Stiffness Mapping of a 12 dof Parallel Manipulator with Flexible base and Top Platforms

V. R. Mishra

G. L. Bajaj Institute of Technology & Management, India
Corresponding author E-mail: vrm140574@gmail.com

Abstract

A geometric scheme for stiffness mapping of 12 dof parallel manipulator with flexible platforms is presented. The manipulator resembles the construction features of Gough-Stewart platform with a difference that both of its platforms have been made flexible by using mobile knots. It is also shown that the tilt angle of the top platform of the proposed manipulator is more as compared to Stewart-Gough platform which increases the workspace of the proposed manipulator. In this study, comparison is also made with manipulator proposed by other investigators. Upon comparing the tilt angle of proposed manipulator it is found that, a manipulator with flexible base with same size of top platforms yields better reach within the workspace.

Keywords: *Stiffness; parallel manipulator; tilt angle; manipulator*

1. Introduction

The most common robot architecture is the serial manipulator in which the various members are connected in series from the base to the end-effector. One such manipulator is shown in Figure 1(a). This design offers numerous advantages, including a large workspace and reach of gripper near to the base. The serial manipulators have their drawbacks also such as limited load carrying capacity, error propagation along the chain, high sensitivity etc. [1].
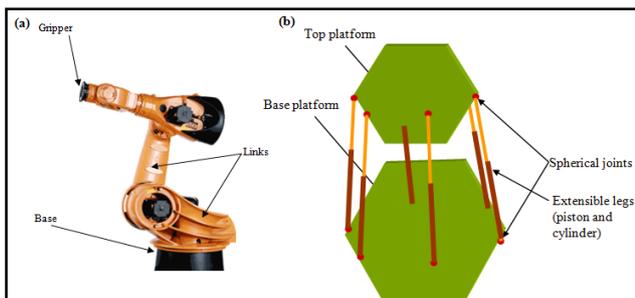

Fig. 1: Types of manipulators; (a) 6-dof Serial manipulator, (b) 6-dof parallel manipulator.

Stiffness is one of the most important factors in the selection of material. Usually, stiffness is defined as resistance against the deformation of body. During the stiffness mapping, mechanical systems containing closed-loop kinematic chains (parallel manipulators) exhibit singularities that result in a loss of controllability of movement of top platform [2]. Therefore, it becomes necessary to control this type of behavior of parallel manipulators. In order to ensure controlled behavior of the proposed manipulator the approach used here is to establish stiffness mapping for the workspace of the proposed manipulator. Stiffness mapping is the process of limiting the positioning of the centre of top platform with

respect to the base platform in order to avoid the singularity problems.

Mechanically stiff workspace of parallel manipulator is the work space of manipulator in which the centre of top platform can be positioned without singularity problems. The criterion for selection of mechanically stiff workspace is the minimization of moment of forces acting on the joints of top platform about centre of the base platform [3-4].
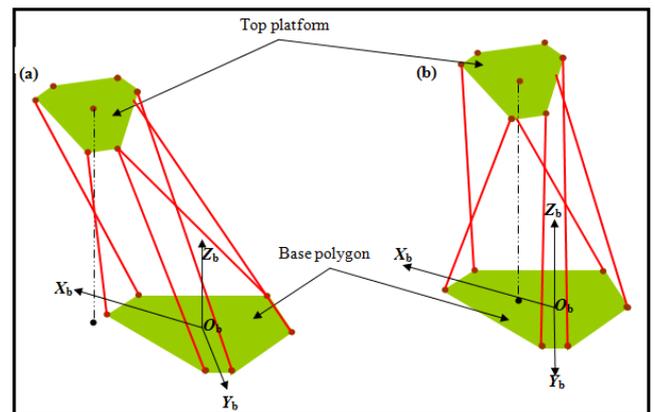

Fig. 2: Stiffness mapping of proposed manipulator; (a) centre of top platform projected outside base polygon, (b) centre of top platform projected inside base polygon [5].

