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Research paper

Solving Fuzzy Game Problem in Octagonal NEUTROSOPHIC Numbers Using Heavy OWA Operator

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Abstract

In this paper, we define octagonal neutrosophic number (ONN) and operations on it. Also, we define α, β andγ-cut sets of ONN. We use octagonal neutrosophic numbers as pay offs of two- person zero sum game and propose a new method to find the best strategies for the two players using heavy ordered weighted averaging (HOWA) operator on these numbers. The applicability of this method is explained through a numerical example

Keywords: *Octagonal Neurosophic numbers, fuzzy game matrix, ordered weighted averaging (OWA) operator.*

1. Introduction

This Neutrosophic logic has been proposed by Florentine Smaradache[5] which is based on non-standard analysis that was given by Abraham Robinson in 1960. In neutrosophic set indeterminacy is quantified explicity whereas the truth membership, indeterminacy membership and falsity membership are independent. Neutrosophic sets have been introduced as a generalization of crisp sets, fuzzy sets & Intuitionistic fuzzy sets to represent uncertain, inconsistent & incomplete information about a real world problem. In 1998, Smarandache[6] introduced neutrosophic sets as an extension of classical sets, fuzzy sets & intuitionistic fuzzy sets. The components of neutrosophic set, namely truth membership degree, indeterminacy-membership degree & falsity-membership degree were suitable to represent indeterminacy and inconsistent information. Wang et al introduced the idea of single valued neutrosophic set in many practical problems.

In recent years, many interesting applications of game theory have been expanded by embedding the ideas of fuzzy sets.

2. Priliminaries

2.1 Fuzzy Neutrosophic Set

A fuzzy neutrosophic set A on the universe of a discourse X is defined as

A={x, $w_A(x)$, $u_A(x)$, $y_A(x)$: $x \in X$} where w,u,y: X→[0,1] and $0 \leq w_A(x) + u_A(x) + y_A(x) \leq 3$ where $w_A(x)$ is membership, $u_A(x)$ is inderministic function and $y_A(x)$ is non-deterministic function.

3. Octagonal Neutrosophic Number (ONN)

A Octagonal neutrosophic number $\tilde{a}$ =($a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$); $w_{\tilde{a}}$, $u_{\tilde{a}}, y_{\tilde{a}}$) is a special neutrosophic set on the set of real numbers ℝ, whose truth-membership, indeterminacy-membership and falsity-membership functions are respectively defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) & for a_1 \leq x \leq a_2 \\ k & for a_2 \leq x \leq a_3 \\ k + (w_{\tilde{a}} - k)\left(\frac{x-a_3}{a_4-a_3}\right) & for a_3 \leq x \leq a_4 \\ w_{\tilde{a}} & for a_4 \leq x \leq a_5 \\ k + (w_{\tilde{a}} - k)\left(\frac{a_6-x}{a_6-a_5}\right) & for a_5 \leq x \leq a_6 \\ k & for a_6 \leq x \leq a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) & for a_7 \leq x \leq a_8 \\ 0 & for x > a_8 \end{cases}$$

$$\vartheta_{\tilde{A}}(x) = \begin{cases} 1 & for x < a_1 \\ 1 + (1-k)\left(\frac{a_1{}' - x}{a_2{}' - a_1}\right) & for a_1 \leq x \leq a_2 \\ k & for a_2 \leq x \leq a_3 \\ k + (k - u_{\tilde{a}})\left(\frac{a_3{}' - x}{a_4{}' - a_3{}'}\right) & for a_3 \leq x \leq a_4 \\ 0 & for a_4 \leq x \leq a_5 \\ u_{\tilde{a}} + (k - u_{\tilde{a}})\left(\frac{x - a_5{}'}{a_6{}' - a_5}\right) & for a_5 \leq x \leq a_6 \\ k & for a_6 \leq x \leq a_7 \\ k + (1-k)\left(\frac{x - a_7{}'}{a_8{}' - a_7{}'}\right) & for a_7 \leq x \leq a_8 \\ 1 & for x > a_8 \end{cases}$$

