



A Modern Technique to Solve the Problem of the Failure of Layers of Multilayer Cylindrical Shells

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Abstract

Depending on the theory of the discrete-structure of the thin walls structures, an arithmetical model was proposed for elements of thin, multilayered walls for set layers anisotropic rigid. Assuming that the transverse shear stress is equal to the compression stress at the contact boundary. An elastic slip on the contact surface of adjacent layers is assumed. The problem was solved in a geometrically nonlinear formula with allowance for transverse deformations landslide and compression. The stressed-deformed state of two-layer transversally isotropic cylindrical shells with interphase defects of the structure of the material is studied. The numerical results were compared with the experimental results. The study proved numerically and experimentally data the change in the kinetic and static conditions of contact on mating surfaces in the solid layers of the various elements of the fine wall structures greatly influences the distribution of compression stress and transverse shear stress deformations. Its ends are rigidly fixed, the work experience of the internal pressure of the uniform intensity of $P = 1.11$ MPa.

Keywords: Isotropic, Discrete-structural, Compression stress, Transverse shear stress, Stressed-deformed.

1. Introduction

Problems of mechanical contact between layers of multi-layer composite materials have been studied by researchers [L'vov G. I., (2005); E. Carrera and S. Brischetto, (2010); Kim J. S. and Cho M., (2007); Guidault P.A., et.al, (2007); Brischetto S. and Carrera E., (2010)]. Where, based on the theory of discrete structures, a system of mathematical equations was developed to the resolve such problems in non-ideal contact conditions between adjacent layers. A method was proposed to solve the problems of nonlinear contact between two shells in different shapes and layers that are at one distance by the researcher [Wang X. and Zhong Z., (2003)]. A complete analysis of the results can be found in the development of the theory of discrete structures shells and plates for multi-layered composite materials in the works of the researchers. [Sofiyev A.H., (2003); Yu B. and Yang L., (2010); Rahman T. and Jansen E. L., (2010); Li Z. M. and Lin Z. Q., (2010)]. It is known that the destruction of fiberglass shells due to the poor resistance to lateral segregation and interlinear shear happen, commonly, before a long period of time the stresses reached their specified value. Beneath the action of the load, due to the peculiarities of the technical nature and the physic-mechanical properties of reinforced plastics, thin non-uniform interfacial interlayers, various imperfections, for example, detachment or non-glue areas, format the contact boundaries of the bonded layers. In this case, Assuming continuity of stresses and displacements, when moving through the contact boundary between the adjacent layers in the composite material multilayer may turn into an actual fracture.

In this work, two mathematical models will be adopted to solve the contact problem between adjacent layers of the composite material rigid anisotropic layers are considered to simulate areas of weakened contact at interphase boundaries.

From the 1st model, the contact of rigid layers for the thin multilayer walls is carried out using an adhesive layer of a specific thickness with a value greater than zero. It is presupposed that in some native area of the thin multilayer shell there is no adhesive layer; Therefore, consider one side connection between solid layers 2nd model is characterized by the fulfillment of static state of contact over the interface of singular layers. It is believed that the compression stress and transverse shear stress shall be equal to each other at the contact boundary. In this case, elastic slippage along the contact surface of neighboring layers is allowed.

2. Analytical Model

In accordance with the theory of the discrete-structure of the thin walls structures, The mathematical model in this study contains a number of layers (n), of thin wall shells made of composite materials multilayer. Every layer of the unreformed thin wall shell is indicated to a perpendicular curvilinear coordinate system $\alpha^i \rightarrow (i = 1, 2), Z^i$, Coordinate Z^i directed by general normal \bar{m}^i , to the median surface S^i , and equidistant surface $S^i_{\frac{1}{2}}$, l is layer number. The index $\ll Z \gg$ when entering other symbols means that the identical values indicate to the point $(\alpha^1, \alpha^2, Z^i)$ equidistant surface $S^i_{\frac{1}{2}}$. Full displacement vector $U^i_{\frac{1}{2}}$, points of a hard layer, as derived from the theory of S.P. Timoshenko's shells can be represented as:

$$\vec{U}^i_{\frac{1}{2}} = \vec{U}^i + Z^i \cdot \vec{\gamma}^i + \varphi^i(Z) \cdot \vec{\psi}^i \quad (1)$$

