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Research paper

Channel Estimation in OFDM Multipath Fading Channel Systems According to Modulation Schemes

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Abstract

Orthogonal Frequency Division Multiplexing system (OFDM) is a very popular communication technique. It is applied nowadays in several wireless network modern systems. Wireless communication system has some limitations; one of them is Multi Path Fading Channel. This issue leads to impact and deteriorate data signal severely which causes the inter channel interferences (ICI). Therefore, our paper is specifically focused on study of the importance of applying channel estimation in OFDM system which leads to be close to an accurate data. Basically, there are many types of channel estimation techniques, e.g., Least Square Error Estimation (LSE) and Minimum Mean-Square (MMSE). In our study, we utilised LSE and proved the necessity of applying channel estimation in multi path fading channel based on QPSK modulation scheme. We verify that by applying simulation codes in Matlab. Moreover, we make a comparison by sending data, e.g., Baboon image with and without channel estimation technique. Our study is to proof the importance of channel estimation with multi path fading channel in OFDM technique.

***Keywords**: Orthogonal Frequency Division Multiplexing (OFDM), Channel Estimation Technique, Minimum Mean Square Error Estimation (MMSE), Least Square Error Estimation (LSE), Modulation Scheme, Modulation Scheme with Channel Estimation.*

1. Introduction

Wireless communication network is considered a sophisticated artefact happened in the world. It plays a big role in many aspects for instance, social media apps and internet of things (IoT). It is very important to study OFDM which is considered the core of modern wireless technologies and highlighting the drawbacks of wireless communication — multipath fading channel, delay spread, frequency selective fading, inter channel interference, inter symbol interference and scarcity of bandwidth [1]. Actually, OFDM is the solution for all the limitations above. It is a multi-carrier wideband digital communication system is applied in modern communication techniques such as cellular 4G and wireless networks, e.g., (Wimax , LTE & WiFi). It offers a high data rate transmission capability with high bandwidth efficiency and its robustness to multi-path delay [4].Literally, multi path fading channel is the first parameter that effect signal's accuracy. It causes a frequency selective and time-varying for wideband mobile communication systems. Hereafter, in OFDM systems, the channel transfer function of radio channel appears unequal in both frequency and time domains. Therefore, a dynamic estimation of the channel is necessary for the demodulation of OFDM signals [3].Basically, channel execution is described by inserting pilot signals (a single frequency, transmitted over a communications system for supervisory or synchronisation between sender and receiver) for all the subcarriers of OFDM symbols with a specific period or by inserting pilot tones into each OFDM symbol [4].

At first case, block type pilot channel estimation has been progressed under the condition of slow varying channel; in other words, channel is not changing very rapidly. There are two methods for block type pilot channel estimation Least Square (LS) and Minimum Mean-Square (MMSE) [4]. Particularly, MMSE estimator has a good performance and execution combines with high complexity. On the other hand, LS estimator has low complexity but its performance is not at the same level of MMSE estimator [2]. MMSE has some features like robustness when it deals with Additive white Gaussian Noise (AWGN) and inter channel interference (ICI).It has been noticed both of (AWGN) and (ICI) are reduced significantly in fast or slow fading noisy radio channel environment [3]. Most channel estimation methods for OFDM transmission systems have been developed under the assumption of the slow fading channel, where the channel transfer function is assumed stationary within one OFDM data block. In addition, the channel transfer function for the previous OFDM data block is used as the transfer function for the present data block. In practice, the channel transfer function of a wideband radio channel may have significant changes even within one OFDM data block. Therefore, it is preferable to estimate channel characteristic based on the pilot signals in each individual data block [3].We can observe that in paper [2] where the two main methods for estimation in time domain Least Square Error (LSE) and Minimum Mean Square Error (MMSE) is addressed for the first time. Similar to [2] with more expansion the paper in [3] is investigated the estimation in comb-type pilot signal by dividing pilot signals into pilot signals estimation and channel interpolation. Two methods are applied for pilot signals estimation (LS and MMSE) while piecewise linear interpolation or second order polynomial interpolation are used for channel interpolation. While the paper in [4] is interesting on study the various methods of channel estimation: channel estimation based on block-type pilot arrangement, channel estimation at pilot frequencies in comb-type pilot arrangement and interpolation techniques in comb-type pilot arrangement and compares between them by applying 16 QAM (16 Quadrature Amplitude Modulation), QPSK (Quadrature Phase Shift Keying) and DQPSK (Differential Quadrature Phase Shift Keying). Next, OFDM system explanation in details in order to make a clear image about channel estimation with its methods, following it with example by simulating OFDM symbols with and without channel estimation. In our study, we apply image data in order to notice the accuracy and the clarity with estimation technique in multi path fading channel.

