Direction of Arrival Estimation of Uncorrelated Signals Using Root-MUSIC Algorithm for ULAs and UCAs

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Abstract

n this paper, Root-MUSIC algorithm for direction of arrival (DOA) estimation of uncorrelated signals is explored both for uniform linear and uniform circular arrays. The basic problem in Uniform Linear Arrays (ULAs) is Mutual coupling between the individual elements of the antenna array. This problem is reduced in Uniform Circular Arrays (UCAs) because of its symmetric structure. The DOA estimation of uncorrelated signals that have different power levels is simulated on a MATLAB environment. And the noise consider is white across all the array elements. The factors considered for simulation are number of number of snapshots, array elements, radius of circular array, array length, and signal to noise ratio.

Keywords: Antenna array, Array length, Root-MUSIC, Snapshots, Signal to Noise Ratio, UCA, ULA.

1. Introduction

The performance of smart antenna systems greatly depends on how effectively the DOA estimation algorithms estimate directions of the incoming signals. Hence DOA estimation is a very important and interesting research problem in smart antenna systems [1]. In mobile communication DOA estimation is the process of determining the direction of an incoming signal from mobile units to the Base Transceiver Stations [2]. The parameters generally consider for analysing the smart antenna systems are type of antenna array, number of array elements (N), spacing between the elements (d), array length (L=N*d), number of snapshots [3], signal to noise ratio and mutual coupling among the array elements [4].

In smart antenna systems by using DOA algorithms the direction of arrival of incoming signals is estimated. It includes the direction of original, interfering and multipath signals. Next from this unwanted signals are separated and the required user signal is estimated. Then by using beamforming algorithms a beam is steered in the required signal's direction by keeping nulls in the direction of interfering signals. This DOA estimation is used for locating and tracking signal sources in wireless and mobile communication applications. Smart antennas use number of smart signal processing algorithms to estimate the incoming signals DOAs. Mainly DOA estimation algorithms can be classified into conventional and subspace algorithms [5]. Delay-Sum and Capon's Minimum Variance algorithms are the two most important conventional algorithms which are based on beamforming approaches. The subspace methods such as MUltiple SIgnal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariant Techniques (ESPRIT) can split the received signal space into subspaces such as signal subspace and noise subspace. The conventional MUSIC algorithm

fails to estimate the DOAs of highly correlated incoming signals because the covariance matrix obtained by it is singular. To decrease the computational complexity in conventional MUSIC algorithm, Root-MUSIC algorithm is developed by Barabell in 1983 based on polynomial roots provides high resolution.

These subspace algorithms exploit the orthogonality between these two subspaces to estimate the DOAs of the incoming signals. Each DOA method has strengths and weakness relative to each other. Root-MUSIC is one of the variant of MUSIC method that belongs to subspace methods [6, 7]. The Root-MUSIC algorithm shows higher accuracy and requires less number of computations than the conventional MUSIC algorithm [8].

2. Uniform Linear Arrays

Uniform linear arrays are popularly used antenna array for cellular communication systems. They are formed with a large number of identical array elements that are arranged in a single line. As shown in figure1 the ULA elements are arranged along the y-axis.
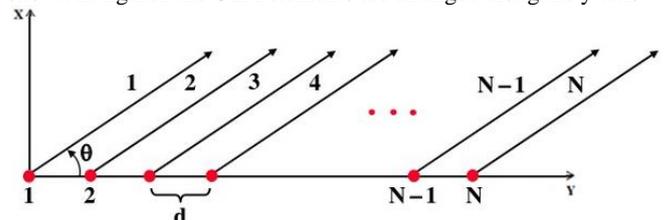

Figure 1. Geometry of N element uniform linear array

For an N-element ULA the elements are spaced equally at distance d from each other. The currents fed to these elements are equal in magnitude and with uniform progressive phase shift. For an N-element ULA the normalized array factor is given by,

$$AF(\theta,\phi) = \frac{1}{N}\left[\frac{\sin\left(N\frac{\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}\right]$$

And the resultant field due to N element ULA at distant point is given by,

$$E_T = E_0 \left[\frac{\sin\left(N\frac{\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}\right] \cdot e^{j\left(\frac{N-1}{2}\right)\Psi}$$

Where phase angle, $\Psi = \beta d\cos\emptyset + \alpha$. And α is the progressive phase shift between two adjacent elements.

