International Journal of Engineering & Technology, 7 (4.36) (2018) 674-677

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Strehl Ratio for Annular Eccentric Circular Aperture with Presence of Seidel Aberrations

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Abstract

In this research the equation of point spread function (PSF) has been derived for annular eccentric aperture optical system which is consisting of circular perture with non-central circular obscuration. The study was done with diffraction limited system and when different 1st and 3rd orders aberrations were present. Also trying to show how the presence of some 1st order errors with some of 3rd order aberrations were affected on Strehl ratio and make a balancing of aberrations which gives in turn a better image.

Keywords: *Point spread function, optimum balance, eccentric circular aperture, seidel aberrations.*

1. Introduction

The image of a point source object formed by a lens system is known as the point spread function (PSF) of the lens, which is the Fourier transform of the exit pupil function. It is one of the most complete functions for describing the performance of an optical system and can be extended to include the effects of obstructed apertures, a podization or any factor external to the optical system. PSF depends on diffraction produces by the lens aperture and the amount of aberrations[1]. Obscuring aperture center of an optical system means that several important effects have been obscured in the practical application, where this is used in optical systems imaging and astronomical telescopes.
A good history of the use of an annular pupil in optical imaging has been investigated by Colin J.R. Sheppard and Amarjyoti Choudhury[2], then In 2007 Guangming Dai and Virendra N. Mahajan considered imaging through atmospheric turbulence by systems with annular pupils. They derived a set of analytical results, including the Fourier transform and functions of Zernike annular polynomials[3]. In 2013 Virendra N. Mahajan and José Antonio Díaz performed The general equations for the PSF and OTF of a system with circular and annular pupils. They found that the symmetry properties of systems with annular pupils aberrated by an annular polynomial aberration are the same as those for a circular pupil aberrated by a corresponding circle polynomial aberration[4].
In this research, the annular aperture with eccentric obstruction (non- circular obstruction) is taken to study its PSF with diffraction limited and with the presence of several types of first and third order aberrations. There were some researchers studied this aperture, where in 2005 Linda Lundstromet.al. designed a Hartmann–Shack sensor to measure peripheral wave front aberrations in subjects using eccentric viewing[5]. In 2007 Yong Liu and Jiabi Chen studied eccentric photo refraction using Fourier optics, where usually eccentric photo refraction is used as early eye sight diagnostic test of infants[6]. In 2012 Suha Al-Awsi gives a detailed explanations to the eccentricity related errors of human eye[7]. Also In 2012 Chaonan Wang Ming and Bai Ming Jin investigated transmission properties through an array of concentric or eccentric double-overlapped annular apertures[8].In 2016 Shih-Hsiang Liu and Jui-che Tsai showed that using eccentric Fresnel projection lenses guarantees intrinsically jointed exit pupils, which allow viewers to adjust the viewing zones based on their requirements[9].
In next section a derivation of the Point Spread Function PSF equation for annular eccentric circular aperture was performed, and in section 3, Strehl ratio S.R. were discussed with three approximation of it were stated, and the relation with variation V and standard deviation S.D. were clarified. Then the results with discussion were performed in section 4.

2. Deriving the Equation of Point Spread Function (PSF) for eccentric circular aperture

The pupil function for any point in exit pupil of any shape can be written as:

$$f(x, y)=\boldsymbol{\tau}(\mathbf{x}.\mathbf{y}) \,.\, \mathbf{e}^{\mathbf{ikw(x,y)}} \tag{1}$$

Where $\tau(x.y)$represents the real amplitude function distributed inexit pupil and it is called "pupil transparency" or "transmission function" and it is equal to1 if there is no apodization, where w(x,y) represents aberration function.
The PSF for incoherent illumination is the square absolute value of Fourier transform for the pupil function [10], i.e.

$$\text{PSF}=|\boldsymbol{\mathcal{F}}\{\mathbf{f}(\mathbf{x},\mathbf{y})\}|^{\mathbf{2}} \tag{2}$$

Figure (1) represents the eccentric aperture that consists of circular aperture of radius 1 unit with non-central obscuration of radius □.