2. Definition of OFDM terms

(Symbols, Symbol Rate, Symbol duration time,
Impulse Response & Cyclic Extension)

We heard a lot about symbols, so what does it mean? **Symbol** is considered the main component in OFDM system since it represents the transmitted data. It may be described as either a pulse in digital baseband transmission or tone in passband transmission using modems [1]. A symbol is a waveform, a state or significant condition of the communication channel that persists for a fixed period of time. A sending device places symbols on the channels at a fixed and known **symbol rate** (which is the number of symbol changes, waveform changes, signaling events across the transmission medium per time unit using a digitally modulating or a line code) and the receiving device has the job of detecting the sequence of symbols in order to reconstruct the transmitted data each symbol may encode one or several binary digits or bits [1]. The time between one transition and the other we call it **symbol duration time** or **unit interval**. Furthermore, OFDM system is designed to have a finite length channel of impulse response where unit Impulse is a signal which has a value of 1 at the sample number 0, so that impulse response is the response of the system for the unit impulse. Add to that ,it has a **cyclic extension** which is a small percentage of the total symbol length put between consecutive blocks of symbols in order to avoid inter block interference and preserve orthogonality of the tones[1].

3. OFDM System Description

In previous section we give a definition for each term includes inside OFDM system in order to simplify it for new researchers. Fig (1) illustrates the environment of OFDM technique by taking a benefit from the property of channel impulse response, where x_k are the transmitted symbols, $g(t)$ is the channel impulse response,$n^{\sim}(t)$,is the white complex Gaussian channel noise and y_k are the received symbols. The transmitted symbols x_k are taken from a multi-amplitude signal constellation. In the other words, OFDM system is taken the advantages of *QAM* modulation (each symbol may include more than one bit depending on the type of modulation scheme). The D/A, A/D converters contain ideal lowpass filters with bandwidth l/*Ts*. A cyclic extension of time length *TG* (not shown in Fig.1. for reasons of simplicity)[2]. We treat the channel impulse response $g(t)$ as a time- limited pulse train of the form.

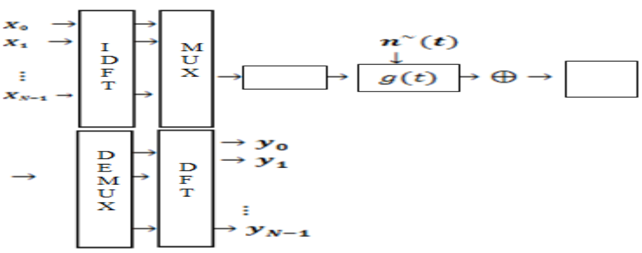


Fig.1: illustrates OFDM Model with channel $g(t)$ and (AWGN) $n^{\sim}(t)$ [2].

Equation (1) represents the process of Inverse Discrete Fourier Transform (IDFT).

$$x(k) = 1/N \sum_{L=0}^{N-1} e^{j2\pi kl/N} \tag{1}$$

Where
$x(k) \textit{ is the transmitted symbols at time} - \textit{doamin}$.
N is total number of subcarriers (***IFFT/FFT SIZE***),
k=0,……,***N-1*** and ***L***=0,…….,***N-1***
k is a sample of symbols at time-domain and ***L*** is the loaded channel is sampled according to N points.

$$g(t) = \sum_m \propto m\ \delta(t - TmTs) \tag{2}$$

Where the amplitudes $\propto m$ are complex valued and $0 \leq TmTs \leq TG$, so, i.g., the entire impulse response lies inside guard space [2]. Literally, we can call (2) circular convolution process. It's a linear system where satisfies the three mathematical properties:

1) Scaling property: if there is any change in input signals amplitude then the corresponding change should replicate at the output.
2) Additive property: if for any two input signals *X1(t)* and *X2 (t),* i.e. the output corresponding to the sum of any two inputs is the sum of the two outputs.
3) Shift invariant property: implies that the response of the system to $x_1(t - t_0)$ is given by $y1(t - t_0)$ for all values of t and t_0.

Actually, if we know the impulse response of the linear system then we can find the output for any input.

Therefore, in convolution process, we are going to send only one sample at the time instead of all the samples, this is usually called impulse decomposition.

Literally, OFDM technique contains three stages:

1) First stage, we call it transmitter side.
2) Second stage, we call it transmitted process.
3) Third stage, we call it receiver side.

First stage represents the Modulation scheme when carrier signal is prepared to carry data signal based on modulation schemes, e.g., PSK ,QAM. While, second stage represents the signal linearly by applying circular convolution process with cyclic prefix in order to prevent inter symbol interference (ISI).

Third stage (receiver side) appoints typically the opposite process of transmitter side. Particularly, it is acted by demodulation schemes, DFT and estimation techniques in multi path fading channel.

According to aforementioned interpretation, we focus on analyzing the signal at receiver side by considering multi path fading channel.