Where: \vec{U}^i , is displacement vector of mid-surface points S^i and $\vec{\gamma}^i$, function of rotation angles and crimping of fibers vertical to the non-deformed middle surface S^i and $\vec{\psi}^i$, are the

nonlinear continuous distribution function for tangential displacements across layer thickness, this was reported in the study [L'vov G. I., (2005)]; $\vec{U}^l, \vec{\gamma}^l$ and $\vec{\psi}^l$, written using the following expressions:

$$\begin{aligned} \vec{U}^l &= \vec{r}^l_i \cdot U^l_i + \vec{m}^l_i \cdot W^l_i; \quad \vec{\gamma}^l = \vec{r}^l_i \cdot \gamma^l_i + \vec{m}^l_i \cdot \gamma^l_i; \\ \vec{\psi}^l &= \vec{r}^l_i \cdot \psi^l_i \end{aligned} \quad (2)$$

The components of the finite strain tensor at $(\alpha^1, \alpha^2, Z^1)$ defined as the semi-difference tensors In the case of pre-deformation and in case of deformation, write the equation as follows:

$$\begin{aligned} 2\varepsilon_{ij}^{li} &= g_{ij}^{l*} - g_{ij}^l; \quad 2\varepsilon_{i3}^{li} = g_{i3}^{l*} - g_{i3}^l; \\ 2\varepsilon_{33}^{li} &= g_{33}^{l*} - 1 \end{aligned} \quad (3)$$

Assuming, that in the orientation of the normal to the middle surfaces of singular shell layers for composite materials, the pivotal lines of the common and topical coordinate systems are combined and local coordinate surfaces are also combined with the middle surfaces of the layers, the variation equation of the Reissner principle for a multilayer written:

$$\begin{aligned} \delta R &= \sum_{i=1}^n \delta R^i = \sum_{i=1}^n \delta A_R^i - \sum_{i=1}^n \iiint_{V^i} \delta \\ (\sigma_i^{\alpha\beta} \varepsilon_{\alpha\beta}^l - F^l) dV &= 0, \quad ((\alpha, \beta = 1, 2, 3)) \end{aligned} \quad (4)$$

The numbering of the layers begins on the side of negative values of the coordinate Z , from one to n . At the same time F^l , is specific deformation work; $\sigma_i^{\alpha\beta}$ and $\varepsilon_{\alpha\beta}^l$, are components of stress tensor and strain tensor. If the conditions of ideal contact are fulfilled on the interfaced front surfaces of the layer:

$$U_{\beta}^{li,l-1} = U_{\beta}^{l-1,l}; \quad \chi_{i,l-1}^{\beta} = \chi_{i-1,l}^{\beta} \quad (5)$$

Or in vector form is:

$$\begin{aligned} \vec{U}_Z^l \left(\alpha_i^l, -\frac{h^l}{2} \right) &= \vec{U}_Z^{l-1} \left(\alpha_i^{l-1}, -\frac{h^{l-1}}{2} \right); \\ \vec{\chi}_i^l \left(\alpha_i^l, -\frac{h^l}{2} \right) &= \vec{\chi}_{i-1}^{l-1} \left(\alpha_i^{l-1}, -\frac{h^{l-1}}{2} \right), \quad (i = 1, 2) \end{aligned} \quad (6)$$

It can be written of the external forces δA_R , as follows:

$$\begin{aligned} \delta A_R &= \sum_{i=1}^n \delta A_R^i = \sum_{i=1}^n \iint_{S_i} \{ \vec{\chi}_i \cdot \vec{\varepsilon}_i^l + \\ M_i^i \vec{r}_i^l \delta \vec{\gamma}^l + B_i^i \vec{r}_i^l \delta \vec{\psi}^l + M_i^3 \delta \varepsilon_{33}^l \} dS + \\ + \sum_{i=1}^n \int_{K_i^l} \{ \vec{\Phi}_i^s \delta \vec{u}^l + \vec{G}_i^s \delta \vec{\gamma}^l + \vec{L}_i^s \delta \vec{\psi}^l \} dL + \\ \sum_{i=1}^n \int_{K_2^l} \{ \vec{\Phi}_i \cdot \delta \vec{u}^l + \vec{G}_i \cdot \delta \vec{\gamma}^l + \vec{L}_i \cdot \delta \vec{\psi}^l + (\vec{u}^l - \vec{u}_5^l) \delta \vec{G}_i \\ + (\vec{\gamma}^l - \vec{\gamma}_5^l) \delta \vec{\Phi}_i + (\vec{u}^l - \vec{u}_5^l) \delta \vec{L}_i \} dL \end{aligned} \quad (7)$$