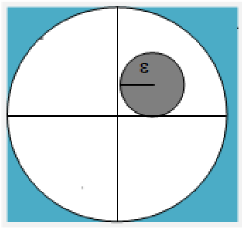

Fig 1: Eccentric aperture

The equation of the outer circle is given by the relation:

$$\mathbf{x^2 + y^2 = 1} \quad (3)$$

While the equation of the obscuration circle is:

$$(x - x_0)^2 + (y - y_0)^2 = \varepsilon^2 \quad (4)$$

Where (x_0,y_0) is the center of the obscuration circle.
Let $x' = x - x_0$ and $y' = y - y_0$ or

$$\mathbf{x = x' + x_0} \quad \text{and} \quad \mathbf{y = y' + y_0} \quad (5)$$

Then eq.(3) becomes

$$x'^2 + y'^2 = \varepsilon^2 \quad (6)$$

From eq.2 and using the limits of eq.s3 and 6 for eccentric aperture the normalized PSF can be written as:

$$\mathrm{PSF} = \mathrm{n.f.}\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)e^{i2\pi(ux+vy)}\,dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} f(x,y)e^{i2\pi(u(x'+x_0)+v(y'+y_0))}\,dx'\,dy'\right|^2$$
(7)

Where u and v are the special frequencies in the focal plane related to the angular distances with x and y axes, and n. f. is the normalizing factor which is for uniform illumination equal to reciprocal of the square of aperture area or the reciprocal of diffraction limited PSF (no aberration), i.e.

$$\mathrm{n.f.} = \left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} dx'\,dy'\right|^{-2}$$

Taking one coordinate of exit pupil (i.e. v=0)

$$\mathrm{PSF} = \mathrm{n.f}\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)e^{i2\pi(ux)}\,dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} f(x'+x_0, y'+y_0)e^{i2\pi(u(x'+x_0))}\,dx'\,dy'\right|^2$$
(8)

3. Strehl ratio

Central value of the diffraction pattern of a point object is affected by the amplitude and phase variations across the pupil. The central intensity value is maximum when the amplitude and phase are uniform. The ratio of central irradiances with and without phase or amplitude variations represents the Strehl ratio, which describes the effect of apodization(amplitude variations) and aberrations (phase variations)[11].The Strehl ratio can be expressed as theratio of central value PSF (u=0) with and without aberration, i.e. for uniform illumination, $f(x,y)=e^{ikw(x,y)}$, then Strehl ratio becomes

$$\mathrm{S.R.} = \frac{\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{ikw(x,y)}\,dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} e^{ikw(x'+x_0,y'+y_0)}\,dx'\,dy'\right|^2}{\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} dx'\,dy'\right|^2} \quad (9)$$

If the aberrations are small enough, the Strehl ratio can be either linearly related to the variance of the phase error as Nijboer approximation (S_N) or quadratically related as Marechal approximation (S_M), While a better approximation is that of Mahajan(S_{MH}) that was given by the exponential of the variance. These approximations can be written as[12, 13]

$$S_N = 1 - k^2\sigma^2\,,\quad S_M = \left(1 - \tfrac{1}{2}k^2\sigma^2\right)^2,\quad S_{MH} = \exp[-k^2\sigma^2] \quad \mathbf{(10)}$$

Where$\sigma^2$is the variance (which is the squared value of standard deviation S.D.), whichis the expected value of the squared differences of the mean value from each aberration value.

$$\sigma^2 = \langle(\Delta w)^2\rangle = \langle(w - \langle w\rangle)^2\rangle = \frac{\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (w-\langle w\rangle)\,dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y''^2}}^{\sqrt{\varepsilon^2-y'^2}} (w-\langle w\rangle)\,dx'\,dy'}{\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} dx'\,dy'} \quad (11)$$

and

$$\langle w\rangle = \frac{\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} w\,dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} w\,dx'\,dy'}{\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy - \int_{-\varepsilon}^{}\int_{-\sqrt{\varepsilon^2-y'^2}}^{\sqrt{\varepsilon^2-y'^2}} dx'\,dy'} \quad (12)$$

4. Results and discussion

In this research, a circular aperture with unit radius obscured with an obstacle of radius 0.3 centered in (0.3,0.3) of the outer aperture has been analyzed.

Fig.(2) shows the curves of PSF changed with different values of 1st and 3rd order aberrations like, focus error, tilt, spherical aberration, and comma respectively. The effect of these aberrations is obvious where the peak value becomes lower as the value of the aberration increased, i.e. the value of Strehl ratio is decreased.

