

Common Fixed Points of Weakly Compatible Mappings in ε -Chainable Intuitionistic Fuzzy Metric Spaces

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Abstract

The aim of this paper is to introduce the notion of ε -chainable intuitionistic fuzzy metric spaces and prove a common fixed point theorem for four weakly compatible mappings in this newly defined space. Our result is intuitionistic fuzzy version of Cho and Jung's [1] and Mukherjee's [9] results in ε -chainable fuzzy metric space.

Keywords: *Intuitionistic fuzzy metric space, ε -chainable intuitionistic fuzzy metric space, compatible mappings, weakly compatible mappings, common fixed point.*

1 Introduction

In 1965, the concept of fuzzy sets was introduced by Zadeh [15]. Many authors have introduced the concept of fuzzy metric space in different ways ([2], [3], [6], [8]). George and Veeramani [3] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [8] and defined a Hausdorff topology on this fuzzy metric space. Jungck [7] gave the more generalized concept compatibility than commutativity and weak commutativity in metric space and proved common fixed point theorems.

Singh and Chauhan [13] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sense of George and Veeramani [3]. Park [10] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t -norm and continuous t -conorm as a generalization of fuzzy metric space due to George and Veeramani [3].

Recently, Cho and Jung [1] introduced the concept of ε -chainable fuzzy metric space and proved common fixed point theorems for four weakly compatible mappings of ε -chainable fuzzy metric space. In this paper, we introduce the concept of ε -chainable intuitionistic fuzzy metric space and prove common fixed point theorems for four weakly compatible mappings in this newly defined space. Our result is intuitionistic fuzzy version of Cho and Jung's [1], and Mukherjee's [9] results in ε -chainable fuzzy metric space.

2 Preliminary Notes

Definition 2.1 ([12]). A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -norm if it satisfies the following conditions:

- (1) $*$ is associative and commutative,
- (2) $*$ is continuous,
- (3) $a * 1 = a$ for all $a \in [0, 1]$,
- (4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Example 2.2. Two typical examples of continuous t -norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 2.3 . A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -conorm if it satisfies the following conditions :

- (1) \diamond is associative and commutative,
- (2) \diamond is continuous,
- (3) $a \diamond 0 = a$ for all $a \in [0, 1]$,
- (4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Example 2.4. Two typical examples of continuous t -conorm are $a \diamond b = \min(a+b, 1)$ and $a \diamond b = \max(a, b)$.

Definition 2.5 [107]. A 5-tuple $(X, M, N, *, \diamond)$ is called a intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t -norm, \diamond a continuous t -conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$, satisfying the following conditions:

for each $x, y, z \in X$ and $t, s > 0$,

$$(a) M(x, y, t) + N(x, y, t) \leq 1,$$

- (b) $M(x, y, t) > 0$,
- (c) $M(x, y, t) = 1$ if and only if $x = y$,
- (d) $M(x, y, t) = M(y, x, t)$,
- (e) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
- (f) $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.
- (g) $N(x, y, t) > 0$,
- (h) $N(x, y, t) = 0$ if and only if $x = y$,
- (i) $N(x, y, t) = N(y, x, t)$,
- (j) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
- (k) $N(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

Then (M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t , respectively.

Remark 2.6. Every fuzzy metric space (X, M, \cdot) is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \diamond)$ such that t -norm $*$ and t -conorm \diamond are associated, i.e., $x \diamond y = 1 - [(1 - x) * (1 - y)]$ for any $x, y \in X$.

Example 2.7. (Induced intuitionistic fuzzy metric)

Let (X, d) be a metric space. Denote $a * b = ab$ and $a \diamond b = \min(a + b, 1)$ for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{ht^n}{ht^n + md(x, y)}$$

$$N_d(x, y, t) = \frac{d(x, y)}{kt^n + md(x, y)}$$

for all $h, k, m, n \in \mathbb{R}^+$. Then $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric space.

Definition 2.8. A sequence $\{x_n\}$ in a intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ converges to x if and only if $M(x_n, x, t) \rightarrow 1$ and $N(x_n, x, t) \rightarrow 0$ as $n \rightarrow \infty$, for each $t > 0$.

It is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ and $N(x_n, x_m, t) < \varepsilon$ for each $n, m \geq n_0$.

An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if every cauchy sequence is convergent.

