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Research paper

Study the Influence of Atmospheric Drag and J_2 Effect in a Close-proximity Operation at LEO

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Abstract

In this paper is to assess the mission stability and the influence of J_2 effect and aerodynamic forces. To maintain the relative motion of satellites by using a feedback control law for tracking error bound in the presence of J_2 perturbation. A constant relative orbit under the effect of earth oblateness and conservative forces is referred as J2 and targeting the presence of atmospheric drag. Although, Schweighart and Sedwick control strategy for satellite relative motion is considering both lift and drag forces. The simulation result shows a better performance with high accuracy than an elliptical orbit under J_2 perturbation and atmospheric drag along in-track formation. The algorithm and control strategies is useful tools for analysing a future space mission.

Index Terms- *Cartosat-2C, J_2 Effect, Atmospheric Drag, Close-Proximity, Low Earth Orbit.*

1. Introduction

Space agencies are inspired by many enabling science application and to suggest a small satellite formation flying mission (SFF) in an upcoming research direction (Healy et al 2015 and Sinclair et al., 2015). In recent research, space industries are envisaging to replace a monolithic satellite into nanosatellites to perform a convinced operation (Chamberlin et al., 1964, Koenig et al. 2016). The launch cost and fuel confines a size of a single spacecraft/satellites and it can be configured into unprecedented resolution of much larger virtual dishes (Scharf et al., 2004, Alfriend et al., 2005 and Russel., 2012.).
The fuel expenditure is not only for the constraints of mission success. It is also extending the life of spacecraft/satellites (Chamberlin et al., 1964, Bevilacqua et al., 2014). In a particular mission (CARTOSAT-2C) accomplish the objective of much larger satellite also allows more versatility and a greater redundancy. According to Koenig et al., investigated the drifting forces to many perturbation forces like non-spherical nature of Earth (J_2 effects), atmosphere drag, solar radiation pressure and some other effects for maintaining the satellite position (Koenig et al., 2016).
The Atmospheric drag forces can be quite and noteworthy in Low Earth Orbit (LEO). A drag force is acting a very large for the undesired drift. To maintain the relative position of satellites is to be hold in a certain formation geometry among the group of satellites (Chamberlin et al., 1964, Gaias et al., 2015).

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Although, aerodynamic forces are being a negative effect, it can be actuating the aerodynamic panel by using the drag for station keeping. Herewith, streamlined linearized dynamic model is developed for better understanding of both J_2 and atmospheric forces as well as formulation of stability analysis (Biria et al., 2016). The mission data has been configured through MATLAB/Simulink and obtained the numerical results are validate with real-time mission application.

2. Relative Motion Equation for Circular Orbits

In this chapter is to refer the effect of aerodynamic forces in a circular orbit. Consider the model development of $J_2$and atmospheric drag with a potential of linear equations motion. There are two steps involved for developing a linearized dynamic model 1) selecting a co-ordinate frame 2) equation of motion to be incorporate.

2.1 Co-ordinate Frame

The coordinate frames Hill (x, y, z) and Earth Centered Inertial (ECI, X, Y, Z) are used in the relative motion shows in figure 1. The Earth Centered Inertial frame is fixed in the origin at the center of the Earth and the plane X and Y is to be direct by the Earth's equator. Vernal equinox is referred as x-axis similarly y axis shows a right hand of coordinate system and z axis represent allied in Earth rotation.
According to Chamberlin the x axis is represent a radial direction, y axis represents in-track or along-track direction, and z direction represent the cross-track or out-of-plane direction (Chamberlin, 2012, Bevilacqua et al., 2008, Gaias et al., 2015). The coordinate frames are well designed for the study of relative motion analysis through the reference of Schweighart and Sedwick. (Schweighart et al., 2002).
The relative motion equation has been used extensively since 1960s for spacecraft close proximity operation modelling, with the heritage on the manned Gemini, Apollo, and Shuttle mission (Chamberlin et al., 2012, Sinclair et al., 2014, Goodman et al., 2006).

It is simplifying the effect of J_2 potential in a dynamic model. In addition, identify the effect of atmospheric drag in the CARTOSAT-2C mission, author added a modified equation. To approach the method is to carry out a preliminary stability analysis of mission. The simulation results are demonstrating the effect of J_2 perturbations and aerodynamic drag.
The author is considering a nanosatellites parameter among the group of monolithic satellites in the Cartosat-2C mission. The selected nanosatellites are travel in a pre-defined reference orbit and the deputy satellites will follow the fictitious of chief.