3. Uniform Circular Arrays

In modern wireless communication systems UCAs are recognized as alternative antenna array for ULAs in recent years. A UCA is constructed with sensor elements of same type and placed equally in the form of a circle. The main advantage of UCAs is that it can scan beam in any direction. They provide 360^0 azimuth coverage and pattern does not change very much when scanned azimuthally. Like ULAs the UCAs can utilize basic phasing techniques but it cannot resolve closely spaced sources in its basic form. The better performance in UCAs can be obtained with high resolution techniques such as MUSIC and its variants particularly for the estimation of signals DOA. The angle of separation between each element in an N element UCA is,

$$\emptyset = \frac{360}{N}$$

In UCA, N identical array elements are placed equally around a circle of radius a as shown in figure 2. The centre of the circle is placed at the origin of the coordinate system and the circle lies on xy-plane. Because UCAs does not have edge elements, the directional patterns synthesized by them can be electronically scanned in the azimuthal plane without the significant change in beam shape. The directional patterns synthesized by UCAs can be scanned in the azimuthal plane without considerable change in beam shape. This is because UCAs does not have edge elements,
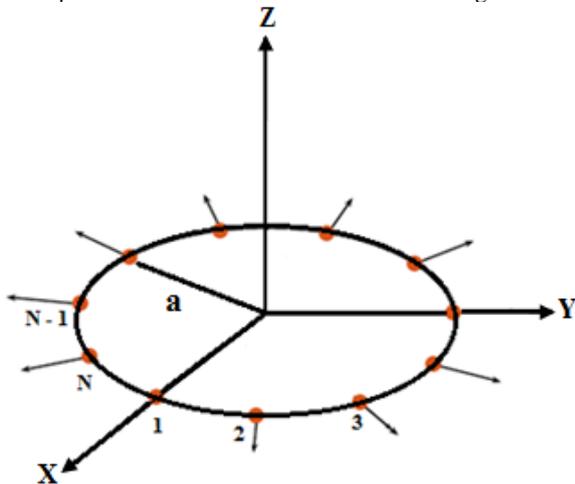

Figure 2. Geometry of N element uniform circular array

For the array of odd number of elements the centre element lies on the *x*-axis. For the even number of elements the midpoint between the two centre elements lies on the *x*-axis.

For an N element array the azimuth angle at mth element is:
$$\phi_m = (-(N-1)/2 + m - 1)\cdot 360/N$$
Where, m = 1, 2, 3 ... N.

4. Root-Music Algorithm

The search free DOA estimation methods are used to achieving lowest computational complexities in subspace methods. Root-

MUSIC is the most popular and frequently used search free DOA estimation technique in smart antenna systems. The root-MUSIC algorithm was initially developed for ULAs in finding the direction of arrival estimation in smart antenna systems. For real time applications the computational complexity of MUSIC algorithm is too high.

For circular arrays in this paper modified Root-MUSIC is used estimate the DOA of uncorrelated signals. When UCAs are used this algorithm uses phase mode excitation technique to transforms the inputs of UCA to a ULA like structure. For the phase mode excitation technique the elevation angles for all signals must be taken the same. For ULAs N defines the number of sensor elements. But for UCA the phase mode excitation technique defines N as the size of the internal ULA like array structure. By taking the centre of UCA as reference, the phase shift produced by each element of UCA can be calculated.

Let UCA of radius a, mth sensor is located at (x_m, y_m, z_m):

$$x_m = a\sin\left(2\pi\frac{m-1}{N}\right), y_m = a\cos\left(2\pi\frac{m-1}{N}\right), z_m = 0$$

Where, m = 1, 2, 3 . . . N.
Then the array steering vector is given by,
$$a(\theta,\phi) = [a_1(\theta,\phi), a_2(\theta,\phi), \ldots, a_N(\theta,\phi)]^T$$
$$a_m(\theta,\phi) = e^{jk_o a\cos\left(2\pi\frac{m-1}{N} - \theta\right)}$$