Definition 2.9. Let A and B be mappings from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ in to itself. The mappings A and B are said to be compatible if

$$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1,$$

$$\lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z \text{ for some } z \in X.$$

Definition 2.10. Self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be weakly compatible if $ABx = BAx$ when $Ax = Bx$ for some $x \in X$.

It is easy to see that if self mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are compatible then they are weakly compatible.

The following example shows that the converse of above statement does not hold.

Example 2.11. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, Where $X = [0, 2]$ with t -norm and t -conorm defined by $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$, for all $a, b \in [0, 1]$ and

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

For all $t > 0$ and $M_d(x, y, t) = 0$ and $N_d(x, y, t) = 1$, for all $x, y \in X$.

Define self maps A and B on X as follows:

$$Ax = \begin{cases} 2 & \text{if } 0 \leq x \leq 1 \\ \frac{x}{2} & \text{if } 1 < x \leq 2 \end{cases}$$

$$Bx = \begin{cases} 2 & \text{if } x = 1 \\ \frac{x+3}{5} & \text{otherwise} \end{cases}$$

And $x_n = 2 - \frac{1}{(2n)}$. Then we have

$$A(1) = B(1) = 2 \text{ and } A(2) = B(2) = 1.$$

Also $AB(1) = BA(1) = 2$. Thus (A, B) is weak compatible. Again

$$Ax_n = 1 - \frac{1}{4n}, Bx_n = 1 - \frac{1}{10n}$$

$$\text{Thus } Ax_n = 1, Bx_n = 1$$

Hence $z = 1$.

$$\text{Further } ABx_n = 2, BAx_n = \frac{4}{5} - \frac{1}{20n}$$

$$\text{Now } \lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = \lim_{n \rightarrow \infty} M(2, \frac{4}{5} - \frac{1}{20n}, t) = \frac{t}{t + \frac{6}{5}} \neq 1,$$

$$\lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = \lim_{n \rightarrow \infty} N(2, \frac{4}{5} - \frac{1}{20n}, t) = \frac{\frac{6}{5}}{t + \frac{6}{5}} \neq 0, t > 0$$

Hence (A, B) is not compatible.

Definition 2.12. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\varepsilon > 0$. A finite sequence $x = x_0, x_1, \dots, x_n = y$ is called ε -chain from x to y if $M(x_i, x_{i-1}, t) > 1 - \varepsilon$ and $N(x_i, x_{i-1}, t) < \varepsilon$ for all $t > 0$ and $i = 1, 2, \dots, n$.

An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is called ε -chainable if for any $x, y \in X$, there exists an ε -chain from x to y .

Lemma 2.13([5], [11]). In intuitionistic fuzzy metric space $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

In all that follows N stands for the set of natural numbers and X stands for an intuitionistic fuzzy metric space X with the following properties :

$$\lim_{t \rightarrow \infty} M(x, y, t) = 1, \lim_{t \rightarrow \infty} N(x, y, t) = 0 \text{ for all } x, y \in X. \quad (I)$$

Lemma 2.14 [14]. If X is an intuitionistic fuzzy metric space with $s * s \geq s$, $r \diamond r \leq r$ for all $s, r \in [0, 1]$ and if $\{x_n\}$ is a sequence in X such that

$$M(x_{i+1}, x_{i+2}, kt) \geq M(x_i, x_{i+1}, t) * M(x_{i+1}, x_{i+2}, t),$$

$$N(x_{i+1}, x_{i+2}, kt) \leq N(x_i, x_{i+1}, t) \diamond N(x_{i+1}, x_{i+2}, t),$$

Where $0 < k < 1$, $i = 0, 1, 2, \dots$ and $t > 0$, then

$$M(x_{i+1}, x_{i+2}, kt) \geq M(x_i, x_{i+1}, t),$$

$$N(x_{i+1}, x_{i+2}, kt) \leq N(x_i, x_{i+1}, t).$$

Lemma 2.15. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ and*

$N(x, y, qt) \leq N(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.

Proof. It is immediate from (I) and Definition 3.

3 Main Results

These are the main results of the paper.

Theorem 3.1. *Let $(X, M, N, *, \diamond)$ be a complete ε -chainable intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:*

- (1) $AX \subset TX$ and $BX \subset SX$,
- (2) A and S are continuous,
- (3) the pairs $[A, S]$ and $[B, T]$ are weakly compatible,
- (4) there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq \{M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)\},$$

$$N(Ax, By, qt) \leq \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(Ax, Ty, t)\}$$

for every $x, y \in X$ and $t > 0$.