3.1 Signal's Pattern in Multi Path Fading Channel

Multi path fading channel unlike a wired channel which uses a fixed path. The signals in wireless channel can reach a user using multiple path. All these signals known as multipath component. This issue leads to have different channel gain and time delay which combined effect causes what we know as multiple fading channel [1-8]. Equation (2) represents multi path fading channel. As the consequence of multipath propagation, the duration of a symbol gets extended. This may interfere with the next symbol which is called Inter Symbol Interference (ISI) or crosstalk. Thus, the idea of adding cyclic prefix to OFDM channel models is utilized. In fact, two models are described the cyclic system at receiver side [2]:

First model, it is implemented by equation (3) [2].

$$y = DFT_N\ (IDFT_N(x) \odot \frac{g}{\sqrt{N}} + n^{\sim} \tag{3}$$

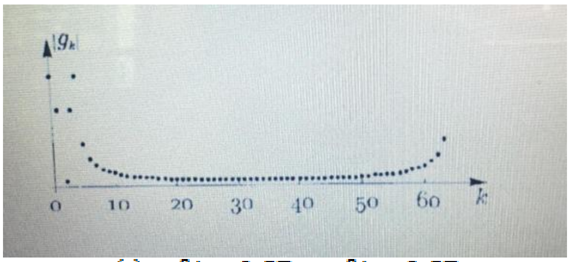
Where is DFT_N Discrete Fourier Transform for N points independent Gaussian channels, is $IDFT_N$ Inverse Discrete Fourier Transform for N points for sampling x , $\frac{g}{\sqrt{N}}$ is a vector of observed channel impulse response after sampling the frequency response of $g(t)$, $g = [g_0,\ g_1, \dots\dots g_{N-1}]^T$ is determined by the cyclic equivalent of sinc-functions. The receiver side determines the observation vector **y**, so that it could calculate the channel estimation. In other words, the receiver side couldn't observe the real channel **g.** Refer to Fig.2,

$$x_0 \quad \downarrow h_0 \quad \downarrow n_0$$
$$\rightarrow \otimes \rightarrow \oplus \rightarrow\ y_0$$
$$\vdots$$
$$x_{N-1} \quad \downarrow h_{N-1} \quad \downarrow n_{N-1}$$
$$\rightarrow \otimes \rightarrow \oplus \rightarrow\ y_{N-1}$$

Fig.2 shows the decomposition samples is loaded on channel impulse response with (AWGN) and the result is the observation y vector [2].
Second model, which is implemented by equation (4) [2].

$$g_{k=1/\sqrt{N}\sum \alpha m\, e^{-j\pi}/N(k+N-1)Tm} \quad \frac{\sin(\pi T_m)}{\sin(\pi/N(T_m-k))} \tag{4}$$

It is the same idea of (3) with analyzing the DFTN process and IDFTN process for each observation vector **y** into g_k channel in frequency-domain. Depending on T_m guard space between symbols.
A good question comes into our mind, how can we determine the validity of the cyclic model described by (3)? It depends on the guard space (time) between the symbol blocks in order to eliminate the issue of inter block interference.Fig.3 describes the leakage between channel taps for specific, e.g., T_m=0.5 and 3.5



$$g(t) = \delta(t-0.5T_{s)} + \ \delta(t-3.5T_{s)}$$

Fig.3: describes the degradation between tap channels for Specific

Example [2]
That it means, there is a degradation between tap channels.
That degradation depends on the amplitude for each symbol duration time during symbols transmitted .
We can rewrite the equation (3) as a set of N independent Gaussian channels

$$y = h_k x_k \qquad \text{Where } k = 0, \dots\dots, N-1 \tag{5}$$

h_k is a complex channel after taken $DFT_N\ (g)$
, $h = [h0, h1, \dots\dots, h_{N-1}]^T$=$DFT_N\ (g)$,
$n^{\sim} = [n0, n1, \dots\dots, n_{N-1}]^T = DFT_{N-1}(n^{\sim})$ is an Independent identical Distribution **i.i.d.** complex zero-mean Gaussian noise vector.

Remark: we take DFT depending on N point (number of subcarriers).

For simplicity, we can rewrite the equation (5) in matrix notation

$$y = XFg + n, \tag{6}$$

Where X is a diagonal matrix of x symbols , F is a unitary matrix or DFT matrix and **g** is a **Sinc**-functions and finally n is **i.i.d.** complex of additive white Gaussian noise vector with **zero-mean** and standard deviation is equal to one, **σ=1.**

$$\begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \tag{7}$$

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{nk}{N}} \tag{8}$$

(7) and (8) describe the DFT-Unitary Matrix F with its component.