$$\lambda_{\tilde{A}}(x) = \begin{cases} 1 & for x < a_1 \\ 1 + (1-k)\left(\frac{a_1{}' - x}{a_2{}' - a_1}\right) & for a_1 \leq x \leq a_2 \\ k & for a_2 \leq x \leq a_3 \\ k + (k - u_{\tilde{a}})\left(\frac{a_3{}' - x}{a_4{}' - a_3{}'}\right) & for a_3 \leq x \leq a_4 \\ 0 & for a_4 \leq x \leq a_5 \\ u_{\tilde{a}} + (k - u_{\tilde{a}})\left(\frac{x - a_5{}'}{a_6{}' - a_5}\right) & for a_5 \leq x \leq a_6 \\ k & for a_6 \leq x \leq a_7 \\ k + (1-k)\left(\frac{x - a_7{}'}{a_8{}' - a_7{}'}\right) & for a_7 \leq x \leq a_8 \\ 1 & for x > a_8 \end{cases}$$

where $0 < k < 1$ respectively.

3.1 α-Cut Set of ONN

The classical set $\tilde{A}^I{}_{\alpha}$ called alpha cut set is the set of elements whose degree of membership is the set of elements whose degree of membership in $\tilde{A}^I{}_o$=([a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]; $w_{\tilde{a}}$, $u_{\tilde{a}}$, $y_{\tilde{a}}$) is defined as

$$\tilde{A}^I{}_{\alpha} = \{x \in X \mid \mu_{\tilde{A}^I{}_o}(x) \geq \alpha\} = \begin{cases} l_1(\alpha), l_2(\alpha) \; for \; \alpha \in [0,k] \\ s_1(\alpha), s_2(\alpha) \; for \; \alpha \in [k, w_{\tilde{a}}] \end{cases}$$

Where $l_1(\alpha) = a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1)$, $l_2(\alpha) = a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)$

$$s_1(\alpha) = a_3 + \left(\frac{\alpha - k}{w_{\tilde{a}} - k}\right)(a_4 - a_3), \quad s_2(\alpha) = a_6 - \left(\frac{\alpha - k}{w_{\tilde{a}} - k}\right)(a_6 - a_5)$$

3.2 β-Cut Set of ONN

The classical set $\tilde{A}^I{}_{\beta}$ is the β-cut set of elements whose degree of non -membership is the set of elements whose degree of non-membership in $\tilde{A}^I{}_o$=([a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]; $w_{\tilde{a}}$, $u_{\tilde{a}}$, $y_{\tilde{a}}$] is defined as

$$\tilde{A}^I{}_{\beta} = \{x \in X \mid \upsilon_{\tilde{A}^I{}_o}(x) \geq \beta\} = \begin{cases} l'_1(\beta), l'_2(\beta) \; for \; \beta \in [k, 0] \\ s'_1(\beta), s'_2(\beta) \; for \; \beta \in [1, u_{\tilde{a}}] \end{cases}$$

Where $l'_1(\beta) = a_1 - \left(\frac{\beta - 1}{1 - k}\right)(a_2 - a_1)$, $l'_2(\beta) = a_7 + \left(\frac{\beta - k}{1 - k}\right)(a_8 - a_7)$,

$$s'_1(\beta) = a_3 - \left(\frac{\beta - k}{k - u_{\tilde{a}}}\right)(a_4 - a_3), \quad s'_2(\beta) = a_5 - \left(\frac{\beta - u_{\tilde{a}}}{k - u_{\tilde{a}}}\right)(a_6 - a_5)$$

3.3 γ-Cut Set of ONN

The classical set $\tilde{A}^I{}_{\gamma}$ is the γ-cut set of elements whose degree of non -membership is the set of elements whose degree of non-membership in $\tilde{A}^I{}_o$=([a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]; $w_{\tilde{a}}$, $u_{\tilde{a}}$, $y_{\tilde{a}}$] is defined as

$$\tilde{A}^I{}_{\gamma} = \{x \in X \mid \lambda_{\tilde{A}}(x) \geq \gamma\} = \begin{cases} l'_1(\gamma), l'_2(\gamma) \; for \; \gamma \in [k, 0] \\ s'_1(\gamma), s'_2(\gamma) \; for \; \gamma \in [1, u_{\tilde{a}}] \end{cases}$$