Then A, B, S and T have a unique common fixed point in X .

Proof : Since $AX \subset TX$ and $BX \subset SX$, for any $x_0 \in X$, there exists $x_1 \in X$ such that $Ax_0 = Tx_1$ and for this $x_1 \in X$, there exists $x_2 \in X$ such that $Bx_1 = Sx_2$.

Inductively, we can find a sequence $\{y_n\}$ in X as follows:

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \text{ and } y_{2n} = Sx_{2n} = Bx_{2n-1} \text{ for } n = 1, 2, \dots$$

Step 1. By taking $x = x_{2n}$ and $y = x_{2n+1}$ in (4), we have

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, qt) &= M(Ax_{2n}, Bx_{2n+1}, qt) \\ &\geq \{M(Sx_{2n}, Tx_{2n+1}, t) * M(Ax_{2n}, Sx_{2n}, t) \\ &\quad * M(Bx_{2n+1}, Tx_{2n+1}, t) * M(Ax_{2n}, Tx_{2n+1}, t)\} \end{aligned}$$

$$\begin{aligned}
&= \{M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n+2}, y_{2n+1}, t) \\
&\quad * M(y_{2n+1}, y_{2n+1}, t)\} \\
&\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t). \\
N(y_{2n+1}, y_{2n+2}, qt) &= N(Ax_{2n}, Bx_{2n+1}, qt) \\
&\leq \{N(Sx_{2n}, Tx_{2n+1}, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \\
&\quad \diamond N(Bx_{2n+1}, Tx_{2n+1}, t) \diamond N(Ax_{2n}, Tx_{2n+1}, t)\} \\
&= \{N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n}, t) \diamond N(y_{2n+2}, y_{2n+1}, t) \\
&\quad \diamond N(y_{2n+1}, y_{2n+1}, t)\} \\
&\leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t).
\end{aligned}$$

From lemma 2.14 and 2.15 , we have that

$$\begin{aligned}
M(y_{2n+1}, y_{2n+2}, qt) &\geq M(y_{2n}, y_{2n+1}, t) \\
N(y_{2n+1}, y_{2n+2}, qt) &\leq N(y_{2n}, y_{2n+1}, t).
\end{aligned} \tag{3.1}$$

Similarly, we have also

$$\begin{aligned}
M(y_{2n+2}, y_{2n+3}, qt) &\geq M(y_{2n+1}, y_{2n+2}, t) \\
N(y_{2n+2}, y_{2n+3}, qt) &\leq N(y_{2n+1}, y_{2n+2}, t).
\end{aligned} \tag{3.2}$$

From (3.1) and (3.2), we have

$$\begin{aligned}
M(y_{n+1}, y_{n+2}, qt) &\geq M(y_n, y_{n+1}, t) \\
N(y_{n+1}, y_{n+2}, qt) &\leq N(y_n, y_{n+1}, t).
\end{aligned} \tag{3.3}$$

From (3.3),

$$\begin{aligned}
M(y_n, y_{n+1}, t) &\geq M(y_n, y_{n-1}, t/q) \geq M(y_{n-2}, y_{n-1}, t/q^2) \\
&\geq \cdots \geq M(y_1, y_2, t/q^n) \rightarrow 1 \\
N(y_n, y_{n+1}, t) &\leq N(y_n, y_{n-1}, t/q) \leq N(y_{n-2}, y_{n-1}, t/q^2) \\
&\leq \cdots \leq N(y_1, y_2, t/q^n) \rightarrow 0 \text{ as } n \rightarrow \infty.
\end{aligned}$$

So $M(y_n, y_{n+1}, t) \rightarrow 1$, $N(y_n, y_{n+1}, t) \rightarrow 0$ as $n \rightarrow \infty$ for any $t > 0$.

