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Research paper

On Certain New Subclass of Sakaguchi Type Function Related to Sigmoid Functions

B. Srutha Keerthi[1*], M. Revathi[2]

[1]*Department of Mathematics, School of Advanced Sciences, VIT Chennai, Vandaloor, Kelambakam Road, Chennai, India.*
[2]*Research Scholar, Department of Mathematics, Bharathiar University, Coimbatore, India.*
E-mail:revathi.joe@gmail.com
**Corresponding author E-mail: sruthilaya06@yahoo.co.in*

Abstract

The object of the present paper is to obtain initial coefficients $|a_2|$,$|a_3|$,$|a_4|$, upper bounds of $|a_3 - \mu a_2^2|$ and second Hankel determinant associated with a class of analytic univalent function of sakaguchi type function related to sigmoid function in the open unit disc Δ. Various authors as Abiodum, Tinuoye Oladipo, Murugusundaramoorthy et. al., and Olatunji have studied sigmoid function for different classes of analytic and univalent functions. Our results serves as a generalisation in this direction and it gives birth some existing subclasses of functions.

Keywords and Phrases: *coefficient estimate, Analytic function, univalent function, Starlike function, convex functionsubordination, upper bound, sigmoid function, differential operator, second Hankel determinant.*

1. Introduction

The hypothesis of unique capacity has over sparkling by different fields like genuine investigation, useful examination, topology, variable based math, differential conditions et cetera. The summed up hypergeometric capacities assumes a noteworthy job in geometric capacity hypothesis after the verification of Bieberbach Conjecture by De-Branges. Despite the fact that the exceptional capacities does not have a particular definition, its application broadly stretch out to material science, PC and so forth.

There are different exceptional capacities however we will worry with one of the enactment work known as sigmoid capacity or basic calculated capacity. It is more mainstream in light of its inclination descendent learning calculation. Sigmoid capacity is the most normally known capacity utilized in feed forward neural systems due to its non-linearity and the computational effortlessness of its subsidiary.

Actuation work is an in data process comprising of countless preparing components (neurons), enlivened by a similar way organic sensory system, (for example, mind), cooperating to settle a particular assignment. The capacity can be learned by precedent, yet can't be customized to do particular errands. It very well may be assessed in various ways, most extraordinarily by truncated arrangement extension. This capacity can be sorted into three, specifically, slope work, edge work and sigmoid capacity. The sigmoid capacity of the shape is differentiable and has the accompanying properties:

$$h(z) = \frac{1}{1+e^{-z}} \qquad (1.1)$$

- It outputs real numbers between 0 and 1.
- It maps a very large input domain to a small range of outputs.
- It never loses information because it is an injective function.
- It increases monotonically.

Sigmoid function is perfectly useful in geometric function theory with all the four properties.

Let A be the class of all univalent analytic functions f of the for

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (1.2)$$

defined in the open unit disk $\Delta = \{z : z \in C \text{ and } |z| < 1\}$, and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$ for $f \in A$. Recall that S^* and C denotes the class of star like and convex functions which their geometric condition satisfies $Re\left\{\frac{zf'(z)}{f(z)}\right\} > 0$ and $Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0$, $z \in \Delta$. Several authors have used the above two classes of functions in different ways of perspectives.

An analytic function f is subordinate to an analytic function g, written $f(z) \prec g(z)$, if there is an analytic function $w : \Delta \to \Delta$ with $w(0) = 0$ satisfying $f(z) = g(w(z))$. It follows from Schwarz lemma that $f(z) \prec g(z)$ $(z \in \Delta) \Rightarrow f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$. If $g(z)$ is univalent, then $f(z) \prec g(z)$, $(z \in \Delta) \leftrightarrow f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$. Let P be the class of Caratheodory function with positive real part consisting of all analytic functions $p : \Delta \to C$ satisfying $p(0) = 1$ and $Re\, p(z) > 0$. We need the following results about the functions belonging to the class P. If the function $p \in P$ is given by the series

$$p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \ldots, \quad (1.3)$$

then the following sharp estimates holds;

$$|c_n| \le 2 \ (n = 1,2,3,\ldots). \quad (1.4)$$

In 1976, Noonan and Tomas [5] stated the qth Hankel determinant for $q \ge 1$ and $n \ge 1$ are defined by

$$H_q(n) = \begin{bmatrix} a_n & a_{n+1} & \ldots & a_{n+q-1} \\ a_{n+1} & a_{n+2} & \ldots & a_{n+q} \\ \vdots & \vdots & & \vdots \\ a_{n+q-1} & a_{n+q} & \ldots & a_{n+2q-2} \end{bmatrix}$$

In recent years, several authors have investigated bounds for the Hankel determinant of functions belonging to various subclasses of univalent and multivalent functions. The Hankel determinant $H_2(1) = a_3 - a_2^2$ is the well known Fekete-Szegö functional. For results related to this functional, see. The second Hankel determinant $H_2(2)$ is given by $H_2(2) = a_2 a_4 - a_3^2$.

For the purpose of our results, the following lemma shall be necessary.