Here S_i , is the middle surface of the layer; K_1^l, K_2^l , contour parts K^l , External force vectors $\vec{\chi}_i$, moment \vec{M}_i , and extra moments \vec{B}_i , which are included in equation (7), are determined by the

equations:

$$\begin{aligned} \vec{\chi}_i &= \vec{\chi}_i^+ - \vec{\chi}_i^- + \int_{-\frac{h^l}{2}}^{\frac{h^l}{2}} \vec{P}^l dz, \\ \vec{M}_i &= \frac{h^l}{2} \{ \vec{\chi}_i^+ - \vec{\chi}_i^- \} + \int_{-\frac{h^l}{2}}^{\frac{h^l}{2}} \vec{P}^l Z^l dz \\ \vec{B}_i &= \varphi^l \{ \frac{h^l}{2} \} \{ \vec{\chi}_i^+ - \vec{\chi}_i^- \} + \int_{-\frac{h^l}{2}}^{\frac{h^l}{2}} \vec{P}^l \varphi^l(z) dz \end{aligned} \quad (8)$$

Where are the vectors $\vec{\chi}_i^+ - \vec{\chi}_i^-$, include contravariant components of the contact stress tensor $\vec{\sigma}_i^{i3+}, \vec{\sigma}_i^{i3-}$ (($i = 1, 2$)).

$$\begin{aligned} \vec{\chi}_i^+ &= \sigma_i^{i3+} \vec{\rho}_i^{l*} + \sigma_i^{33+} \vec{m}_i^{l*}, \\ \vec{\chi}_i^- &= \sigma_i^{i3-} \vec{\rho}_i^{l*} + \sigma_i^{33-} \vec{m}_i^{l*}, \quad ((i = 1, 2)) \end{aligned} \quad (9)$$

Include contravariant components of the contact stress tensor in expressions (8) and (9), the indexes $\ll + \gg$ and $\ll - \gg$ indicate the upper code and lower code front surfaces of the layer. There is an external load of similar vectors \vec{q}_i^+, \vec{q}_i^- , as in the following relationships:

$$\begin{aligned} \vec{q}_n^+ &= q_n^{i3+} \vec{\rho}_i^{l*} + q_n^{33+} \vec{m}_i^{l*}, \\ \vec{q}_1^- &= q_1^{i3-} \vec{\rho}_i^{l*} + q_1^{33-} \vec{m}_i^{l*}, \quad ((i = 1, 2)), \end{aligned}$$

Vector \vec{P}_i , the effect of weight is taken into account.

Formula (7) contains force vectors $\vec{\Phi}_i^s$, moment \vec{G}_i^s , additional moment \vec{L}_i^s , which grow from the effect of given external poor-ness forces on L_1^l . Force vectors $\vec{\Phi}_i$, moment \vec{G}_i , additional moment \vec{L}_i , take place at contour points L_2^l , in the presence of a given vector of contour points displacements \vec{u}_i^l .

The following formula can be expressed for part II of Equation (4):

$$\begin{aligned} \delta \Pi_R &= \sum_{i=1}^n \{ \delta \Pi_{1R}^i + \delta \Pi_{2R}^i \} = \sum_{i=1}^n \iiint_{V^i} \{ \sigma_i^{\alpha\beta} \delta \eta_{ij}^z \} dV - \\ - \sum_{i=1}^n \iiint_{V^i} \{ \frac{\partial F^l}{\partial \sigma_i^{\alpha\beta} - \eta_{\alpha\beta}^l} - \partial \eta_i^{\alpha\beta} \} dV, \end{aligned} \quad (10)$$