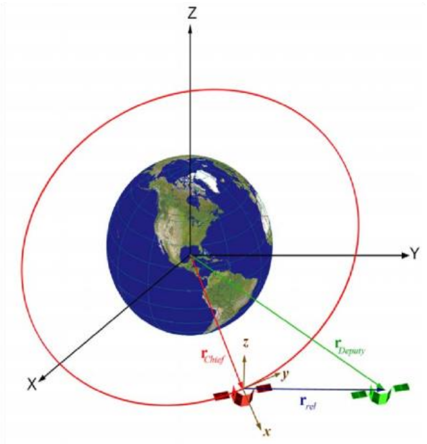

Fig 1 Hill coordinate frame and Earth Centered Inertial (ECI) Frame

The chief orbit is lies at the center of mass. Hence proved that, the orbital rate ($\dot{\theta}$) and rotating rate of chief satellites are same in the trajectory path. A very common method is follow to identify the position of a satellites by using the six quantities of orbital elements. [1, 21]. The following equation is described the state of X in the ECI reference frame.

$$X=[X\ Y\ Z\ \dot{X}\ \dot{Y}\ \dot{Z}]^T \quad [1]$$

The state of X in the ECI frame can be rewritten in the form of standard orbital elements as follows:

$$X=[a\ e\ i\ \Omega\ \omega\ f\ ]^T \quad [2]$$

2.1.1 Equation of Motion

This chapter is to study of linear dynamic motion for a circular orbit. The results of this equation represent the matrix vector form. The specified equation is used to examine the effect of drag and unperturbed reference orbit in a particular space mission. The chief and deputy satellites relative model of equation is respect with reference frame between the chief and deputy satellites. Therefore, to determine the drag effect by using the equation as given below.

$$\Delta \hat{f}_{drag} = \hat{f}_{drag,Deputy} - \hat{f}_{drag,Chief} \quad [3]$$

Noteworthy, the spacecraft characteristics as well as local atmospheric density are not necessarily the same. Generally, differential drag in the dynamical model is more challenge for arbitrary formation. It is dependence for the uncertain atmospheric density model, spacecraft geometry and orientation (Gaias et al, 2015, Gaias et al., 2016). The equation (3) is approximating in (4).

$$\Delta \hat{f}_{drag} \approx \frac{1}{2}\frac{d}{r_{ref}}\upsilon_{0,y}\left\{\left(\frac{1}{\beta}\begin{bmatrix}\upsilon_{1,x}\\ \upsilon_{0,y}+2\upsilon_{1,y}\\ \upsilon_{0,z}+\upsilon_{1,z}\end{bmatrix}\right)_{Deputy} - \left(\frac{1}{\beta}\begin{bmatrix}\upsilon_{1,x}\\ \upsilon_{0,y}+2\upsilon_{1,y}\\ \upsilon_{0,z}+\upsilon_{1,z}\end{bmatrix}\right)_{Chief}\right\} \quad [4]$$

In order to simplify the following assumption is to be considered in the first case of physical parameters, it is shown in table1. The Cartosat-2C (20 satellites) is launched among the group of twenty satellites into an elliptical orbit and the same radii consequently. The linearized differential drag expression was simplified based on the mission data.

$$\Delta \hat{f}_{drag} \approx -\frac{1}{2}\frac{1}{\beta}\left(\frac{d}{r_{ref}}\right)\upsilon_{0,y}\begin{bmatrix}\Delta\upsilon_{1,x}\\ 2\Delta\upsilon_{1,y}\\ \Delta\upsilon_{1,z}\end{bmatrix} \quad [5]$$

Hence, from the equation 5 cancelling the constant terms and defined as $\Delta\upsilon$.