For each $\varepsilon > 0$ and each $t > 0$, we can choose $n_0 \in \mathbb{N}$ such that

$$M(y_n, y_{n+1}, t) > 1 - \varepsilon, N(y_n, y_{n+1}, t) < \varepsilon \text{ for all } n > n_0.$$

For $m, n \in \mathbb{N}$, we suppose $m \geq n$. Then we have that

$$\begin{aligned}
M(y_n, y_m, t) &\geq M(y_n, y_{n+1}, t/m - n) * M(y_{n+1}, y_{n+2}, t/m - n) \\
&\quad * \cdots * M(y_{m-1}, y_m, t/m - n) \\
&> \overbrace{(1 - \varepsilon) * (1 - \varepsilon) * \cdots * (1 - \varepsilon)}^{m-n} \geq 1 - \varepsilon,
\end{aligned}$$

$$\begin{aligned}
N(y_n, y_m, t) &\leq N(y_n, y_{n+1}, t/m - n) \diamond N(y_{n+1}, y_{n+2}, t/m - n) \\
&\quad \diamond \cdots \diamond N(y_{m-1}, y_m, t/m - n) \\
&\leq \overbrace{\varepsilon \diamond \varepsilon \diamond \cdots \diamond \varepsilon}^{m-n} \leq \varepsilon
\end{aligned}$$

and hence $\{y_n\}$ is a Cauchy sequence in X .

From completeness, $y_n \rightarrow z$ for some $z \in X$, and so $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z .

Since X is ε -chainable, there exists ε -chain from x_n to x_{n+1} , that is, there exists a finite sequence $x_n = y_1, y_2, \cdots, y_\lambda = x_{n+1}$ such that

$$M(y_i, y_{i-1}, t) > 1 - \varepsilon, N(y_i, y_{i-1}, t) < \varepsilon \text{ for all } t > 0 \text{ and } i = 1, 2, \cdots, \lambda.$$

Thus we have

$$\begin{aligned}
M(x_n, x_{n+1}, t) &\geq M(y_1, y_2, t/\lambda) * M(y_2, y_3, t/\lambda) * \cdots * M(y_{\lambda-1}, y_\lambda, t/\lambda) \\
&> (1 - \varepsilon) * (1 - \varepsilon) * \cdots * (1 - \varepsilon) \geq 1 - \varepsilon
\end{aligned}$$

$$\begin{aligned}
N(x_n, x_{n+1}, t) &\leq N(y_1, y_2, t/\lambda) \diamond N(y_2, y_3, t/\lambda) \diamond \cdots \diamond N(y_{\lambda-1}, y_\lambda, t/\lambda) \\
&< \varepsilon \diamond \varepsilon \diamond \cdots \diamond \varepsilon \leq \varepsilon.
\end{aligned}$$

$$\begin{aligned}
\text{For } m > n, M(x_n, x_m, t) &\geq M(x_n, x_{n+1}, t/m-n) * M(x_{n+1}, x_{n+2}, t/m-n) \\
&\quad * \cdots * M(x_{m-1}, x_m, t/m-n) \\
&> (1-\varepsilon) * (1-\varepsilon) * \cdots * (1-\varepsilon) > 1-\varepsilon
\end{aligned}$$

$$\begin{aligned}
N(x_n, x_m, t) &\leq N(x_n, x_{n+1}, t/m-n) \diamond N(x_{n+1}, x_{n+2}, t/m-n) \diamond \cdots \diamond N(x_{m-1}, x_m, t/m-n) \\
&< \varepsilon \diamond \varepsilon \diamond \cdots \diamond \varepsilon < \varepsilon
\end{aligned}$$

and so $\{x_n\}$ is a Cauchy sequence in X and hence there exists $x \in X$ such that $x_n \rightarrow x$.

From (2), $Ax_{2n-2} \rightarrow Ax$ and $Sx_{2n} \rightarrow Sx$. Since X is Hausdorff, $Ax = z = Sx$.

Because $[A, S]$ is weakly compatible, $ASx = SAx$ and so $Az = Sz$.

From (2), $ASx_{2n} \rightarrow ASx$ and hence $ASx_{2n} \rightarrow Sz$. Also, from continuity of S , $SSx_{2n} \rightarrow Sz$.