Comparing the two positions of top platform it can be shown that sum of moment produced for position shown in Figure 2(a) is more as compared to that for position shown in Figure 2(b). Thus it is concluded that the position shown in Figure 5.1b is mechanically stiffer as compared to that shown in Figure 2(a) and also the manipulators will be stable. For better stiffness of proposed manipulator the coordinates of centre of top platform (X and Y) and distance of base mobile knots (A, B and C) should have a relation

such that the projection of centre of top platform lies inside the polygon formed by mobile knots of base platform

2. Formulation for Stiffness Mapping

For a particular position of top platform X and Y coordinates will be assigned a value and then feasible solution will be obtained by moving the base mobile knots along the slide ways. The movement of base mobile knots will cause the expansion or contraction of base polygon. As X and Y coordinates are the desired values, the variables here are the distance of mobile knots from base centre.

It becomes necessary to impose a limit on the size base polygon which varies with the distance of base mobile knots from the centre of base platform(A, B and C).The limit on size of base platform is decided by the desired coordinates of centre of top platform (X and Y). The relation between size of base polygon and coordinates of centre of top platform for stiffness conditions is defined as Stiffness bound. For determining this relationship, it has been assumed that the distance of all base mobile knots from centre of base will remain same (A = B = C) for each assigned position (Figure 3).

Thus, the condition for stiffness mapping has been defined as "*projection of centre of top* platform should lie within the inscribed circle to base polygon".

[Note: It has been discussed earlier that for stiffness conditions the projection of centre of top platform should lie within the base polygon. For simplicity of relation between projected coordinates of centre of top platform (X, Y) and position of base mobile knots (A, B and C) the stiffness condition is modified as stated above.]

Let R be the radius of the inscribed circle in the base polygon. The distance of base mobile knots are:

$OO_1 = A$, $OO_2 = B$ and $OO_3 = C$ and
Angles $O_1OO_2 = O_2OO_3 = O_3OO_1 = 120^0$
OP is the perpendicular bisector of side a_1a_2
Angle $O_1OP = 60^0$ and
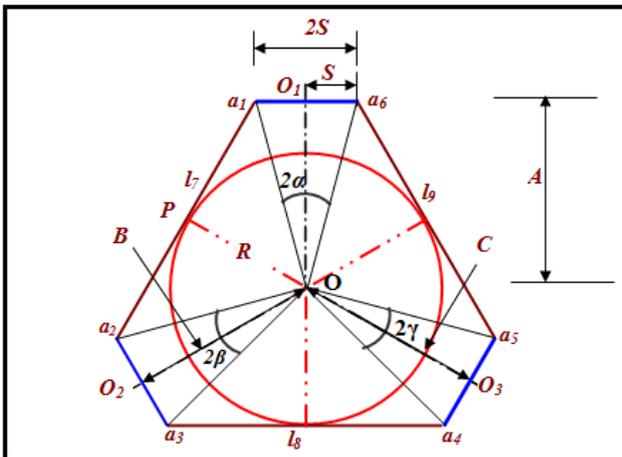Angle $a_1OP = 60-\alpha$

Fig. 3: Geometric scheme for stiffness mapping [5]

Now for A = B = C, the angles $2\alpha = 2\beta = 2\gamma$. Then as per stiffness conditions the X and Y coordinates of top platform have the relationship as:

$$X, Y \leq R \tag{1}$$

From geometry of right angle triangle a_1O_1O:
$a_1O = A*Sec\alpha$
Also from geometry of right angle triangle a_1OP
$R = a_1O\ Cos\ (60-\alpha)$ hence radius of inscribe circle is:

$$R = (A*Sec\alpha) *Cos\ (60-\alpha) \tag{2}$$

From equation (1) and (2)

$$X, Y < \{A*Sec\alpha *Cos\ (60-\alpha)\} \tag{3}$$

Equation (3) is defined as the condition for stiffness bound
For maximum distance of base mobile knots from centre of base platform i.e. A= B = C =800 mm and S = 64mmvalue of X and Y obtained by Equation 3 are749.36mm. While the minimum values of X and Y are zero when centre of top and base platform lies on a line perpendicular to base plane.