Where T denotes transpose, free space propagation constant k$_0$ = 2π/λ and λ is wavelength of the signal to be radiated.
Let γ_m be angle made by mth element with the x-axis then,

$$\gamma_m = \frac{2\pi(m-1)}{N}$$

Where, m = 1, 2, 3 . . . N.
The UCA can be excited with mth phase-mode by the weight vector,

$$w_m^H = \frac{1}{N}\left[e^{jm\gamma_1}, e^{jm\gamma_2}, \ldots, e^{jm\gamma_N}\right]$$
$$= \frac{1}{N}\left[1, e^{jm\gamma_2}, \ldots, e^{jm\gamma_N}\right]$$

The excitation of mth phase mode by a signal having steering vector a(θ,ϕ) is given by
$$f_m(\theta,\phi) = w_m^H a(\theta,\phi)$$

Then from the above equations the resulting array pattern $f_m(\theta,\phi)$ can be expressed as,

$$f_m(\theta,\phi) = \frac{1}{N}\sum_{n=0}^{N-1} e^{jm\gamma_0} e^{jk_o r\sin\theta\cos(\phi-\gamma_0)}$$

5. Simulation Results

For simulation two uncorrelated signals with different power levels are considered. Assume that first signal with 3Watts power is arriving at the array with an azimuth angle of 30^0. And the second signal with 1Watts power is coming to the array with an azimuth angle of 60^0. Assume noise is white across all the elements of the array.

Here the Root-MUSIC algorithm is used to estimate DOAs of these two signals both for ULA and UCA. For ULA 15 isotropic antenna elements are considered and are separated by half of the incoming signals wavelength spacing. Similarly for UCA the number of isotropic elements considers are 15 and radius of array be 1.5. Also assume that at each antenna element SNR be 10dB. DOA estimation ability of Root-MUSIC algorithm for both the arrays are analysed for varying SNR and number of snapshots. Finally the results are compared for both the arrays in the presence of strong noise environment. Assume that these uncorrelated power signals are arriving from 0^0 elevation angle.

(a) **Performance of Root-MUSIC algorithm with increasing SNR:**
As shown in Figure (3) average error of Root-MUSIC based on UCA is compared with average error of Root-MUSIC based on

ULA. It is observed that as the signal to noise ratio increases the accuracy of Root-MUSIC is better for UCAs in the presence of low and medium noise and can accurately estimate the DOAs of incoming signals. For SNR less than 10dB, Root-MUSIC algorithm performance better for ULAs. The experimental results for varying SNR are tabulated in tables (1) and (2).

Table I. The experiment results of Root-MUSIC based on ULA at different SNR

S.No	Snr (Db)	Estimated Doa In Degrees		Error In Estimated Doa In Degrees (^{0})		Average Error(^{0}) Of Rootmusic Based On Ula
		30^0	60^0			
1	0	29.9558	60.0368	0.0442	0.0368	0.0405
2	1	29.9495	60.0163	0.0505	0.0163	0.0334
3	10	29.9155	59.9189	0.0845	0.0811	0.0828
4	20	29.9026	59.8859	0.0974	0.1141	0.1057
5	30	29.8985	59.8758	0.1015	0.1242	0.1128
6	40	29.8972	59.8726	0.1028	0.1274	0.1151
7	50	29.8968	59.8716	0.1032	0.1284	0.1158
8	100	29.8967	59.8713	0.1033	0.1287	0.1160
9	∞ (No Noise)	29.8966	59.8712	0.1034	0.1288	0.1161