4. Related Work

In section **3** we mentioned about fundamental stages of OFDM system. Therefore, we classify the literature review according to these stages:
1) Transmitter stage: this stage includes ***Modulation*** part which considers the most important part in this stage. 2) Receiver stage:this stage includes ***Estimation Channel Technique*** which considers the most powerful technique to estimate the accurate data by compromising with the complexity. Since the receiver side gets the observation vector y which the complex channel h is entangled with in multi path fading channel. Therefore, the taxonomy of the literature review is built based ***on Modulation Schemes*** and ***Estimation Techniques.*** Since OFDM system is applying the idea of Multi-Amplitude (Multi-Levels) signaling scheme,it's important to track in which point the signal is goingto fade. In other words,OFDM system is executed by implementing different types of modula- tion technique e.g., Quadrature Phase Shift Keying QPSK, Differential Quadrature Phase Shift Keying DQPSK and Quad- rature Amplitude Modulation 16QAM [4].Previously,in old transmission technique ,each symbol consists one bit. In contrast, nowadays each symbol consists of several bits (Multi- Amplitude).Thus, according to that the rate of error is increased. Which leads to get an accurate data. So that, it's very important to use estimation channel technique to approximate the trans- mission data and reduce the error at receiver side. The paper in [2] is proposed methods for the first time in time-domain statis- tical channel estimation .These methods are minimum mean- square error (MMSE) and least-squares (LS) channel estimators then a method for modifications. These techniques make trade- off between complexity and their performance. Literally, OFDM system is designed to have a finite length channel of impulse response where unit Impulse is a signal which has a value of 1 at the sample number 0 then impulse response is the response of the system for the unit impulse. Add to that , it has cyclic extension which is a small rate of the total symbol length put between consecutive blocks of symbols. That leads to avoid inter block interference and preserve orthogonality of the tones.Paper [2] takes a benefit from channel impulse response property when applying channel estimation method. For that reason, the authors in paper [2] depicts the utilization of property for channel impulse response in details. Literally, we can see that very clearly in ***Modified MMSE Estimators*** *and* ***Modified LSE Estimators*** [2].The authors try to take the advantages of MMSE and LSE by proposing these new techniques.Simply, the idea of ***Modified MMSE Estimators*** and ***Modified LSE Estimators*** is about complexity reduction for ***MMSE*** and accuracy for ***LSE*** since ***MMSE*** suffers from complexity and ***LSE*** suffers from accuracy(Npoints) Basically, in order to do that, dimensionality of sub carriers is reduced by exploiting the first L columns of impulse channel response where the energy is specifically existed in L area. The first L taps = TG/Ts, therefore QMMSE is modified to calculate the L*L tap channels instead of calculating N*N.Consequently, ***QMMSE*** matrix contains ***L*L*** the ***first L*** columns taps instead of

N*N points, refer to Fig.4. While in [8], the idea of binding detection, estimation and modulation technique is considered the fundamental of multi-level amplitude signalling schemes. It's explained in details since this process needs to keep track of the signal for each subcarrier in order to get an accurate data.

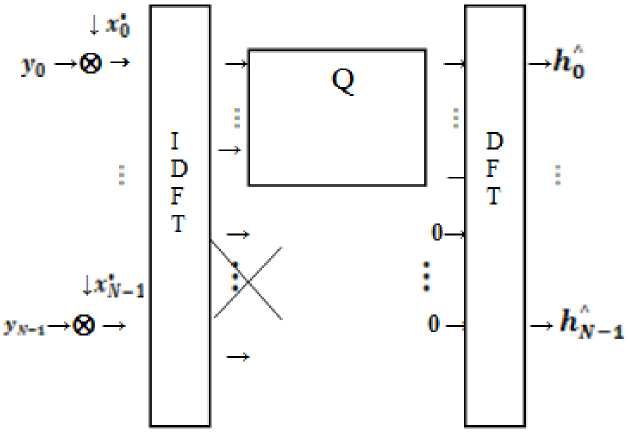


Fig.4: illustrates the modified estimator structure [2].

4.1. Modulation Schemes and Estimation Techniques

Before we delve in Estimation methods, let's take a look brief for Modulation technique in order to understand well the idea behind estimation technique [8].

4.1.1 Modulation Schemes

Generally, the basic idea of any modulation technique is to prepare a signal for transmission on a wireless medium. Since it's difficult to send the baseband signal or "message" directly to an antenna, so it is converted into a form in order to be carried over a carrier signal [8].

We can divide the constellation Mapper (Modulation Technique) into:

- Amplitude Modulation (AM): it carries the information (data) in the amplitude of the signal. Generally, communication doesn't work very well when it uses any form of amplitude modulation because noise affects amplitude most.
- Frequency Modulation (FSK): it carries the information (data) in frequency of the carrier, e.g., if data consists of (0) bit, the carrier frequencies will be low. Contrary, if data includes (1) bit, the carrier frequencies will be high. So that, in this case the amplitude of carrier signal doesn't change just the carrier signal frequencies.
- Phase Shift Keying (PSK) and Differential Phase Shift Keying (DPSK): In this case, amplitude and frequencies are kept constant however phase of carrier signal is changed according to the data binary bits if it's zero or one.DPSK is significantly simpler to execute than PSK since it doesn't need for demodulator to keep track of a reference wave. In contrast, PSK is complicated in execution, it needs for CPSK (coherent system demodulator) in order to implement [11].