Step 2. By taking $x = Sx_{2n}$ and $y = x_{2n-1}$ in (4), we have

$$\begin{aligned}
M(ASx_{2n}, Bx_{2n-1}, qt) &\geq \{M(SSx_{2n}, Tx_{2n-1}, t) * M(ASx_{2n}, SSx_{2n}, t) \\
&\quad * M(Bx_{2n-1}, Tx_{2n-1}, t) * M(ASx_{2n}, Tx_{2n-1}, t)\} \\
N(ASx_{2n}, Bx_{2n-1}, qt) &\leq \{N(SSx_{2n}, Tx_{2n-1}, t) \diamond N(ASx_{2n}, SSx_{2n}, t) \\
&\quad \diamond N(Bx_{2n-1}, Tx_{2n-1}, t) \diamond N(ASx_{2n}, Tx_{2n-1}, t)\}.
\end{aligned}$$

Taking limit as $n \rightarrow \infty$

$$\begin{aligned} M(Sz, z, qt) &\geq \{M(Sz, z, t) *M(Sz, Sz, t) *M(z, z, t) *M(Sz, z, t)\} \\ &\geq M(Sz, z, t) \\ N(Sz, z, qt) &\leq \{N(Sz, z, t) \diamond N(Sz, Sz, t) \diamond N(z, z, t) \diamond N(Sz, z, t)\} \\ &\leq N(Sz, z, t). \end{aligned}$$

From lemma 2.15, we get $Sz = z$, and hence

$$Az = Sz = z. \tag{3.4}$$

Since $AX \subset TX$, there exists $v \in X$ such that $Tv = Az = z$.

Step 3. By taking $x = x_{2n}$ and $y = v$ in (4), we have

$$\begin{aligned} M(Ax_{2n}, Bv, qt) &\geq M(Sx_{2n}, Tv, t) *M(Ax_{2n}, Sx_{2n}, t) \\ &\quad *M(Bv, Tv, t) *M(Ax_{2n}, Tv, t) \\ N(Ax_{2n}, Bv, qt) &\leq N(Sx_{2n}, Tv, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \\ &\quad \diamond N(Bv, Tv, t) \diamond N(Ax_{2n}, Tv, t). \end{aligned}$$

Letting $n \rightarrow \infty$, we have

$$\begin{aligned} M(z, Bv, qt) &\geq M(z, Tv, t) *M(z, z, t) *M(Bv, Tv, t) *M(z, Tv, t) \\ &= M(z, z, t) *M(z, z, t) *M(Bv, z, t) *M(z, z, t) \\ &\geq M(Bv, z, t) \\ N(z, Bv, qt) &\leq N(z, Tv, t) \diamond N(z, z, t) \diamond N(Bv, Tv, t) \diamond N(z, Tv, t) \\ &= N(z, z, t) \diamond N(z, z, t) \diamond N(Bv, z, t) \diamond N(z, z, t) \\ &\leq N(Bv, z, t) \end{aligned}$$

and so $Bv = z$ and hence $Tv = Bv = z$. Since $[B, T]$ is weakly compatible,

$$TBv = BTv \text{ and hence } Tz = Bz. \tag{3.5}$$

Step 4. By taking $x = x_{2n}$ and $y = z$ in (4), we have

$$\begin{aligned} M(Ax_{2n}, Bz, qt) &\geq \{M(Sx_{2n}, Tz, t) *M(Ax_{2n}, Sx_{2n}, t) *M(Bz, Tz, t) \\ &\quad *M(Ax_{2n}, Tz, t)\} \\ N(Ax_{2n}, Bz, qt) &\leq \{N(Sx_{2n}, Tz, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \diamond N(Bz, Tz, t) \\ &\quad \diamond N(Ax_{2n}, Tz, t)\} \end{aligned}$$

which implies that taking limit as $n \rightarrow \infty$

$$\begin{aligned} M(z, Bz, qt) &\geq \{M(z, Tz, t) *M(z, z, t) *M(Bz, Tz, t) *M(z, Tz, t)\} \\ &= \{M(z, Bz, t) *M(z, z, t) *M(Bz, Bz, t) *M(z, Bz, t)\} \\ &\geq M(z, Bz, t) \end{aligned}$$

$$\begin{aligned}
N(z, Bz, qt) &\leq \{N(z, Tz, t) \diamond N(z, z, t) \diamond N(Bz, Tz, t) \diamond N(z, Tz, t)\} \\
&= \{N(z, Bz, t) \diamond N(z, z, t) \diamond N(Bz, Bz, t) \diamond N(z, Bz, t)\} \\
&\leq N(z, Bz, t)
\end{aligned}$$

which implies that $Bz = z$. (3.6)

From (3.4), (3.5) and (3.6), A, B, S and T have common fixed point z in X .