3. Comparison of Tilt Angle of Top Platforms of Stewart-Gough Platform with Proposed Manipulator

As the base polygon of proposed manipulator can be varied in size greater reach of the top platform near to the plane of base platform can be achieved. The reach of top platform near to the plane of the base is discussed in the form of angle (ξ) shown in Figure 4:
This is to be noted that the proposed manipulator will behave like Stewart-Gough platform when the base mobile knots are considered fixed. For comparison the distance of mobile knots from the centre of base platform is taken as 800mm.
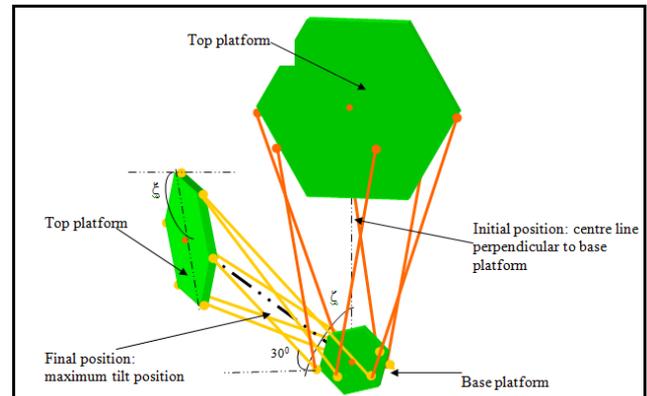

Fig. 4: Schematic representation for Tilt angle of top platform [5]

Angle between the lines joining centers of top and base platform at initial and final positions is denoted as ζ. Greater angle ζ means that top platform reaches nearer to the base. Angle between the horizontal plane and the final position of the top platform is denoted asξ. This angle is known as tilt angle of top platform. The larger value of ξ indicates that the gripper mounted on the top platform can grip the objects which are placed nearer to the plane of base platform.

In order to determine angles, ζ and ξ for Stewart-Gough platform and the manipulator proposed by author following assumptions have been made:

1. In case of Stewart-Gough platform the distance of base mobile knots from centre of base platform is fixed (800mm) while in the proposed manipulator the distance can be varied as mobile knots can be moved closer to centre of base.

2. The size of top platform is considered fixed and same for both cases as it is decided by the size of object (distance of mobile knots from centre of top platform is taken as 400mm)

3. In this study the stiffness mapping is not considered as it can be avoided for very light weight objects.

4. Tilt of any one leg connecting base and top platform is considered for comparison and here this angle is taken to be 30^0 which is the angle between plane of base and the projection of axis of one leg on the plane in which the line joining centers of base and top platforms also lies (vertical plane).

5. The initial position of base mobile knot is considered to be at maximum position (800mm from the centre of base) and it is

assigned a new position towards centre of base till it reaches minimum position (36.95 form centre of the base).

6. The distance between centre of top platform and that of base platform is taken as 1000mm and will remain unchanged for all positions of base mobile knots.

Using the values of Table 1 the variation of tilt angle ξ is plotted against the position of base mobile knots from the base centre. It is observed from the Figure that ξ increase when the mobile knots are brought nearer to centre of base platform. The above analysis of tilt angle ξ is done without considering the stiffness bounds mentioned in Equation 3. The maximum tilt angle ξ (122.17^0) can be utilized for lighter loads without considering uncontrolled behavior of the proposed manipulator due to singularity problems.

Table 1: variation in Tilt angle of top platform with distance of base mobile knots from centre of base

Position of top mobile knot, A_t(mm)	Position of base mobile knot, A (mm)	Tilt angle of top platform (ξ) (Degrees)
600	800	10.88
600	770	17.36
600	740	22.74
600	710	27.48
600	680	31.81
600	650	35.84
600	620	39.66
600	590	43.31
600	560	46.85
600	530	50.30
600	500	53.69
600	470	57.04
600	440	60.37
600	410	63.71
600	380	67.07
600	350	70.48
600	320	73.96
600	290	77.53
600	295	76.93
600	260	81.24
600	230	85.12
600	200	89.22
600	170	93.63
600	140	98.45
600	110	103.85
600	80	110.14
600	36.95	122.17

The tilt angle ξ by considering the stiffness bounds (Equation 3) is determined as 76.93^0.