PSK is classified into six types:

I. Binary Phase Shift Keying (BPSK) which applies both the amplitude and the phase. In this case, it consists of two points on the n phase axis opposite the quadrature axis. It maps one bit to one symbol.

II. Quadrature Phase Shift Keying (QPSK) is the most robust modulation which applies both the amplitude and the phase. In this case, we have 4 possible symbols that we can transmit. Therefore, we still transmit a sinusoidal

Carrier of fixed amplitude and fixed frequency. While, the phase of that carrier keeps changing according to the user bits. Consequently, the transmitted bits are grouped pairs into phaser with the same amplitude and single frequency. Refer Fig.7.

8-PSK: Each symbol includes 3 bits.
16-QAM: Each symbol includes 4 bits.
32-QAM: Each symbol includes 5 bits.
64-QAM: Each symbol includes 8 bits.

Actually, all aforementioned Modulation Schemes for PSK showing robust performance to noise and ISI .We consider QPSK in our simulation since it's the most robust modulation and it's applied in modern technology LTE.

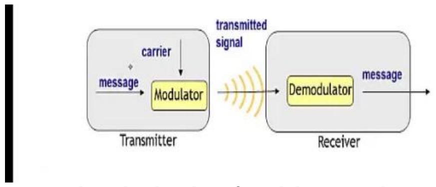


Fig.5: describe the idea of Modulation Technique [12].

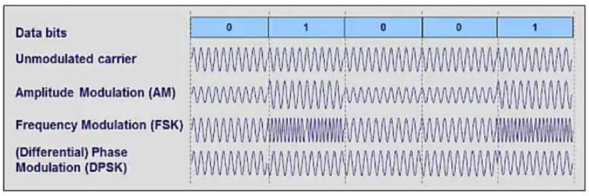


Fig.6: describes different types of Modulation Technique [12].

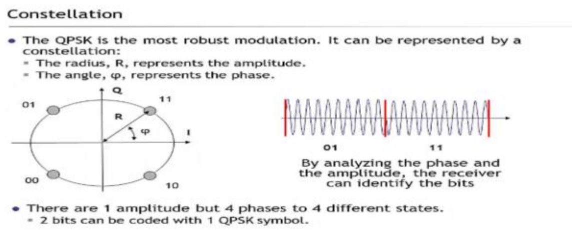


Fig.7: describes QPSK Modulation Schemes [8].

4.1.2. Estimation Technique

According to [2] Estimation channel technique can be classified into two kinds Minimum Mean Square Error Estimation ***MMSE*** and Least Square Error Estimation ***LSE***.

Primarily, let's understand that any estimation technique depends on probability density function (PDF) because we don't know the exact value that carries the data .In other words the observation vector **y** is include random variables correlated with noise.

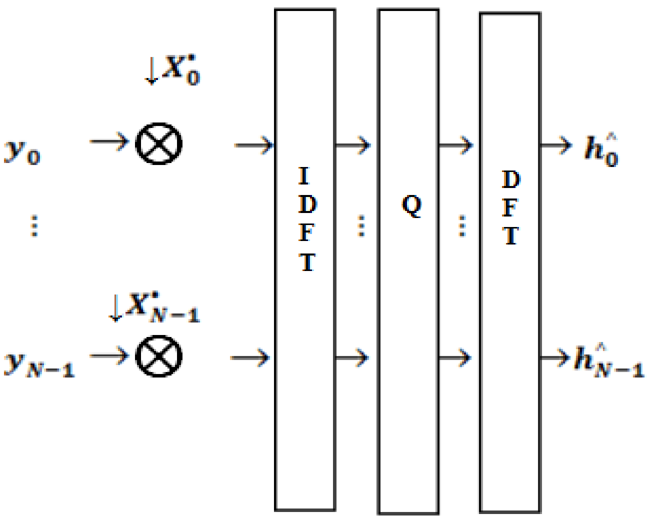


Fig.8: depicts the structure of general estimator [2].

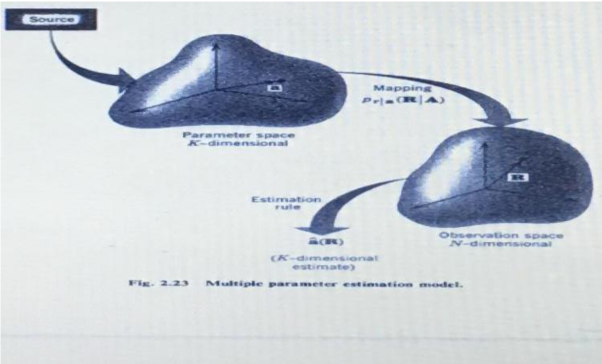


Fig.9: shows the multiple parameter estimation model [8]