Where $l'_1(\gamma) = a_1 - \left(\frac{\gamma - 1}{1 - k}\right)(a_2 - a_1)$, $l'_2(\gamma) = a_7 + \left(\frac{\gamma - k}{1 - k}\right)(a_8 - a_7)$,

$$s'_1(\gamma) = a_3 - \left(\frac{\gamma - k}{k - u_{\tilde{a}}}\right)(a_4 - a_3), \quad s'_2(\gamma) = a_5 - \left(\frac{\gamma - u_{\tilde{a}}}{k - u_{\tilde{a}}}\right)(a_6 - a_5)$$

4. Ranking of Average Index of the Truth-Membership, Indeterminacy-Membership and Falsity-Membership for ONN

The average Index of the truth-membership $\mu_{\tilde{a}}(x)$, the indeterminacy-membership $\vartheta_{\tilde{a}}(x)$ and falsity-membership $\lambda_{\tilde{a}}(x)$ for ONN are defined as

$$R_{\mu}(\tilde{a}) = w_{\tilde{a}}\left[\frac{k(a_1 + a_2 + a_7 + a_8) + (1 - k)(a_3 + a_4 + a_5 + a_6)}{4}\right]$$

$$R_{\vartheta}(\tilde{a}) = (1 - u_{\tilde{a}})\left[\frac{k(a_1 + a_2 + a_7 + a_8) + (1 - k)(a_3 + a_4 + a_5 + a_6)}{4}\right]$$

$$R_{y}(\tilde{a}) = (1 - y_{\tilde{a}})\left[\frac{k(a_1 + a_2 + a_7 + a_8) + (1 - k)(a_3 + a_4 + a_5 + a_6)}{4}\right]$$

5. Heavy Owa Operators for Fuzzy Games With Payoff as ONN

Let us consider a matrix game with payoffs of ONN and two sets of pure strategies for players I and II respectively. The Payoff matrix for Player I is given by $\mathcal{M} = (\tilde{a}_{ij})_{mxn}$, where $\tilde{a}_{ij} = ([a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]; w_{\tilde{a}_i}, u_{\tilde{a}_i}, y_{\tilde{a}_i})$, (i=1, 2…m; j=1, 2…n) is a ONN.

The heavy OWA operators of ONN are used to solve the corresponding Game problem. Assume that the payoff matrix is characterized by a ONN $\tilde{a}_{ij} = ([a_{ij}^{(1)}, a_{ij}^{(2)}, a_{ij}^{(3)}, a_{ij}^{(4)}, a_{ij}^{(5)}, a_{ij}^{(6)}, a_{ij}^{(7)}, a_{ij}^{(8)}], w_{\tilde{a}_i}, u_{\tilde{a}_i}, y_{\tilde{a}_i})$

The Normalized decision matrix for player I and II is

$$\tilde{r}_{ij}^{(k)} = \frac{a_{ij}^{(k)} - \min a_{ij}^{(1)}}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}}, \text{ for j=1, 2...n}$$

$$\tilde{r}_{ij}^{(k)} = \frac{\max a_{ij}^{(8)} - a_{ij}^{(9-k)}}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}}, \text{ for i=1, 2…m}$$

where k=1, 2, 3, 4,5,6,7,8

6. Procedure for Heavy Owa Operators for Neutrosophic Fuzzy Games

Step (1) Compute the given payoff-matrix can be converted into Normalized payoff matrix according to the equation

Step (2) Use the Heavy OWA operator of NN to aggregate normalized decision matrix for player I. Suppose the weighting vector is W= (w_1, w_2… w_n), then the collective overall fuzzy number of player I is $H(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{j=1}^{n} w_j a_{ij}$

$$= \left(\left[\sum_{j=1}^{n} w_j a_{1j}, \sum_{j=1}^{n} w_j a_{2j}, \sum_{j=1}^{n} w_j a_{3j}, \ldots \sum_{j=1}^{n} w_j a_{nj}\right]; \min_{1 \leq i \leq n} w_{\tilde{a}_i}, \max_{1 \leq i \leq n} u_{\tilde{a}_i}, \max_{1 \leq i \leq n} y_{\tilde{a}_i}\right)$$