Strehl ratio is affected by variance (V) or standard deviation (S.D.), as illustrated in fig. (3) which shows the three types of Strehl ratios, S_N, S_M. and S_{MH}, as the standard deviation changed compared with exact S.R. of eq. (10) for different types of aberrations focus w_{20}, tilt w_{11}, spherical aberration w_{40}, and coma w_{31}. The figure shows that as standard deviation is increased, Strehl ratio is decreased for S.D. less than 1.4λ/2π, and it is obvious that Mahajan approximation is the better one that describing Strehl ratio, i.e. it is the nearest curve to that of actual Strehl ratio. The three approximation are nearly the same and equal to that of actual S.R. when the value of S.R. more than 0.8 or for small aberrations, while when the aberrations are significant then S_N becomes smaller than S_M and this in turn smaller than S_{MH}.

To improve the value of Strehl ratio we will use a method used by Cosmas M. Afusire and T Jaart P. J. K ruger[14], (where they used aberrations in the form of Zernike polynomials) by letting different values of 1st order aberration balanced with each value of 3rd order aberration and find the maximum Strehl ratio, as follows:

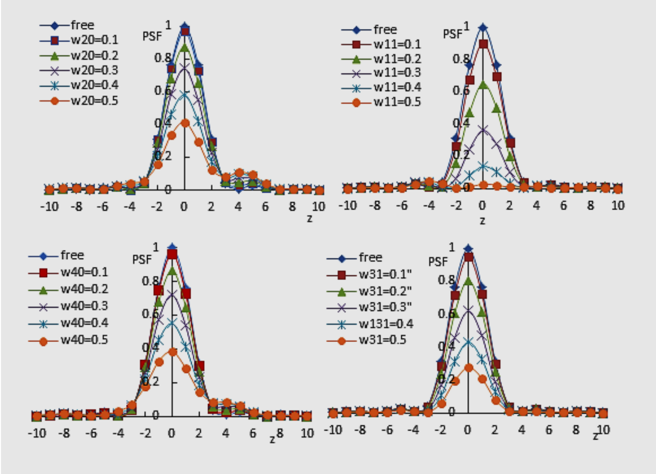

Fig. 2: Point spread function for eccentric annular aperture (ε=0.3, a=b=0.3) with present of different values of 1st and 3rd order aberrations: a)focus error w_{20} b) tilterror w_{11} c) spherical aberration w_{40} d) comma w_{31}. (x- axis z=2πu).

First the spherical aberration (3rd order) balanced with focus error (1st order) will be taken, as:$w = w_{40}(x^2 + y^2)^2 + w_{20}(x^2 + y^2)$ Inserting this in aberration factor of eq.(10) and find S.R. from eq.(10), the results is shown in fig. (4), and it is clear that the values of w_{20} were equal with the balanced spherical aberration values. i.e. when w_{40}=0.2, w_{20} must equal 0.2 to get maximum S.R. and it's clear that the results of S.R. is better than that with present spherical aberration alone, as shown in table (1). According to Marechal[12], the accepted value of S.R. is 0.8, that's mean before balancing the accepted value of w_{40} is 0.2λ/2π, while after balancing it becomes 0.3λ/2π.

Table 1: Balanced Values of w_{40} and w_{20} with the values of S.R. Before and after Balancing, for Eccentric Annular Aperture (ε=0.3,a=b=0.3)

w_{40}	w_{20}	S.R. before B.	S.R. after B.
0	0	1	1
0.1	0.1	0.965	0.984
0.2	0.2	0.867	0.939
0.3	0.3	0.721	0.867
0.4	0.4	0.552	0.775
0.5	0.5	0.386	0.668