4.1.2.1. MMSE Estimator

Suppose channel g^2 is Gaussian uncorrelated channel with noise n, therefore we can estimate it by applying [2]:

$$\tilde{g} = R_{gy} R_{yy}^{-1} y \quad (9)$$

Where $R_{gy} = E\{gy^H\} = R_{gg}F^H X^H$ let's elaborate that clearly: Since the parameter g is random in nature, so we estimate it based on y.
The assumption is that $E\{g\}=E\{y\}=0$ zero-mean of the probability density function **PDF** where g is the parameter joint with observation vector y.
$E\{g^2\}=\sigma^2_g$, $E\{y^2\}=\sigma^2_y$,this is the variance of parameter g and observation vector y respectively. Therefore, the term covariance or the cross covariance means:
$E\{gy\} = p\sigma_g\sigma_y$
Where, p is the correlation coefficient between g and y.
$p = \frac{E\{gy\}}{\sigma_g\sigma_y}$;$p = \frac{E\{gy\}}{\sqrt{\sigma^2_g\sigma^2_y}}$; $p = \frac{E\{gy\}}{\sqrt{E\{g^2\}E\{y^2\}}}$
The most interesting property of magnitude p is always less than 1 ;it lies between **1** and **-1**.It indicates the correlation between $g$and y.For Random Gaussian variable g and y if p =0 then g and y parameter is uncorrelated and that it means g and y are independent.
$R_{yy} = E\{yy^H\} = XFR_{gg}F^H X^H + \sigma^2 I_N$
Both expression above are cross covariance matrix between g and y and auto-covariance matrix of y.
Furthermore, R_{gg} is the auto-covariance matrix between **g** and its variance σ^2denotes the noise $E\{|n_k|^2\}$,these two quantities are assumed to be known. Where Covariance is the expectation of the product of two random variables minus the product of their mean and auto-covariance is the covariance between a stochastic (having a random probability distribution) process at different times. E is a usual notation for the expectation average operator of Gaussian noise vector. The ***MMSE*** concept is to minimize the error between $\tilde{g}$ and g by subtracting and square the distance. So that, the idea of ***MMSE*** estimation is to reduce the average estimation error not just the error estimation itself [2].

$$R = \begin{bmatrix} \sigma 2_g & p\sigma_g\sigma_y \\ p\sigma_g\sigma_y & \sigma 2_y \end{bmatrix} \quad (10)$$

Where R represents the covariance matrix (cross correlation) of vector $\begin{bmatrix} g \\ y \end{bmatrix}$.
Since all columns in F are $orthonormal^1$, therefore $\tilde{g}_{MMSE}$ generates the frequency- domain ***MMSE*** estimate for

$$\hat{h}_{mmse} = F\tilde{g}_{MMSE} = FQ_{MMSE}X^H F^H y \quad (11)$$

Where,

$$Q_{MMSE}=R_{gg}\left[(FX^H XF)+R_{gg}\right]^{-1}(F^H X^H XF)^{-1} \quad (12)$$

Remark[1] **:** We can call any matrix is orthonormal when the length of its vectors is equal to **1** or they have been normalized. Add to that all the vectors are orthogonal to each other.
Fig.7 & Fig.8 shows the estimation pattern.

Remark[2] : If g isn't Gaussian complex channel, $\tilde{h}$ has two choices either Minimum Mean Square Error estimator or the best linear estimator.

4.1.2.2. LSE Estimator

Corresponding to **Euclidean** distance and orthogonal projection matrix, ***LSE*** Minimizes the distance between y and g by generating $(y - XFg)(y - XFg)$ [2].

$$\tilde{h}_{Ls} = FQ_{Ls}F^H X^H y \quad (13)$$

$$Q_{Ls} = (F^H X^H XF)^{-1} \quad (14)$$

From (11) we can get:

$$\tilde{h}_{Ls} = X^{-1}y \quad (15)$$

Let's explain that in more details;
If we have some matrix
$A_{n\times k}\,\vec{x} = \vec{b}$, in this case $\vec{x} \in R^k$
We have k columns here and $\vec{b}$ is a member of R^n .
In this case, there is no solution since $\vec{x}$ belongs to R^k and $\vec{b}$ belongs to R^n.In other words there is no set of weights here for $\vec{b}$ on the column vectors of A. So that, there is no linear combination of the columns vector of A will be equal to $\vec{b}$.However, there is no solution like we mentioned above, we can find the best solution for this situation. Therefore, we consider LSE is the best approximate solution which is close to exact solution.
Thus, we are going to we find $\vec{x}^*$ where $A\vec{x}^*$ is as close to $\vec{b}$ as possible. In other way, we want to minimize the length of $\|\vec{b} - Ax^*\|$.
In terminology, we suppose that A$\vec{x}^*$ = V

