2 2c(\delta-1)}.$$

This complete the proof.

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