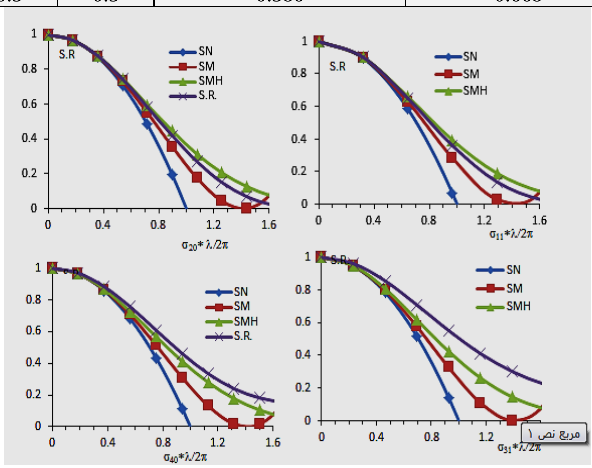

Fig. 3: Shows the three types of Strehl ratios, S_N, S_M. andS_{MH}, for eccentric annular aperture (ε=0.3,a=b=0.3), as the standard deviation changed compared with exact strehl ratio S.R. for different types of S.D., focus σ_{20}, tilt σ_{11}, spherical aberration σ_{40}, and coma σ_{31}.

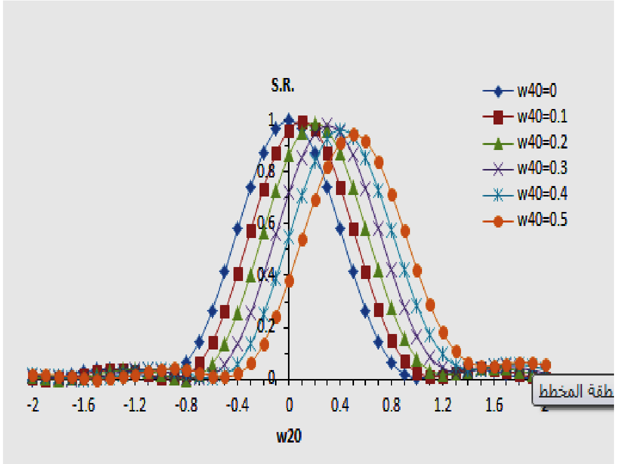

Fig. 4: Strehl ratios for fixed values of w_{40} with different values of w_{20} for eccentric annular aperture with (ε=0.3, a=b=0.3)

The same thing done with coma (3rd order aberration) and tilt (1st order) as follows:

$$w = w_{31}(x^2 + y^2)x + w_{11}x$$

The result is shown in fig. (5). Here the values of w_{31} and w_{11} for optimum balance are not equal,as that for w_{40} and w_{20}, but it found to be as in table (2), and it seems to be that the difference between them is increased as the value of coma aberration increased. The value of Strehl ratio becomes better with the combination of these two than if apresence of w_{31}alone. Here The value of accepted w_{31} becomes 0.4 λ/2πafter balancing, while before balancing it is equal to 0.2 λ/2π.

Table 2: Balanced Values of w_{31} and w_{11} with the values of S.R. Before and after Balancing, for Eccentric Annular Aperture (ε=0.3,a=b=0.3)

W31	W11	S.R. before B.	S.R. after B.
0	0	1	1
0.1	0.1	0.948	0.984
0.2	0.3	0.807	0.953
0.3	0.4	0.619	0.912
0.4	0.6	0.431	0.825
0.5	0.7	0.277	0.765

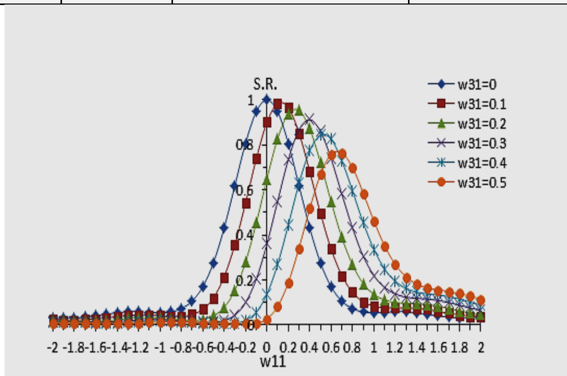

Fig. 5: Strehl ratio for fixed value of w_{31} with different values of w_{11} for eccentric annular aperture with (e=0.3, a=b=0.3)

It can be shown that the spherical aberration does not balanced with tilt error and also comma does not balanced with focus error as shown in fig. (6), that sowed the best value of Strehl ratio for the presence of w_{40} and w_{31} are when w_{11} and w_{20} are equal to zero respectively.