Lemma 1.1: *Let h be a sigmoid function and*

$$\phi(z) = 2h(z) = 1 + \sum_{m=1}^{\infty} \frac{(-1)^n}{z^m}\left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} z^n\right)^m \quad (1.5)$$

then $\phi(z) \in P$, $|z| < 1$ *where* $\phi(z)$ *is modified sigmoid function*

Lemma 1.2: *Let h be a sigmoid function and*

$$\phi_{m,n}(z) = 2h(z) = 1 + \sum_{m=1}^{\infty} \frac{(-1)^n}{z^m}\left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} z^n\right)^m \quad (1.6)$$

then $|\phi_{m,n}(z)| < 2.$

Lemma 1.3: *If* $\phi(z) \in P$ *and it is starlike, then f is normalised univalent function of the form (1.2), taking m=1, Fadipe et al remarked that*

$$\phi(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$$

Where $c_n = \frac{(-1)^{n+1}}{2n!}$ *then* $|c_n| \le 2, (n = 1,2,3,\ldots)$, *the result is sharp for each* n.

Definition 1.4.

Let the class $L^{\lambda,n}(\tau, \phi_{m,n}), \tau \in \setminus C \ \{0\}$ denote the subclass of A consisting of functions f of the form (1.2) satisfying

$$Re\left[1 + \frac{1}{\tau}\left[\frac{(1-t)\left[\lambda z^3 (D_{n-1}f(z))''' + (1+2\lambda) z^2 (D_{n-1}f(z))'' + z(D_{n-1}f(z))'\right]}{\lambda z^2\left[(D_{n-1}f(z))'' - t^2 (D_{n-1}f(tz))''\right] + z\left[(D_{n-1}f(z))' - t(D_{n-1}f(tz))'\right]} - 1\right]\right] > 0, \quad (1.7)$$

for $0 \le \lambda \le 1$, $|t| \le 1, t \ne 1$ and $\phi_{m,n}(z)$ is a simple logistic sigmoid activation function [9] and

$$D_{n-1}f(z) = \frac{z}{(1-z)^{n+1}} * f(z) \quad (1.8)$$

when $n + 1 > 0$.

2. Coefficient estimates

Various authors as Abiodum[6], Tinuoye Oladipo[3], Murugusundaramoorthy et. al.[4], and Olatunji [7,8] have studied sigmoid function for different classes of analytic and univalent functions. In this paper, we obtain few coefficient bounds for the class $M^{\lambda,n}(\tau, \phi_{m,n})$

Theorem 2.1. If $f(z)$ given by (1.2) belongs to the class $M^{\lambda,n}(\tau, \phi_{m,n})$, $m \ge 2$ then

$$|a_2| \le \frac{|\tau|}{4(n+1)(1+\lambda)(1-t)}, \quad (2.1)$$

$$|a_3| \le \frac{(1+t)|\tau|^2}{6(n+1)(n+2)(1+2\lambda)(1-t)(2-t-t^2)} \quad (2.2)$$

$$|a_4| \le \frac{|\tau|}{2(n+1)(n+2)(n+3)(1+3\lambda)(4-(1+t)(1+t^2))}\left[\frac{3(1+t)(1+t+t^2)\tau^2}{8(1-t)(2-t-t^2)} - \frac{1}{8}\right]. \quad (2.3)$$

(2.3)

Proof: If $f(z) \in M^{\lambda,n}(\tau, \phi_{m,n})$, then

$$1 + \frac{1}{\tau}\left[\frac{(1-t)\left[\lambda z^3 (D_{n-1}f(z))''' + (1+2\lambda) z^2 (D_{n-1}f(z))'' + z(D_{n-1}f(z))'\right]}{\lambda z^2\left[(D_{n-1}f(z))'' - t^2 (D_{n-1}f(tz))''\right] + z\left[(D_{n-1}f(z))' - t(D_{n-1}f(tz))'\right]} - 1\right] = \phi_{m,n}(z), \quad (2.4)$$

where Taylor's series expansion of $\phi_{m,n}(z)$ gives

$$\phi_{m,n}(z) = 1 + \frac{1}{2}z - \frac{1}{24}z^3 + \frac{1}{240}z^5 - \frac{17}{40320}z^7 + \ldots, \quad (2.5)$$

from (2.4) we have

$$1\frac{1}{\tau}\Bigg[2(n+1)(1+\lambda)(1-t)a_2 z + \left[\frac{3}{2}(n+1)(n+2)(1+2\lambda)(2-t-t^2)a_3 - 4(n+1)^2(1+\lambda)^2(1+t)(1-t)a_2^2\right] z^2 + \left[\frac{2}{3}(n+1)(n+2)(n+3)(1+3\lambda)(4-(1+t)(1+t^2)a_4 - 3(n+1)^2(n+2)(1+\lambda)(1+2\lambda)[3(1+t) - 2t(1+t+t^2)]a_2 a_3 + 8(n+1)^3(1+\lambda)^3(1+t)^2(1-t)a_2^3\right] z^3 + \ldots\Bigg] = \phi_{m,n}(z).$$

Equating the coefficients of z, z^2 *and* z^3, we obtain

$$a_2 \le \frac{\tau}{4(n+1)(1+\lambda)(1-t)}, \quad (2.6)$$

$$|a_3| \le \frac{(1+t)\tau^2}{6(n+1)(n+2)(1+2\lambda)(1-t)(2-t-t^2)}, \quad (2.7)$$

$$|a_4| \le \frac{\tau}{2(n+1)(n+2)(n+3)(1+3\lambda)(4-(1+t)(1+t^2))}\left[\frac{3(1+t)(1+t+t^2)\tau^2}{8(1-t)(2-t-t^2)} - \frac{1}{8}\right] \quad (2.8)$$

Results (2.1),(2,2) and (2.3) can be obtained from (2.6), (2.7) and (2.8) respectively.