$$\Delta\upsilon_{1,z} = \Delta\hat{x}' - \Delta\hat{y}(\hat{n}c-\omega_e\cos i) - \Delta\hat{z}\,\omega_e\cos\theta\sin i \quad [6]$$

$$\Delta\upsilon_{1,z} = \Delta\hat{z}' + \Delta\hat{x}\,\omega_e\cos\theta\sin i - \Delta\hat{y}\,\omega_e\sin\theta\sin i \quad [7]$$

$$\Delta\upsilon_{1,y} = \Delta\hat{y}' - \Delta\hat{x}(\hat{n}c-\omega_e\cos i) - \Delta\hat{z}\,\omega_e\sin\theta\sin i \quad [8]$$

where, $$\Delta\hat{x} = x_{Deputy} - x_{Chief}, \Delta\hat{y} = y_{deputy} - y_{chief} \quad [9]$$

The next step is involving for consider the linearized differential drag model from the reference of Schweighart and Sedwick (2002) into the equation of motion. The linearized equations are arranged and described in the relative motion between the chief and deputy satellites. The added equation is valid for circular and elliptical orbit for considering the effect of both J_2 and aerodynamic drag [Gurfil et al., 2004, Gaias et al., 2015]. These relative equations are given by:

$$\Delta\hat{y}'' - 2\hat{n}c\Delta\hat{x}' - \Delta f_{drag,_{N.D.,}y} = 0 \quad [10]$$

$$\Delta\hat{z}'' + \hat{q}^2\,\Delta\hat{z} - \Delta f_{drag,_{N.D.,}z} = 2\hat{q}\left(\frac{l}{n_0 d}\right)\cos(\hat{q}\tau+\phi) \quad [11]$$

$$\Delta\hat{x}'' - 2\hat{n}c\Delta\hat{y}'(5c^2-2)\hat{n}^2\,\Delta f_{drag,_{N.D.,}x} = 0 \quad [12]$$

The above equation is written in the matrix form as follows:

$$M\Delta\hat{x}'' + C\Delta\hat{x}' + k\hat{x} = f \quad [13]$$

where,

$$\hat{\Delta x} = \begin{bmatrix} \Delta \hat{x} \\ \Delta \hat{y} \\ \Delta \hat{z} \end{bmatrix}, \quad f = \begin{bmatrix} 0 \\ 0 \\ 2\hat{q}\left(\frac{l}{n_0 d}\right)\cos(\hat{q}\tau + \varphi) \end{bmatrix} \qquad [14]$$

Here, M is the inertia matrix and identity matrix I:

$$M = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [15]$$

the stiffness matrix K is found to be:

Table1: The orbital parameters of Cartosat-2C mission for validating in a circular orbit at Low Earth Orbit.

Test Cases	a	e	i, rad	Ω, rad	ω, rad	v , rad.	J_2 Force	Co-efficient of Drag
Chief	9467.5	0.00	97.50	18.2	136.47	45.36	0.00	0.00
Deputy (Behind 1Km)	9467.5	0.00	97.50	18.2	136.47	-0.00016	0.00	0.00
Chief	9467.5	0.00	97.50	0.34	0.17	0	1	2.3
Deputy(1km)	9467.5	0.00	97.50	0.34	0.17	-0.00003	1	2.3

$$K = \begin{bmatrix} -(5c^2-2)\hat{n}^2 & -\frac{1}{2}\frac{1}{\beta}\hat{\sigma}^2 & -\frac{1}{2}\frac{1}{\beta}\hat{\sigma}\hat{\varsigma}\cos\theta \\ \frac{1}{\beta}\hat{\sigma}^2 & 0 & \frac{1}{\beta}\hat{\sigma}\hat{\varsigma}\sin\theta \\ \frac{1}{2}\frac{1}{\beta}\hat{\sigma}\hat{\varsigma}\cos\theta & -\frac{1}{2}\frac{1}{\beta}\hat{\sigma}\hat{\varsigma}\sin\theta & \hat{q}^2 \end{bmatrix} \qquad [16]$$

and the damping matrix C is found to be:

$$K = \begin{bmatrix} \frac{1}{2}\frac{1}{\beta}\hat{\sigma} & -2\hat{n}c & 0 \\ 2\hat{n}c & \frac{1}{\beta}\hat{\sigma} & 0 \\ 0 & 0 & \frac{1}{2}\frac{1}{\beta}\hat{\sigma} \end{bmatrix} \qquad [17]$$

The system is easily adopting in the state-space form. It is noted here in the following equations

$$\Delta X^1 = A\Delta X + W \qquad [18]$$

where,

$$\Delta X = \begin{bmatrix} \Delta x \\ \Delta x^1 \end{bmatrix}; \quad A = \begin{bmatrix} 0_{3X3} & M \\ -K & -C \end{bmatrix}, \quad W = \begin{bmatrix} 0_{3X1} \\ f \end{bmatrix} \qquad [19]$$

3. Test Case in Circular Orbit

Identifying the relative problems of conservative forces and J_2 effect by using a numerical analysis. These assessments are helpful to understand both the orbit in LEO. The figure 2 shows the comparative results for a cumulative delta V vs time throughout a complete orbit in a circular and elliptical orbit.