This is equivalent to

$$\left\| \begin{matrix} b_1 - v_1 \\ b_2 - v_2 \\ b_n - v_n \end{matrix} \right\|^2$$

Which is equal to
$(b_1 - v_1)^2 + (b_2 - v_2)^2 + \cdots + (b_n - v_n)^2$
So that, we want to minimize this assumption or we want to get the least square estimate. For that reason, $\vec{x}^*$ is called the least square estimate (LS) for the equation $A_{n\times k}\,\vec{x} = \vec{b}$ .Now the question comes into our mind, what's the closest vector in any subspace? Well, the closest vector to it is the projection of $\vec{b}$ on the subspace and that's the closest vector.
Hence, Ax^*= $proj_{c(A)}\,\vec{b}$ **.** Then, we are going to subtract $\vec{b}$ from both side of equation. According to previous process ,we will get $Ax^* - \vec{b}$=$proj_{c(A)}\vec{b} - \vec{b}$.
That it means $proj_{c(A)}\vec{b} - \vec{b}$ is orthogonal to my subspace or column space. Therefore, $Ax^* - \vec{b} \in C(A)^{\perp}$ is the orthogonal complement of the column space (which is the set of any type of data).So, $C(A)^{\perp} = N(A^T)$ where N the null space of A.
So when we substitute previous assumption, we will get $Ax^* - \vec{b} \in N(A^T)$ .This leads to$A^T(A\vec{x}^* - \vec{b}) = \vec{0}$

Now, we will simplify this:

$$A^T A\vec{x}^* - A^T\vec{b} = \vec{0} \rightarrow A^T A\vec{x}^* = A^T\vec{b}$$

Literally, we multiply the both side of equation with A^T in order to get the best solution not the exact solution for our original equation $Ax^{\rightarrow *} = b^{\rightarrow}$.Therefore, the left side of original equation $Ax^{\rightarrow *}$ always has a solution and this solution is the least square solution. We can notice that LSE estimator is simpler than MMSE estimator. Thus, the consequences of LSE implementation is low in complexity than MMSE estimator while MMSE is better in reducing error and improve the channel.
The both estimators (11) and (14) have issues [2]:
MMSE suffers from high complexity on the other hand ***LSE*** suffers from high mean square error.

5. Simulation Result and Discussion

In this section we evaluate OFDM system by applying ***QPSK*** modulation technique (4 possible symbols are transmitted where each symbol includes pair of bits), ***LSE*** estimator technique, OFDM transmitted symbols are generated by applying **Baboon image**, number of subcarriers (***IFFT/FFT*** size) =64 ***N*** points. We are specifying a target ***SNR****=20* dB,10 dB. Add to that, we consider length cyclic prefix extension=16 and number of channel taps **L**=8.Firstly, we read the image and translate it from a three dimensional array along with each ***RGB*** or ***True colour*** (is an array can be of class double where each colour component is a value between 0 and 1) ***MATLAP*** component into a single binary stream. Secondly, this binary stream is mapped from bits 1 & 0 to symbols. These symbols are correspond to the modulation order. Basically, we use ***QPSK*** in our case. After that, ***IDFT*** is implemented to move data to the time domain from the frequency domain.A cyclic prefix extension is added. Then, the data is shifted from parallel to serial Refer to Fig.1.At this stage , the image signal is corrupted with noise. The noise power is calculated based of the target channel ***SNR*** value. Then, the fading channel taps are calculated refer to equation (15) and the time-domain OFDM signal is convolved with impulse response of the channel. All aforementioned steps are related to ***transmitter stage*** , hence at ***receiver stage*** we are going to implement the opposite procedures.AT receiver stage, we start with applying DFT ,therefore the signal is shifted from time-domain to frequency-domain. After that, ***LSE*** first order statistics estimation technique is executed. Then, the data is recovered from the modulated symbols and parsed back into a binary stream. Finally, the binary stream is reshaped to form the image matrix. Once we shape the binary stream, the image is generated.
In the last section of code, we generate all of the plots shown in this paper.
All simulations were run in MATLAB on a PC with a Intel ® core ™ i5-2410 M
CPU @ 2.30 GHZ 2.20 GHZ and 10.0 GB RAM.

$$g(t) = e^{(-(0:L-1))} \quad (16)$$

Our aim from this evaluation is to observe the importance of applying channel estimation and notice the difference when we consider channel estimation and the opposite.

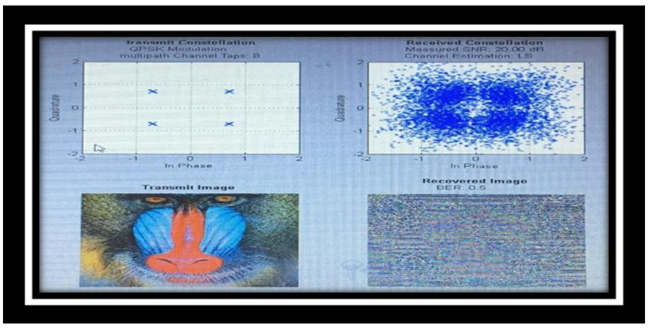


Fig.10: shows the simulation results of OFDM model signal without applying channel estimation, SNR=20.