Corollary 2.2. *If* $f(z) \in C(\tau, \phi)$, *then [10]*

$$|a_2| \leq \frac{|\tau|}{4(n+1)}, \tag{2.9}$$

$$|a_3| \leq \frac{|\tau|^2}{12(n+1)(n+2)}, \tag{2.10}$$

$$|a_4| \leq \frac{|\tau|}{12(n+1)(n+2)(n+3)}\left[\frac{3\tau^2}{8} - \frac{1}{4}\right]. \tag{2.11}$$

3. Fekete-Szegö inequalities

Recently there has been interest to obtain the Fekete-Szegö inequality for various subclasses of S and C. In this section making use of a_2 and a_3, we prove the following Fekete-Szegö result for the function class $M^{\lambda,n}(\tau, \phi_{m,n})$.

Theorem 3.1: *if* $f(z)$ *belongs to the class* $M^{\lambda,n}(\tau, \phi_{m,n})$, *of the form (1.2), then*

$$|a_3 - \mu a_2^2| \leq \frac{|\tau|^2}{16(1+\lambda)^2(1-t)^2(n+1)^2}\left[\frac{8(1-t^2)(1+\lambda)^2(n+1)}{3(n+2)(1+2\lambda)(2-t-t^2)} - \mu\right]. \tag{3.1}$$

Proof: From (2.6) and (2.7)

$$a_3 - \mu a_2^2 \leq \frac{|\tau|^2}{16(n+1)^2(1+\lambda)^2(1-t)^2}\left[\frac{8(1-t^2)(1+\lambda)^2(n+1)}{3(n+2)(1+2\lambda)(2-t-t^2)} - \mu\right] \tag{3.2}$$

hence (3.1) can be easily obtained from (3.2).

Corollary 3.3. *If* $f(z) \in C(\tau, \phi)$, *then [10]*

$$|a_3 - \mu a_2^2| \leq \frac{|\tau|^2}{16(n+1)^2}\left[\frac{4(n+1)}{3(n+2)} - \mu\right]. \tag{3.3}$$

4. Second Hankel determinant

In this section making use of a_2 and a_3, we obtain the following second Hankel determinant result for the function class $M^{\lambda,n}(\tau, \phi_{m,n})$.

Theorem 4.1 If $(z) \in M^{\lambda,n}(\tau, \phi_{m,n})$, then

$$|a_2a_4 - a_3^2| \leq \left|\frac{\tau^2}{64(n+1)^2(n+2)(n+3)(1+3\lambda)\left(4-(1+t)(1+t^2)\right)(1+\lambda)(1-t)}\left[-1 + \frac{3\tau^2(1+t)(1+t+t^2)}{(1-t)(2-t-t^2)} - \frac{16\tau^2(1+t)^2(1+\lambda)(1+3\lambda)(n+3)(4-(1+t)(1+t^2))}{9(n+2)(1+2\lambda)^2(2-t-t^2)^2(1-t)}\right]\right| \tag{4.1}$$

Proof: From (2.6) , (2.7) and (2.8), we have

$$a_2a_4 - a_3^2 \leq \frac{\tau^2}{64(n+1)^2(n+2)(n+3)(1+3\lambda)\left(4-(1+t)(1+t^2)\right)(1+\lambda)(1-t)}\left[-1 + \frac{3\tau^2(1+t)(1+t+t^2)}{(1-t)(2-t-t^2)} - \frac{16\tau^2(1+t)^2(1+\lambda)(1+3\lambda)(n+3)(4-(1+t)(1+t^2))}{9(n+2)(1+2\lambda)^2(2-t-t^2)^2(1-t)}\right], \tag{4.2}$$

which gives the desired inequality (4.1)

Corollary 4.3. *If* $f(z) \in C(\tau, \phi)$, *then [10]*

$$|a_2a_4 - a_3^2| \leq \left|\frac{\tau^2}{192(n+1)^2(n+2)(n+3)}\left[-1 + \frac{3\tau^2}{2} - \frac{4\tau^2(n+3)}{3(n+2)}\right]\right|. \tag{4.3}$$

5. Conclusion

By selecting the values of λ and t we state the interesting Fekete-Szegö inequality and Second Hankel determinant for the subclasses of $C(\tau; \varphi)$ [1,2].The results above serve as a new generalization of subclasses of univalent functions related to sigmoid functions. The investigation of initial coefficients bounds, Fekete-Szegö inequality and Second Hankel determinant for various subclasses can be a scope of future research.

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