For uniqueness, let w be another common fixed point of A, B, S and T .

$$\begin{aligned}
\text{Then } M(z, w, qt) &= M(Az, Bw, qt) \\
&\geq \{M(Sz, Tw, t) * M(Az, Sz, t) * M(Bw, Tw, t) * M(Az, Tw, t)\} \\
&\geq M(z, w, t)
\end{aligned}$$

$$\begin{aligned}
N(z, w, qt) &= N(Az, Bw, qt) \\
&\leq \{N(Sz, Tw, t) \diamond N(Az, Sz, t) \diamond N(Bw, Tw, t) \diamond N(Az, Tw, t)\} \\
&\leq N(z, w, t).
\end{aligned}$$

From lemma 2.14, $z = w$.

Corollary 3.2. *Let $(X, M, N, *, \diamond)$ be a complete ε -chainable intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying (1) - (3) of theorem 3.1 and there exists $q \in (0, 1)$ such that*

$$\begin{aligned}
M(Ax, By, qt) &\geq \{M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) \\
&\quad * M(By, Ty, t) * M(Ty, Ax, t)\},
\end{aligned}$$

$$\begin{aligned}
N(Ax, By, qt) &\leq \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Sx, By, 2t) \\
&\quad \diamond N(By, Ty, t) \diamond N(Ty, Ax, t)\}
\end{aligned}$$

for every $x, y \in X$ and $t > 0$.

Then A, B, S and T have a unique common fixed point in X .

Proof. From definition, we have that

$$\begin{aligned}
&\{M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t)\} \\
&\geq \{M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Sx, Ty, t) \\
&\quad * M(Ty, By, t) * M(Ax, Ty, t)\} \\
&\geq \{M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)\}
\end{aligned}$$

and

$$\begin{aligned}
&\{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, 2t) \diamond N(Ax, Ty, t)\} \\
&\leq \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(Sx, Ty, t) \diamond \\
&\quad N(Ty, By, t) \diamond N(Ax, Ty, t)\}
\end{aligned}$$

$\leq \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(Ax, Ty, t)\}$ hence, from theorem 3.1, A,B, S and T have a unique fixed point in X.

Corollary 3.3. *Let $(X, M, N, *, \diamond)$ be a complete ε -chainable intuitionistic fuzzy metric space and let A,B, S and T be self mappings of X satisfying (1) - (3) of theorem 3.1 and there exists $q \in (0, 1)$ such that*

$$M(Ax, By, qt) \geq M(Sx, Ty, t), N(Ax, By, qt) \leq N(Sx, Ty, t)$$

for every $x, y \in X$ and $t > 0$.

Then A,B, S and T have a unique common fixed point in X.

Proof. We have that

$$\begin{aligned} M(Sx, Ty, t) &= M(Sx, Ty, t) * 1 \\ &= M(Sx, Ty, t).M(Ax, Ax, 5t) \\ &\geq \{M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) \\ &\quad * M(By, Ty, t) * M(Ty, Ax, t)\} \end{aligned}$$

$$\begin{aligned} N(Sx, Ty, t) &= N(Sx, Ty, t) \diamond 0 \\ &= N(Sx, Ty, t) \diamond N(Ax, Ax, 5t) \\ &\geq \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Sx, By, 2t) \diamond N(By, Ty, t) \\ &\quad \diamond N(Ty, Ax, t)\} \end{aligned}$$

and hence, from corollary 3.2, A,B, S and T have a unique fixed point in X.

Let S and T be the identity mapping on X in corollary 3.3 Then we get the next result.

Corollary 3.4. *Let $(X, M, N, *, \diamond)$ be a complete ε -chainable intuitionistic fuzzy metric space and let A and B be self mappings of X satisfying the following condition: there exists $q \in (0, 1)$ such that*

$$M(Ax, By, qt) \geq M(x, y, t), N(Ax, By, qt) \leq N(x, y, t),$$

for every $x, y \in X$ and $t > 0$.

Then A and B have a unique common fixed point in X.

In corollary 3.4, if we take $A = B$, then this result becomes to intuitionistic fuzzy Banach contraction theorem.

4 Conclusion

In this work we have dealt with fixed points of four mappings in which two pairs of two mappings is weak compatible. The future scope is to obtained quartet fixed points for mappings in this space.

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