If we take the 3rd order astigmatism aberration w_{22} and balanced it with x-focus, as

$$w = w_{22}(x^2 - y^2) + w_{20x}(x^2)$$

The result as in fig. (7), and table (3), which shows the values of w_{20x} that balanced with w_{22}, and how the values of S.R. be improved, and the accepted value of w_{22} changed from 0.1 λ/2π before balancing to 0.3 λ/2π after balancing. This balancing is not good because, unlike focus error, the sign of astigmatism in x-axis is not like that in y- axis, so, the balancing can be improved if the x-focus and y-focus taken separately as:

$$w = w_{22}(x^2 - y^2) + w_{20x}x^2 + w_{20y}y^2$$

and letting $w_{20y} = -w_{20x}$, then the results as in 2nd part of fig.(7), which shows the best balancing when $w_{22}=w_{20y} = -w_{20x}$, as shown in table (3), it seems to be that all values of w_{22} can be corrected.

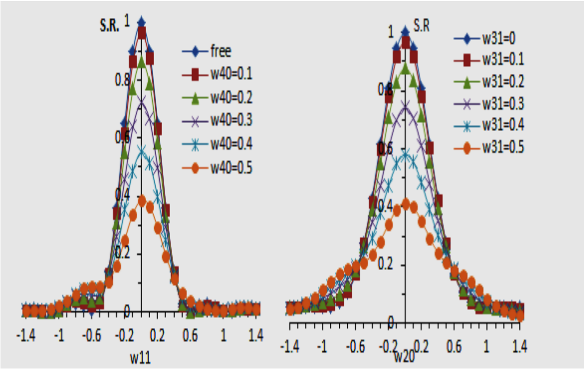

Fig. 6: Strehl ratio for fixed value of (a) w_{40} with different values of $w_{11}$and (b) w_{31} with different values of $w_{20}$for eccentric annular aperture with (ε=0.3, a=b=0).

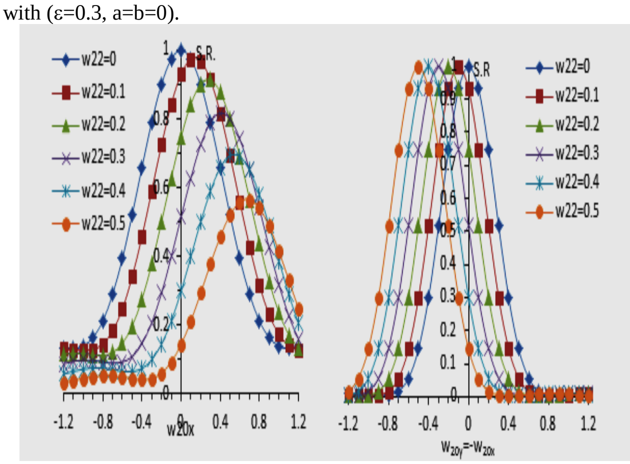

Fig. 7: Strehl ratio for fixed values of w_{22} balanced with different values of (a) w_{20x} and (b) $w_{20y}=w_{20x}$ for eccentric annular aperture with (ε=0.3, a=b=0.3)

Table 3: Values of w_{20x} and $w_{20y}=-w_{20x}$ Needed for Balancing Different Values of w_{22} with Values of S.R. Before and After Balancing for Eccentric Annular Aperture (e=0.3, a=b=0.3)

w_{22}	w_{20x}	S.R. before B.	S.R. after B.with w_{20x}	$w_{20y}=-w_{20x}$	S.R. after B.with $w_{20y}$and w_{20x}
0	0	1	1	0	1
0.1	0.1	0.931	0.974	0.1	1
0.2	0.3	0.748	0.913	0.2	1
0.3	0.4	0.516	0.817	0.3	1
0.4	0.5	0.3	0.694	0.4	1
0.5	0.6	0.145	0.559	0.5	1

5. Conclusions

In this research, the accentric annular circular aperture is analyzed and several conclusions can be written:

- Strehl ratio can be better if 3rd order aberrations balanced with appropriate 1st order. Like spherical aberration with focus error and comma with tilt error.
- Astigmatism can have excellent balance if adding equal amounts of x-focus and y- focus separately.
- In balancing spherical aberration with focus error there values must be equal, while it must not for coma and tilt, where the difference value between them increased as the value of comma aberration increased.
- When adding x-focus to astigmatism for balancing the value of difference value between them is increased as the value of astigmatism increased, while if adding both x and y- focus separately with $w_{20}=-w_{20x}$,they must equal that of astigmatism.

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