Similarly, the collective overall fuzzy number of player II is $H(\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_n) = \sum_{j=1}^{n} w_j a_{ij}$

$$= \left[\sum_{i=1}^{n} w_j a_{i1}, \sum_{j=1}^{n} w_j a_{i2}, \sum_{j=1}^{n} w_j a_{i3}, \ldots \sum_{j=1}^{n} w_j a_{in}\right]; \min_{1 \leq i \leq n} w_{\tilde{a}_i}, \max_{1 \leq i \leq n} u_{\tilde{a}_i}, \max_{1 \leq i \leq n} y_{\tilde{a}_i})$$

Step (3) For each strategy, calculate truth-membership, indeterminacy-membership and falsity-membership functions average indexes for its collective overall Fuzzy number $H(A_i)$ and $H(B_i)$, then the ranking orders of strategies are generated according to the ranking method .

7. Numerical Example

7.1. Game Problem with Payoff as Octagonal Neutrosophic Numbers

Suppose that two companies are competing for business whatever company A gains company B loses. The following problem shows advertising strategies of both companies and the utilities to company A for various market shares in percentage. The company A and B are considered as players A and B respectively.

Company B

Company A	Media	Radio	T.V
Media	([2,3,4,5,6,7,8,10]; 0.5,0.6,0.2)	([3,5,6,8,9,10,11,13]; 0.5,0.1,0.3)	([3,4,5,6,8,9,11,12]; 0.7,0.2,0.4)
Radio	([1,3,4,5,7,9,10,11]; 0.6,0.3,0.2)	([4,5,6,7,8,10,12,13]; 0.9,0.4,0.5)	([2,4,5,7,9,12,14,16]; 0.8,0.2,0.4)
T.V	([2,3,4,6,9,10,11,14]; 0.4,0.2,0.3)	([3,4,6,7,8,10,12,14]; 0.8,0.4,0.5)	([4,6,7,9,10,11,13,15]; 0.4,0.2,0.3)

Best Strategy for player I

Step(1)

According to equation $\tilde{r}_{ij}^{(k)} = \frac{a_{ij}^{(k)} - \min a_{ij}^{(1)})}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}}$, for j=1, 2...n,the given game matrix $\mathcal{M}=(\tilde{a}_{ij})_{mxn}$ can be converted into normalized matrix

Company B

$$\begin{pmatrix} ([0,0.09,0.18,0.27,0.36,0.45,0.54,0.73]; 0.5,0.6,0.2) & ([0.09,0.27,0.36,0.54,0.64,0.73,0.82,1]; 0.5,0.1,0.3) & ([0.09,0.18,0.27,0.36,0.54,0.64,0.82,0.9]; 0.7,0.2,0.4) \\ ([0,0.13,0.2,0.27,0.4,0.53,0.6,0.67]; 0.6,0.3,0.2) & ([0.20.27,0.33,0.4,0.47,0.6,0.73,0.8]; 0.9,0.4,0.5) & ([0.07,0.2,0.27,0.4,0.53,0.73,0.87,1]; 0.8,0.2,0.4) \\ ([0,0.08,0.15,0.31,0.54,0.62,0.69,0.92]; 0.4,0.2,0.3) & ([0.08,0.15,0.31,0.38,0.46,0.62,0.77,0.92]; 0.8,0.4,0.5) & ([0.15,0.31,0.38,0.54,0.62,0.69,0.85,1]; 0.4,0.2,0.3) \end{pmatrix}$$

Step (2)

Use the Heavy OWA operator of ONN to aggregate normalized matrix for player I. Suppose the weighting vector is W= (w_1, w_2… w_n), then the collective overall ONN of player Iis H $(\tilde{A}_1,\tilde{A}_2, \dots, \tilde{A}_n) = \sum_{j=1}^{n} w_j\, a_{ij}$

$= \left[\sum_{j=1}^{3} w_j a_{1j}, \sum_{j=1}^{3} w_j\, a_{2j}, \sum_{j=1}^{3} w_j\, a_{3j}\right]$

; $\min_{1\le i\le n} w_{\tilde{a}_i}, \max_{1\le i\le n} u_{\tilde{a}_i}, \max_{1\le i\le n} y_{\tilde{a}_i})$

The weight of the strategies are W= (0.3, 0.4, 0.3).