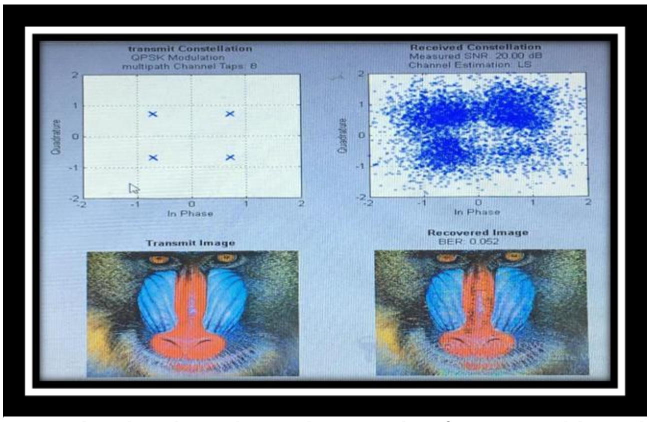


Fig.11: describes shows the simulation results of OFDM model signal with applying Least Square Error LSE estimator, SNR=20.

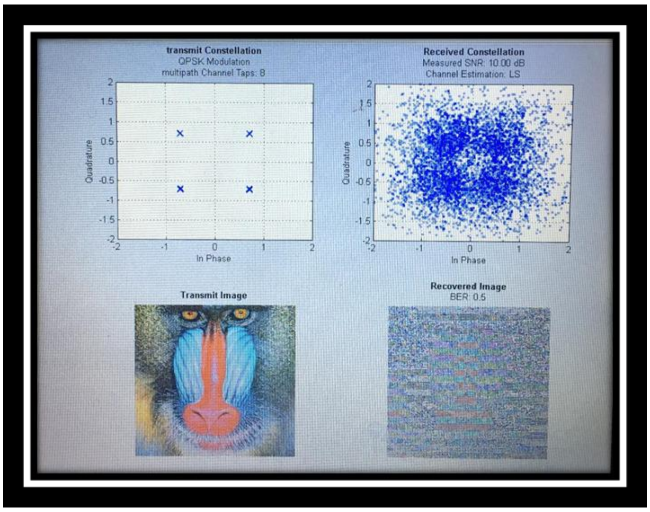


Fig.12: describes shows the simulation results of OFDM model signal with applying Least Square Error LSE estimator, SNR=10.

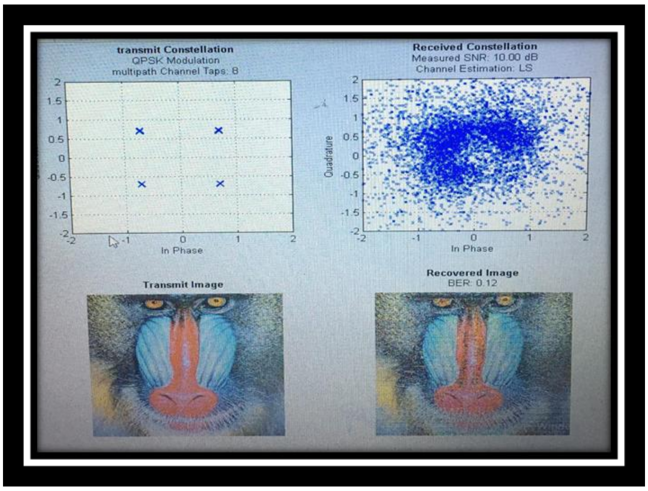


Fig.13: describes shows the simulation results of OFDM model signal with applying Least Square Error LSE estimator, SNR=10.

However, Bit Error Rate (BER) in Fig.11 is lesser than Fig.12, we couldn't recognize the image without applying channel estimation. On the other hand, we could recognize the Baboon image even BER is high by applying channel estimation.

6. Conclusion

We have reviewed the concepts of OFDM system and assigned to the significant properties between modulation techniques and estimation techniques in multipath fading channel. We have explained in details the mechanism of channel impulse response, convolution process and leakage in channel supporting that with equations and graphs. Particularly, we considered QPSK modulation technique with LSE estimation technique in order to recognize the link between them in terms of consistency and accuracy when sending the signal (transmitter side) and recover it (receiver side).In our case, we employ Baboon image as transmitted signal. The performance evaluation is conducted according to our observation the dissimilarity when LSE estimator is applied we can improve the channel and recognize the image even when BER is high while without applying channel estimation we couldn't recognize Baboon image. This article is specifically for new researchers who they want to get clear and sufficient information based on OFDM system in multipath fading channel with modulation and estimation techniques.

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