Where

$$\begin{aligned} \delta \Pi_R &= \iiint_{V^i} \{ \sigma_i^{\alpha\beta} \eta_{\alpha\beta}^l \} dV = \iiint_{V^i} \{ \sigma_i^{ij} \delta \varepsilon_{\alpha\beta}^l + 2\sigma_i^{i3} \delta \varepsilon_{i3}^l + 2\sigma_i^{33} \delta \varepsilon_{33}^l \} dV, \\ \delta \Pi_{2R}^i &= \iiint_{V^i} \delta W_i^f dV = \iiint_{V^i} \{ [\frac{\partial F^l}{\partial \sigma_i^{ij}} - \varepsilon_{ij}^z] \delta \sigma_i^{i3} + \\ + [\frac{\partial F^l}{\partial \sigma_i^{i3}} - 2\varepsilon_{i3}^z] \delta \sigma_i^{i3} + [\frac{\partial F^l}{\partial \sigma_i^{33}} - \varepsilon_{33}^z] \delta \sigma_i^{33} \} dV, \\ ((i = 1, 2)). \end{aligned}$$

The part II of equation (4) should be represented as substituting the geometric relations (3) into (4), (7), (10) on the basis of the Reissner variation principle, it is easy to obtain per layer in thin multi-layer shell for system equations in the case of equilibrium, physical relations, boundary conditions for static and kinematic. general applications of Newton's Laws, the theory nonlinear of the average bend of the thin multi-layer shell [Karash E. T. B.,

(2017)] greatly simplifies the derivation of the boundary conditions and equilibrium equations. In this work was used tensors system to transformation to the physical components, this researches, can find boundary conditions and derive equilibrium equations [Xiang X. , Guoyong J., Wanyou Li., et.al, (2014); Karash E. T. B., (2017)].

Carrying out of the conditions of static and kinematic contact (5) over facial conjugate surfaces using the penalty function method [Karash E. T. B., (2017)], it is easy to solve the problem of communication theory of discrete-structure of the thin walls structures. If between the layers of the thin wall shell to allow the absence of kinematic relations, then on the interface of the interface of these layers $S_z^{i,i+1}$, unknown force vectors may occur $\vec{q}_i = \vec{q}_{i+1}$, contact interaction. Take into consideration the influence of the contact forces of the thin multilayer shell layers, A term must be introduced taking into account the influence of contact forces between layers [Xiang X. , Guoyong J., Wanyou Li., et.al, (2014)] in the variation equation of the Reissner principle (4).

3. Results and Discussions

On the basis of the stated approach, the stress state of cylinders made of fiberglass with a diameter of (90 mm) and a length of (100 mm) under the action of internal pressure was investigated. Cylinders of the 1st type were made of four layers of fiberglass type TG 430 - C (100) (manufacturer - Latvia). A polyester ortho-phthalic resin, low emission of Crislic 2 - 446 PA (manufacturer - United Kingdom). Cylinders of the type 2nd differ from cylinders of the type 1st by the incipient defect in the form of a non-glue section in the form of a ring in the center, which was between the second and third layer. Non-glue areas were created at the time of sample preparation using a polyethylene.

The physic - mechanical characteristics of the fiberglass shells in question were specified in the following concatenation. First, according to the standard GOST 25.601 - 80, the modulus of elasticity and the Poisson's ratio for tensile specimens of fiberglass are determined. The mechanical tests carried out suggest material of the thin plates under consideration can be classified as athwart isotropic $E_z = E_\theta = 1.55 \cdot 10^4 \text{ MPa}$, $\nu_{\theta z} = \nu_{z\theta} = 0.133$. The remaining physicommechanical characteristics of fiberglass were determined fully for the integral package of thin multilayer shell layers based on the dependencies of [Karash E. T. B., (2017)], when the modulus of elasticity of the first kind, Poisson's coefficients of the fibers and the matrix, respectively, are: $E_B = 7.13 \cdot 10^4 \text{ MPa}$, $E_M = 0.362 \cdot 10^4 \text{ MPa}$, $\nu_B = 0.237$, and $\nu_M = 0.362$.

For empirical studies, designed and manufactured a set of tests. Measurement of deformations was carried out using different types of strain gauges. To measure the output signals of the tensor-resistors and to report in digital form, the measuring system SIIT-3 was used.

The mathematical model the designer depends on two rigid transversely isotropic layers with a thickness of $h_1 = h_2 = 1000 \text{ mm}$. Thin layers of an adhesive bond the solid layers together $h_0 = 500 \text{ mm}$ and $h_0 = 250 \text{ mm}$, (first model) [Karash E. T. B., (2017)].

According to the 2nd model, The thickness of the adhesive interlayer was not taken effect into consideration on the plate's stress state, but elastic slip along the contact surface of adjacent layers was allowed. In this case, the solution to the problem was obtained on the basis of the orthogonal sweep method of S.K. Godunov in a geometrically nonlinear formulation.