The heavy ordered weighted value for A_1 is

$H(A_1)$ = .3∗ ([0,0.09,0.18,0.27,0.36,0.45,0.54,0.73]; 0.5,0.6,0.2)
+0.4∗ ([0.09,0.27,0.36,0.54,0.64,0.73,0.82,1]; 0.5,0.1,0.3)
+ 0.3 ∗ ([0.09,0.18,0.27,0.36,0.54,0.64,0.82,0.9]; 0.7,0.2,0.4)
= ([0.063, 0.189, 0.279, 0.405, 0.526, 0.619, 0.736, 0.889] ; 0.5, 0.6, 0.4)

The heavy ordered weighted value for A_2 is

$H(A_2)$ = 0.3∗ ([0,0.13,0.2,0.27,0.4,0.53,0.6,0.67]; 0.6,0.3,0.2)
+0.4∗ ([0.20.27,0.33,0.4,0.47,0.6,0.73,0.8]; 0.9,0.4,0.5)
0.3∗ ([0.07,0.2,0.27,0.4,0.53,0.73,0.87,1]; 0.8,0.2,0.4)
= ([0.101, 0.207, 0.273, 0.361, 0.467, 0.618, 0.733, 0.821]; 0.6, 0.4, 0.5)

The heavy ordered weighted value for A_3 is

$H(A_3)$ = 0.3∗ ([0,0.08,0.15,0.31,0.54,0.62,0.69,0.92]; 0.4,0.2,0.3)
.4∗ ([0.08,0.15,0.31,0.38,0.46,0.62,0.77,0.92]; 0.8,0.4,0.5)+0.3∗ ([0.15,0.31,0.38,0.54,0.62,0.69,0.85,1]; 0.4,0.2,0.3)
= ([0.077, 0.177, 0.283, 0.407, 0.532, 0.641, 0.77, 0.944]; 0.4, 0.4, 0.5)

Step (3):

For each strategy, calculate truth-membership, indeterminacy-membership and falsity-membership average indexes for its collective overall ONN $H(A_i)$ and $H(B_i)$, then the ranking orders of strategies are generated according to the ranking method ONN.

R_μ [H (A_1)] = 0.23 ; R_ϑ [H (A_1)] = 0.18 ; R_y [H (A_1)] = 0.28

R_μ [H (A_2)] = 0.27 ; R_ϑ [H (A_2)] = 0.27; R_y [H (A_2)] = 0.24

R_μ [H (A_3)] = 0.19 ; R_ϑ [H (A_3)] = 0.29; R_y [H (A_3)] =0.24

Then the ranking order of the strategies for Player I according to the truth-membership, indeterminacy-membership and falsity-membership average indexes is given by $A_2>A_3>A_1$, Hence the best strategy for player I is A_2.

Best Strategy for Player II

Step(1)

According to equation $\tilde{r}_{ij}^{(k)} = \frac{\max a_{ij}^{(8)} - a_{ij}^{(9-k)}}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}}$, for i=1, 2…m, the given game matrix □=$(\tilde{a}_{ij})_{mxn}$ can be converted into normalized matrix

$$\begin{pmatrix} ([0.31,0.46,0.54,0.62,0.69,0.77,0.85,0.92];0.5,0.6,0.2) & ([0.09,0.27,0.36,0.45,0.55,0.73,0.82,1];0.5,0.1,0.3) & ([0.29,0.36,0.5,0.57,0.71,0.79,0.86,0.93];0.7,0.2,0.4) \\ ([0.23,0.31,0.38,0.54,0.69,0.77,0.85,1]; 0.6,0.3,0.2) & ([0.09,0.18,0.36,0.54,0.64,0.73,0.82,0.91];0.9,0.4,0.5) & ([0,0.14,0.29,0.5,0.64,0.79,0.86,1]; 0.8,0.2,0.4) \\ ([0,0.23,0.31,0.38,0.62,0.77,0.85,0.92]; 0.4,0.2,0.3) & ([0,0.18,0.36,0.55,0.64,0.73,0.91,1];0.8,0.4,0.5) & ([0.07,0.21,0.36,0.43,0.5,0.64,0.71,0.86];0.4,0.2,0.3) \end{pmatrix}$$