It is noted here that in case of the proposed manipulator the tilt angle (ξ) even after considering the stiffness conditions is larger than that in case of Stewart-Gough platform, when the base mobile knots are place at a fixed distance of 800 mm.

4. Comparison of Proposed Manipulator with other Similar Parallel Manipulators

As already discussed earlier in case of proposed manipulator the top and base platforms are of flexible in size and they are connected through six extensible legs. It is also shown earlier that the maximum tilt angle of 122.17^0 can be achieved for light loads and 76.93^0 for medium to heavy loads when considering the stiffness bound conditions. Due to six legs in the proposed manipulator the load carrying capacity is also high as compared to other parallel manipulators with three legs.

The double tripod manipulator presented by Hertz and Hughes [8] has triangular top and base plat forms with provision of leg movement along the triangular sides of both triangular top and base platforms. Although there is possibility of sufficiently large tilt angle but in case of the proposed manipulator the tilt angle will still larger tilt angle because of the fact that its legs can move

much nearer to centre of base platform. Also the load carrying capacity of proposed manipulator is larger as there are six legs to share the load of the top platform as compared to three legs in case of Double tripod manipulator.

The Stewart platform with improved dexterity presented by Stoughton and Arai [5] consists of circular base & top platforms enabling the tilt angle of top platform more as that of the manipulator proposed by author when the legs are brought nearer to each other. However this manipulator will then result in almost a serial configuration hence its load carrying capacity will become poor as compared to proposed manipulator. Thus the proposed manipulator has better load carrying capacity.

The Tip-Tilt manipulator presented by Tahmasebi [5] consists of linear actuators which enable the movement of legs towards or away from the centre of base platform similar to movement of mobile knots provided in proposed manipulator. However, the legs of Tip-Tilt manipulator are inextensible which results in smaller tilt angle of top platform as compared to proposed manipulator. Thus the workspace of proposed manipulator is larger as compared to Tip-Tilt manipulator. Also there are only three legs in Tip-Tilt manipulator as compared to six in proposed manipulator, this difference results in a better load carrying capacity also.

The 3-PRRR redundant parallel manipulator presented by Ebrahimi and Caretero [5] consists of triangular top platform and base platform that consists of three slide-ways assembled at 120^0 to each other. Each of the slide-ways is provided with one slider that can move towards and away from centre of base platform. Three PRRR serial chains known as legs are connecting the three corners of top platform with the sliders. Due to the fact that the legs can be placed very near to the base through slider movement, the top platform will have a large tilt angle. However due to provision of only three legs in the manipulator the load carrying capacity is less when compared to proposed manipulator.

The kinematic structures proposed by Glozman and Shoham [5] presented two kinematic structures namely RRRS (Revolute Revolute Revolute Spherical) and RRSR (Revolute Revolute Spherical Revolute). The RRRS configuration consists of three limbs, each formed by joining two links with revolute pair. One end of each limb is connected to circular base platform through revolute pair and other end is connected to top platform through spherical pair. Due to this configuration the this manipulator has the advantage of greater tilt angles, but due to circular platform and less number of legs (three in this case) the manipulator have the disadvantage of poor load carrying capacity.

4. Conclusion

Following conclusions can be drawn from the analysis.

1. The top platform can be tilted through an angle of 122.17^0 when the base mobile knots are placed at minimum distance from centre of base platform i.e. 36.95mmwithout considering stiffness bounds. However the top platform can be tilted only through 10.88^0 when the base mobile knots are placed at maximum distance from centre of the base platform i.e. 800mm (Stewart-Gough platform). The tilt angle of the proposed manipulator obtained by considering the stiffness bounds is $76.93^0$when base mobile knots are at a distance of 295mm from centre of the base platform, which is also much larger as compared to that obtained for Stewart-Gough platform (10.88^0). Large tilt angle indicates large workspace of the manipulator.

2. The height of the proposed manipulator required to achieve a tilt angle of 122.17^0 is 1000 while the height required for the Stewart-Gough platform for a tilt angle of 122.17^0 is 1660.3 mm (refer Figure 8 and 9).This fact also confirms that the workspace of the manipulator proposed by the author is larger for its same dimensions of the manipulators.

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