For a qualitative assessment of the proposed model variants, the cylinder calculation using the first model was carried out in a spatial axisymmetric formulation by the finite element method (complex of the FEM No. A8 Y8.0). The model is represented by

(rectangular 8-node elements). The flaw was modeled by a domestic annular non-glue region, taking into account the connect of rigid layers.

The material properties of a rigid fiberglass layer were determined by the following parameters: $E_\theta = 1.55 \cdot 10^4 \text{ MPa}$, $E_z = 1.55 \cdot 10^4 \text{ MPa}$, $E_r = 4.13 \cdot 10^4 \text{ MPa}$, $\nu_{\theta r} = 0.253$, $\nu_{zr} = 0.253$, $\nu_{\theta z} = 0.133$, $G_{zr} = 1.803 \cdot 10^3 \text{ MPa}$, $G_{\theta r} = 1.803 \cdot 10^3 \text{ MPa}$, $G_{\theta z} = 6.133 \cdot 10^3 \text{ MPa}$. Glue was considered an isotropic material: $E = 3.58 \cdot 10^3 \text{ MPa}$, $\nu = 0.353$.

The results of studies of cylinders of the 1st and 2nd type without a defect in the structure of the material and with a local non-glue section in the form of a ring of length $L_d = 500 \text{ mm}$, in the center of the plate are presented in Fig. 1-6. Cylinders, the ends of which are rigidly fixed, experience the action of the uniform internal pressure of intensity $q = 1.11 \text{ MPa}$.

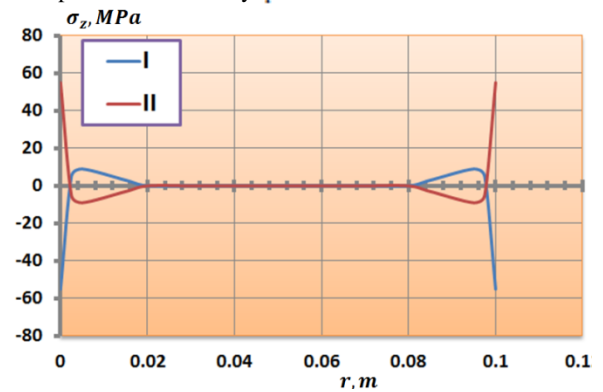


Fig. 1: The dependence of the normal stress σ_z on the front surfaces cylinder 1st type (second model) from the coordinate z (I, II - outer and inner surface, respectively; ° - results experiment).

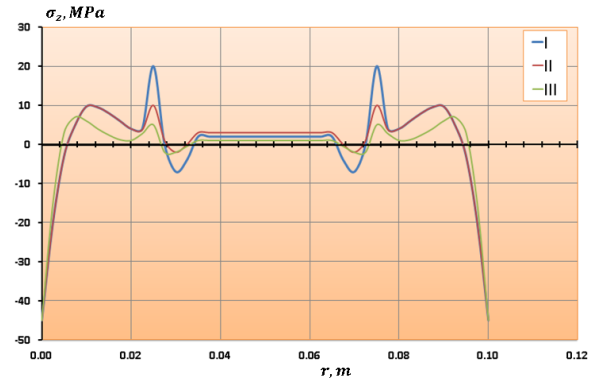


Fig. 2: Dependencies of normal stresses σ_z on the outer surface of a cylinder of 2nd type from the coordinate z (I, II is a two-layer plate with an adhesive layer of thickness $h_0 = 500 \text{ mm}$ and $h_0 = 250 \text{ mm}$, (first model) respectively; III - double - layer plate (second model)).

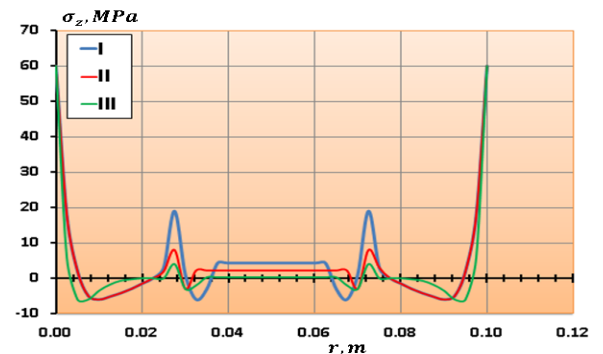


Fig. 3: Dependencies of normal stresses σ_{zr} on the outer surface of a cylinder of 2nd type from the coordinate z (I, II is a two-layer plate with

an adhesive layer of thickness $h_0 = 500 \text{ mm}$ and $h_0 = 250 \text{ mm}$, (first model) respectively; **III** – double - layer plate (second model)).