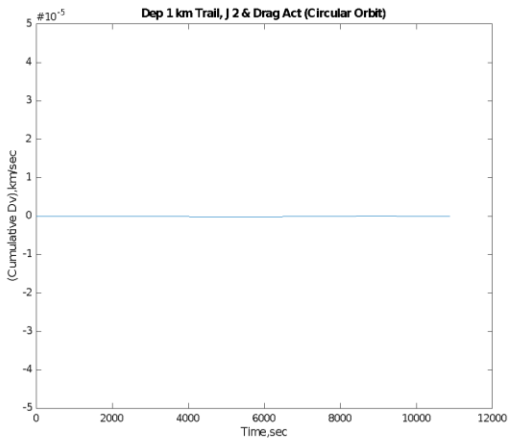

Fig 2 Total Delta V for Circular orbit.

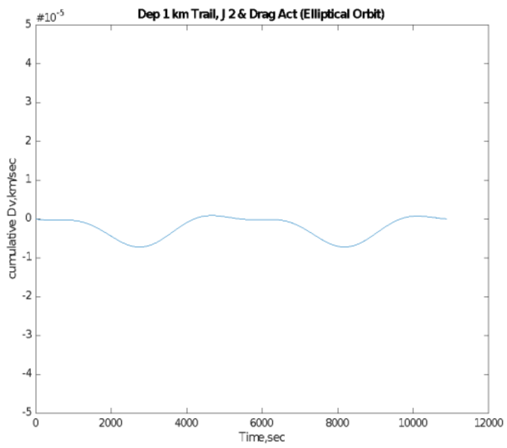

Fig 3 Total Delta V for Elliptical orbit

3.1 Test case 1: With Deputy in 1 km

The proposed dynamic model is validating with the initial condition of chief and deputy satellites in a simple test case 1. The test case is initiated in a circular orbit for the comparison of both the orbits. (eccentricity assigned as 0). In addition, assigning a deputy satellite

are behind 1km from the chief also considering the Mean anomaly (M0) of deputy is translates into -0.00015 rad. from the chief anomaly is 44.760 rad.
The mission orbit radius is assigned a value of 9468.3 km as shown in figure 4. The result of mid trace in a figure represent the velocity (Km/sec) vs true anomaly. Consequently, last trace shows a drift rate of the satellites is very linear with respect time. It is observed that, circular orbit angular rate is equal in a continuous state unlike as an eccentric orbit. The orbital inclination rate of value is 0.719 and the angular momentum in a present of perturbation is -0.0131 as shown in figure 5. The mid of graph is represent the orbit inclination (radians) vs true anomaly.

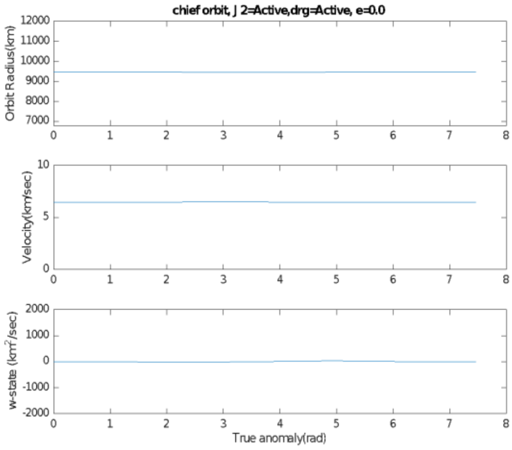

Fig 4 J2 and Drag inactive (Assesment1)

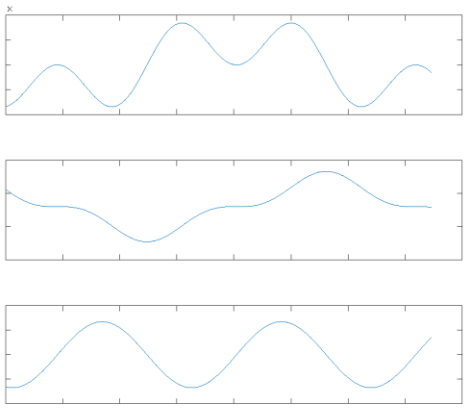

Fig 5 Perturbation effects in a circular orbit.