Step (2)

Use the Heavy OWA operator of ONN to aggregate normalized matrix for player II. Suppose the weighting vector is W= (w_1, w_2… w_n), then the collective overall ONN of player IIis H $(\tilde{B}_1,\tilde{B}_2, \dots, \tilde{B}_n) = \sum_{j=1}^{n} w_j\, a_{ij}$

$$= (\left[\sum_{i=1}^{3} w_j a_{i1}, \sum_{j=1}^{n} w_j\, a_{i2}, \sum_{j=1}^{3} w_j\, a_{i3}\right]$$

; $\min_{1\le i\le n} w_{\tilde{a}_i}, \max_{1\le i\le n} u_{\tilde{a}_i}, \max_{1\le i\le n} y_{\tilde{a}_i})$

The weight of the strategies are W= (0.3,0.4,0.3).

The heavy ordered weighted value for $B_1$is

$H(B_1)$ = 0.3∗([0.31, 0.46, 0.54, 0.62, 0.69, 0.77, 0.85, 0.92]; 0.5,0.6,0.2)
+0.4∗([([0.23, 0.31, 0.38,0.54, 0.69, 0.77, 0.85, 1]; 0.6,0.3,0.2)
0.3∗ ([0, 0.23, 0.31, 0.38, 0.62, 0.77, 0.85, 0.92]; 0.4, 0.2, 0.3)
= ([0.185, 0.331, 0.407, 0.516, 0.669, 0.77, 0.85, 0.952]; 0.4,0.6, 0.3

The heavy ordered weighted value for B_2 is

$H(B_2)$ = 0.3∗ ([0.09, 0.27, 0.36, 0.45, 0.55, 0.73, 0.82, 1]; 0.5,0.1, 0.3)
+0.4∗ ([0.09, 0.18, 0.36, 0.54, 0.64, 0.73, 0.82, 0.91]; 0.9,0.4, 0.5)
+0.3∗ ([0, 0.18, 0.36, 0.55, 0.64, 0.73, 0.91,1]; 0.8,0.4, 0.5)
= ([0.063, 0.207, 0.36, 0.516, 0.613, 0.73, 0.847, 0.964]; 0.5, 0.4, 0.5)

The heavy ordered weighted value for B_3 is

$H(B_3)$ = 0.3∗(([0.29, 0.36, 0.5, 0.57, 0.71, 0.79, 0.86, 0.93]; 0.7,0.2,0.4)
+0.4∗ ([0,0.14, 0.29, 0.5, 0.64, 0.79, 0.86, 1]; 0.8,0.2,0.4)
+0.3∗ ([0.07, 0.21, 0.36, 0.43, 0.5, 0.64, 0.71, 0.86]; 0.4,0.2, 0.3)
= ([0.108, 0.227, 0.374, 0.5, 0.619, 0.745, 0.832, 0.937]; 0.4, 0.2, 0.4)

Step (4)

For each strategy, calculate truth-membership, indeterminacy-membership and falsity-membership average indexes for its collective overall Fuzzy number $H(A_i)$ and $H(B_i)$, then the ranking orders of strategies are generated according to the ranking method .

R_μ [H (B_1)] = 0.24 ; R_ϑ [H (B_1)] = 0.24 ; R_y [H (B_1)] = 0.41

R_μ [H (B_2)] = 0.27 ; R_ϑ [H (B_2)] = 0.32; R_y [H (B_2)] = 0.27

R_μ [H (B_3)] = 0.22 ; R_ϑ [H (B_3)] = 0.43; R_y [H (B_3)] =0.32

Then the ranking order of the strategies for Player II according to the truth-membership, indeterminacy-membership and falsity-membership average indexes is given by $B_3>B_1>B_2$, Hence the best alternative for player II is B_2.

The best strategy of the player I and player II are$A_2$and B_2 respectively.

8. Conclusion

In this paper, we have defined octagonal neutrsophic number (ONN) and operations on it. We proposed a method to find the best strategies for the two players using heavy ordered weighted averaging (HOWA) operator on the octagonal neutrosophic numbers with weights given to the strategies.

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