Table II. The experiment results of Root-MUSIC based on UCA at different SNR

| S.No	Snr (Db)	Estimated Doa In Degrees		Error In Estimated Doa In Degrees(^{0})		Average Error(^{0}) Of Rootmusic Dased On Uca	Average Error(^{0}) Of Rootmusic Based On Ula
		30^0	60^0				
1	0	29.8569	59.9447	0.1431	0.0553	0.0992	0.0405
2	1	29.8710	59.9459	0.1290	0.0541	0.0916	0.0334
3	10	29.9409	59.9833	0.0591	0.0167	0.0379	0.0828
4	20	29.9646	60.0108	0.0354	0.0108	0.0231	0.1057
5	30	29.9717	60.0210	0.0283	0.0210	0.0246	0.1128
6	40	29.9740	60.0244	0.0260	0.0244	0.0252	0.1151
7	50	29.9747	60.0255	0.0253	0.0255	0.0254	0.1158
8	100	29.9749	60.0259	0.0251	0.0259	0.0255	0.1160
9	∞ (No Noise)	29.9750	60.0261	0.0250	0.0260	0.0255	0.1161
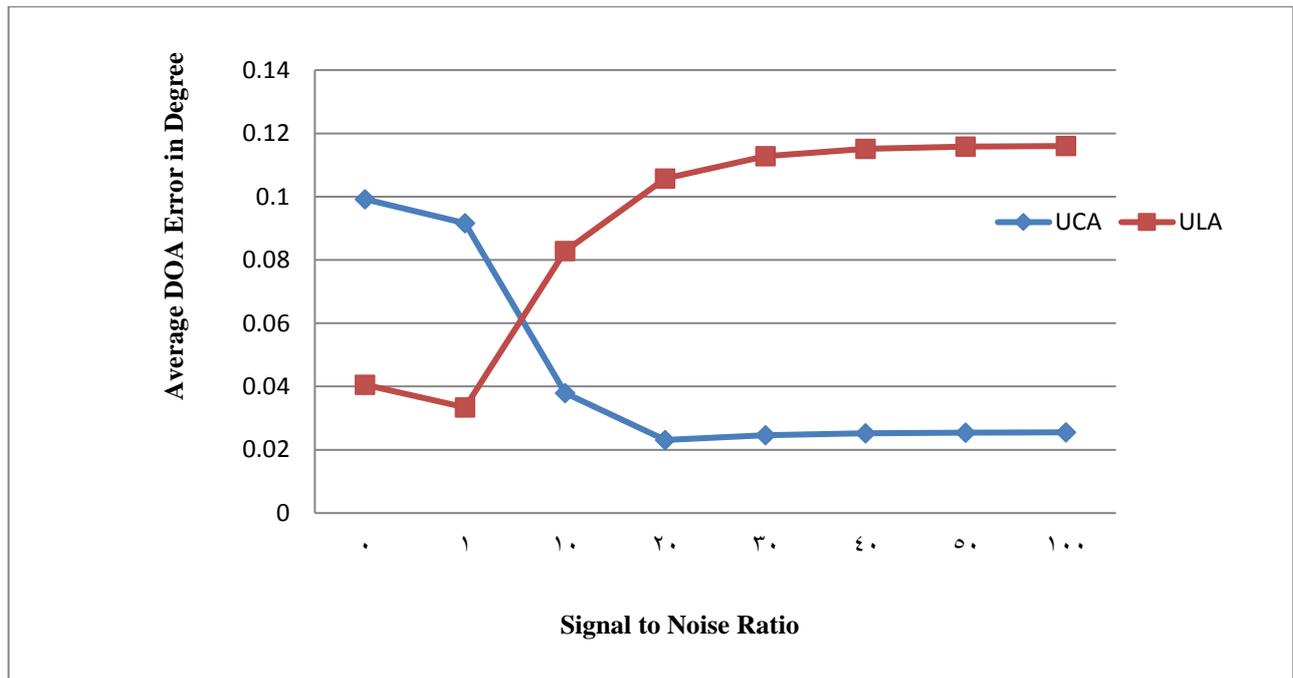
Figure 3.Performance of Root-MUSIC at different SNR

(b) Performance of Root-MUSIC algorithm with number of Snapshots:
As shown in Figure (4) as number of snapshots increases, Root-MUSIC with UCA can accurately estimate the DOAs of incoming signals compared to Root-MUSIC with ULA. The DOA estimation of uncorrelated power signals is carried at SNR of 10dB. With less number of snap shots the performance of Root-MUSIC algorithm for ULAs is poor. The experimental results for increasing number of snap shots are tabulated in tables (3) and (4).

Table III. The experiment results of Root-MUSIC based on ULA at different SNR

S.No	Number Of Snapshots	Estimated Doa In Degrees		Error In Estimated Doa In Degrees (0)		Average Error(0) Of Rootmusic Based On Ula
		30^0	60^0			
1	1	3.0221	47.5888	26.9779	12.4112	19.6995
2	10	29.8698	59.5929	0.1302	0.4071	0.2687

3	50	29.9059	59.9162	0.0941	0.0838	0.0890
4	100	29.9155	59.9189	0.0845	0.0811	0.0828
5	500	29.9108	59.8559	0.0892	0.1441	0.1167
6	1000	29.8965	59.8574	0.1035	0.1426	0.1231