The chief and deputy satellites are maintaining a constant velocity and the results shows there is no perturbation act in the circular orbit. The figure 6 is indicates along track separation of deputy satellites behind 1 km from chief. In addition, shows a constant value for the separation distance vector magnitude vs true anomaly in a complete orbit.
In this assessment is conclude that, the chief and deputy satellite maintains the same radius also other following satellites have no relative velocity difference which is very anticipated results. Although, no maneuvering is required because of zero velocity difference towards each other to keep the orbit.

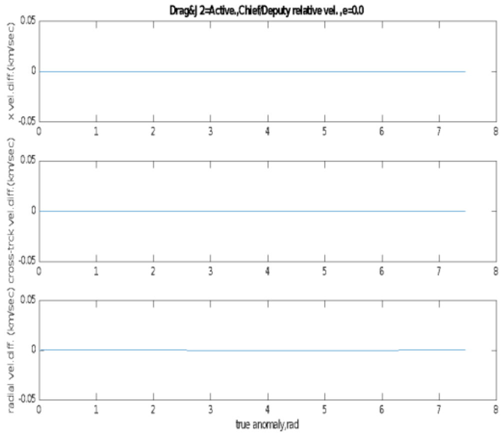

Fig 6 Maintain a Velocity and No perturbation.

3.2 Test Case 2: Active J_2 and Atmospheric Drag

The proposed dynamic model is validating with the initial condition of chief and deputy satellites in a simple test case 1. The test case 2 is also initiated in a circular orbit. (eccentricity assigned as 0). In this case activate the J_2 effect and atmospheric drag for an analysis.
The consequence of adding the effect of atmospheric drag and J_2 force in the equations of motion. Significantly, relative drift may diverge more in apogee and perigee. The figure 7 shows the radial difference between chief and deputy of circular orbit and extend to the lateral drift between the chief and deputy.
The J_2 effect variation slightly higher than the circular orbit for an elliptical by changing the angular momentum, orbit inclination and semi major axis(a). The higher orbit always having a higher angular momentum in order to cause an inclination change as shown in figure 8(a) and (b).
The purpose of investigation, to avoid perturbation and gravity gradient in low earth orbit. The Cartosat-2C mission placed all the satellites in a circular orbit instead of elliptical. In addition, observe the result in figure 9 and 10 for a Repition in J_2 disturbance pattern to maintain the velocity difference for an entire circular orbit. Indian Space Research Organization (ISRO) has considered a major parameter of fuel consumption is achieved by launching the satellites into circular.