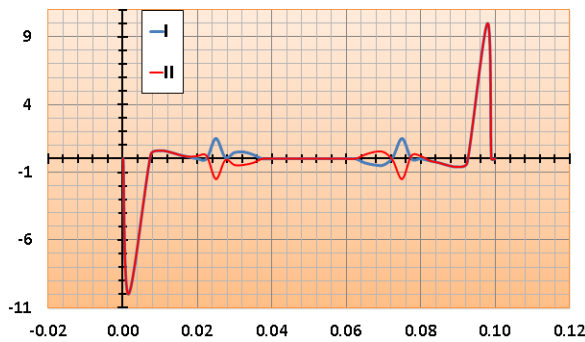


Fig. 4: Dependence of transverse shear stresses σ_{zr} on mated surfaces of a cylinder of the type 2nd from the coordinate z (**I**, **II** surfaces of the inner and outer layer, respectively).

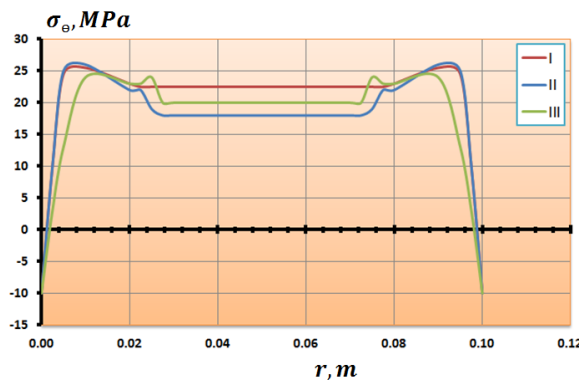


Fig. 5: Dependencies of normal stresses σ_θ on the outer surface of a cylinder of 2nd type from the coordinate z (**I** is a two-layer plate with an adhesive layer of thickness $h_0 = 500 \text{ mm}$ and $h_0 = 250 \text{ mm}$, (second model) respectively; **II**, **III** - double - layer plate first model).

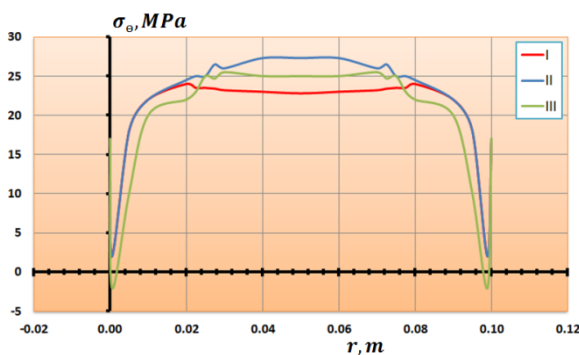


Fig.6: Dependencies of normal stresses σ_θ on the outer surface of a cylinder of 2nd type from the coordinate z (**I** is a two-layer plate with an adhesive layer of thickness $h_0 = 500 \text{ mm}$ and $h_0 = 250 \text{ mm}$, (second model) respectively; **II**, **III** - double - layer plate first model).

4. Conclusion

thus, in this work, the state of stress-strain anisotropic cylindrical shells with defects in the structure of the material as structural elements of high-pressure vessels has been studied on the basis of a geometrically theory of nonlinear discrete-structural of multilayers structure elements. the conjugation of rigid anisotropic layers at interphase boundaries is modeled by three computational models, which take into account the conditions of their ideal and weakened contact. the state of stress-strain of transversely isotropic cylinders without a defect in the structure of the material and with a local portion of the annular non-glue is studied numerically and experimentally. during the analysis, it was established that a change in the conditions static and kinematic of

contact over the mating surfaces of rigid layers of anisotropic thin multilayer elements structures significantly influence the nature of the distribution of influence of the compression stress and transverse shear stress deformations. the model different, when the compression stress and transverse shear stress deformations at the interphase boundaries of the contact are equal to some, but elastic slippage of these layers relative to each other is allowed, adequately be inverted the work large deformations of multi-layered laminate walls

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