Table IV. The experiment results of Root-MUSIC based on UCA at different SNR

S.No	Number Of Snapshots	Estimated Doa In Degrees		Error In Estimated Doa In Degrees		Average Error(^{0}) Of Rootmusic Dased On Uca	Average Error(^{0}) Of Rootmusic Dased On Ula
		30^0	60^0				
1	1	34.2438	58.0456	4.2438	1.9544	3.0991	19.6995
2	10	29.9098	60.4256	0.0902	0.4256	0.2579	0.2687
3	50	29.9396	60.0040	0.0604	0.0040	0.0322	0.0890
4	100	29.9409	59.9833	0.0591	0.0167	0.0379	0.0828
5	500	30.0189	60.0648	0.0189	0.0648	0.0418	0.1167
6	1000	29.9695	60.0612	0.0305	0.0612	0.0459	0.1231
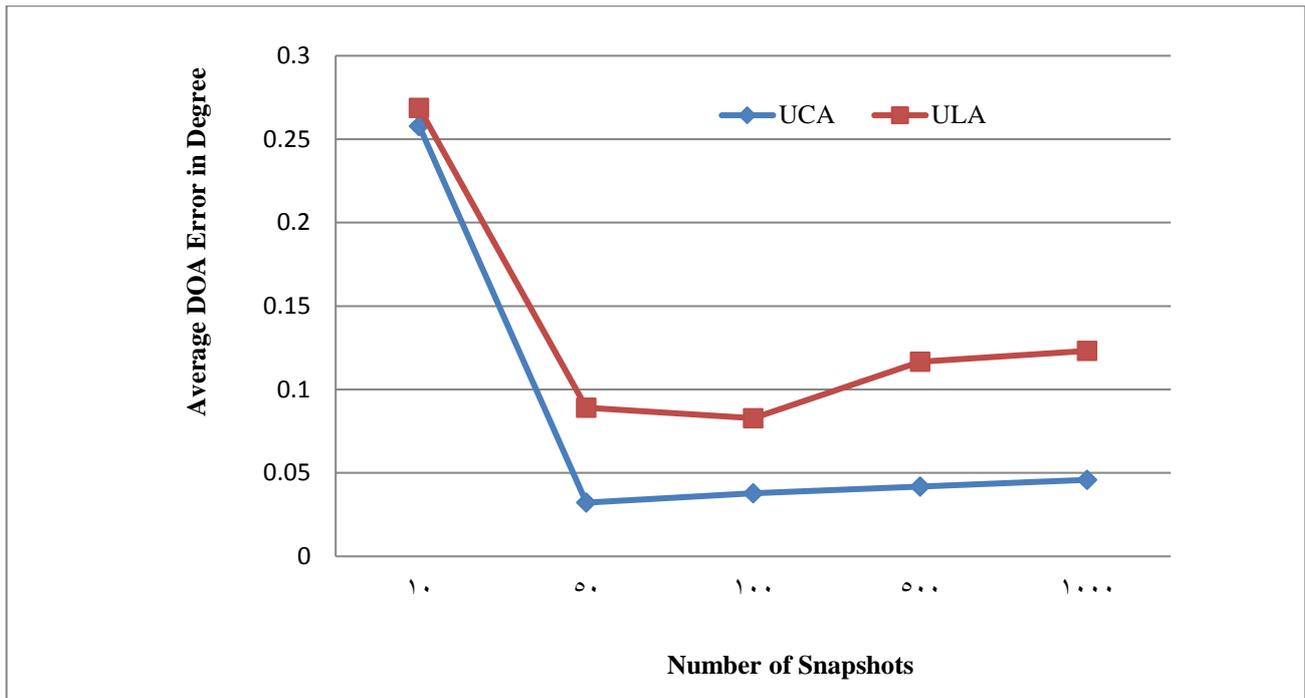
Figure 4. Performance of Root-MUSIC with number of snapshots

6. Conclusion

In this paper the Root-MUSIC algorithm which is the variant of MUSIC algorithm is used to estimate the direction of arrivals of uncorrelated power signals in the presence of strong noise by using both ULAs and UCAs. The results are tabulated and graphically compared. From simulations we conclude that, the DOA estimation capability of Root-MUSIC based on UCA decreases with decreasing the SNR values. Here the simulations are carried out in MATLAB.

Acknowledgment

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