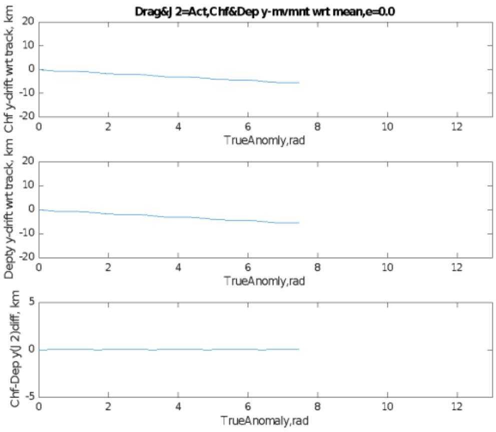

Fig 7 Lateral drift and Radial Difference

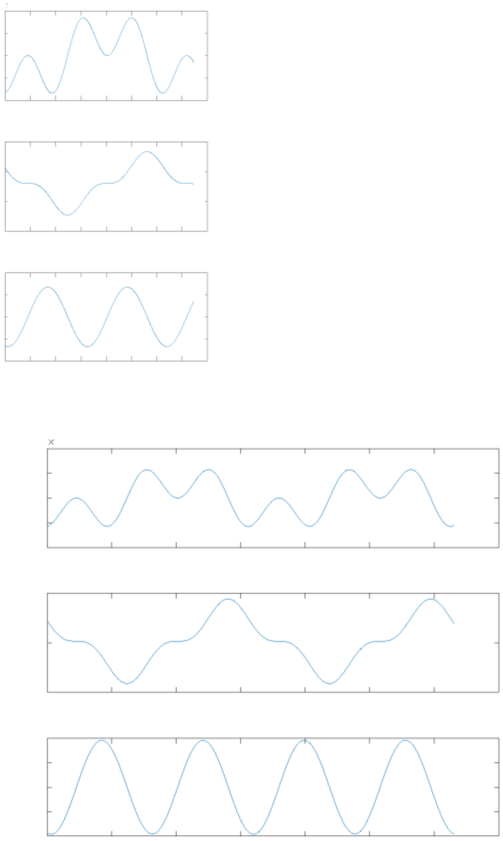

Fig 8(a) and (b) J_2 Effect in a Circular (a) and Elliptical orbit (b)

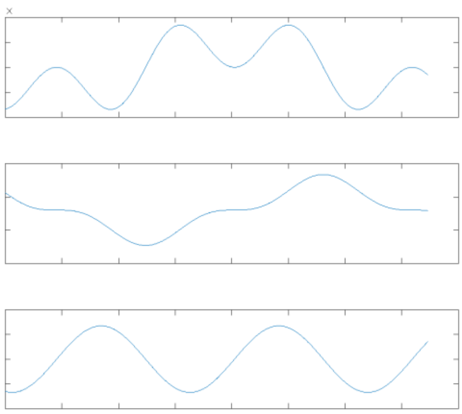

Fig.9. Repition in J_2 Disturbance Patter.

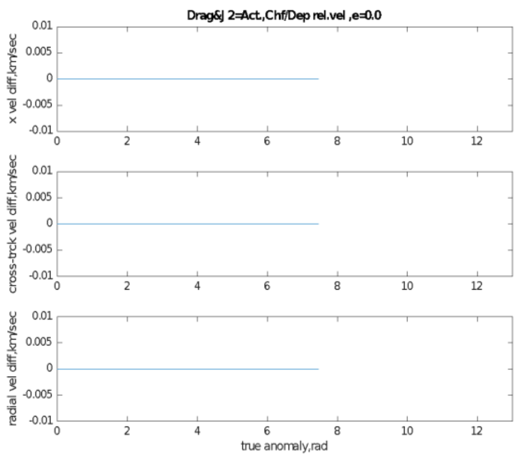

Fig 10 Maneuvered Repition in J_2 Disturbance Pattern

4. Conclusion

The proposed model is to study the stability analysis of J_2 effect and aerodynamic forces in a real-time space mission of Cartosat-2C at LEO. The simulation results are compared in both the orbit in a simple case (Circular and Elliptical). The purpose of examining the presence of atmospheric drag and gravity perturbations is to maintain the number of satellites in formation at LEO. The proposed dynamic model is significantly ease to both the collision rate and drift rate through an extensive range of reference orbits. It is with respect of inclination and altitude of 505.8 km, 0.00158 eccentricity and the inclination 97.5 degree.

The obtained results are originated for evolving a close proximity operation control strategies by defining a " delta-v " budgets. It is also help to manage a satellite in formation of satellites with respect to chief (Refer figure 2). Although, streamline a linearize dynamic model for a better understanding of both J_2 and atmospheric forces. The following findings are listed here during the effort of comparison.

1.Among the group of satellites in Cartosat-2C mission, author has select a six nano independent satellites for testing analysis. The model is defined based on the orbit radius, true anomaly, velocity, angular momentum vector and orbit inclination angle.

2. In each test cases consider for the effect of J_2 gravity perturbation and drag with and without activating of that.

3. The observe result is to establish the features of maintain the satellites in a relative motion with the presence of perturbation. In addition, caused by J2 gravity perturbation satellite which is tracks to drift off a leftward direction, about 7.40 km (Refer in figure 10), during each orbit and maintaining the same velocity in all the time.

4. Simulation results shows a better performance with high accuracy under a J_2 perturbation and atmospheric drag projected in circular and compare with an elliptical in-track formation of Cartosat-2C Satellites.

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