International Journal of Engineering & Technology, 7 (3.28) (2018) 163-169

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Integrating Interval Agreement Approach (IAA) with TOPSIS in Multi-Criteria Group Decision Making (MCGDM)

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Abstract

This paper presents a proposed method to integrate Interval Agreement Approach (IAA) with Technique for order performance by similarity to ideal solution (TOPSIS) for Multi-Criteria Group Decision Making (MCGDM). The proposed method utilises IAA to model the preferences (word) given by decision maker in the initial step of TOPSIS-MCGDM. In real world, problems preferences may be given as imprecise or incomplete information. An issue has been raised about how to precisely modelling the preferences or word in the context of MCGDM. One of the strengths in IAA technique during the modelling process is it distinguished the two types of uncertainties (intra- and inter-) in different degree of freedom. The proposed method is able to capture the information from one or more intervals (with crisp or uncertain interval endpoints) associated with preferences given by the decision maker. This paper provides an example to illustrate the application of the proposed model.

Keywords: *Interval Agreement Approach; TOPSIS; MCGDM.*

1. Introduction

Nowadays, the process of making a decision is very important especially when there are varieties of options and when involving multiple decision makers. An improvement in the decision-making method is essential in determining the best evaluation, especially in a uncertain environment. The preferences made by decision makers in group decision-making process can be view as an agreement between decision makers. Thus, the crucial part in decision making is getting the right preferences or responses from multiple decision makers. More recent attention had focused on the provision of modelling the preferences or word. By drawing on the concept of modelling word or preferences, in [1] has been stated that words mean different things to different people. Since then, researchers have shown an increased interest to precisely model the word or preferences [2-8].

A primary concern in this decision-making field is that the preferences made by decision maker are not necessarily rationally guided. In real-world problems, it may be given as imprecise, incomplete information about the set of feasible alternatives. A key aspect in this field is in determining the right meaning of preferences or responses given by decision maker by making fewer assumptions and minimise the loss of information [9]. Thus, the questions have been raised about how to precisely modelling the preferences or word.

Several attempts have been made to model this preferences or word [10-16]. Zadeh was reported in [17-19] that the numerical value could be linguistic variables. Previous studies have reported that there exists uncertainty in linguistic variables. This linguistic uncertainty can be viewed as uncertainty produced by statements in natural languages [20]. The fundamental process of decision making involves the composition of different phases. It starts with information gathering, analysed the information and choosing the best option or alternatives through different reasoning process from the set of possible alternatives. Conventional decision theory usually used probabilistic models to handle uncertainty in decision problems. However, in many environments commonly a lot of aspects of these uncertainties have a non-probabilistic condition since they are related to imprecision and vagueness of meanings [21]. Linguistic descriptors frequently used by experts to solve the particular type of problem. Therefore, by converting linguistic terms to fuzzy judgments rather than probabilistic values to overcome these difficulties of managing and modelling uncertainties, fuzzy logic and fuzzy set theory arise to natural managing uncertainty and fuzzy linguistic approach transform the linguistic variables indirect way to represent the information. This linguistic information used thus increased the flexibility and reliability of conventional decision models [22]. The research to date has tended to focus on the notion variability and the accuracy in decision making rather than modelling uncertainty in words/preferences.

When a decision problem is solved using linguistic information, it generally implies a need for linguistic computational models for computing with words (CW) [12, 23]. In a classical approach, type-1 fuzzy sets (T1 FS) have been used to describe the semantics element in decision-making problem [12]. However, this T1 FS does not have enough degrees of freedom to represent all the uncertainty collecting from a group of subjects [24]. Several researchers has been proposed to use Interval Type-2 fuzzy sets to handle uncertainty in modelling word/preferences [25-27]. They proposed an approach to encoding words using IT2 FS namely as Interval Approach (IA) [25]. Then, this work has been extended to Enhanced Interval Approach (EIA), which still using IT2 FS with a few modifications on it [26-27]. Then, current work was done by [7], which proposed an Interval Agreement Approach (IAA) for modelling interval-based responses using General Type-2 Fuzzy Set based on *z*-slices. Although all these previous studies have shown great promise, they still cannot adequately represent the uncertainty (the third dimension) in decision-making process which highly related with the decision makers.

In [15], they stated that several sources of uncertainty might exist in a particular situation. There is uncertainty to an approximate human thought that needs a computational model to handle the approximation. There is measurement uncertainty associated with distinguished between mapping the sensory data and concepts. Finally, there is uncertainty in modelling variation of inter- and intra- expert in the process. This study will focus on the last source of uncertainty.

In this study, we propose to integrate IAA with Technique for order performance by similarity to ideal solution (TOPSIS) for Multi-Criteria Group Decision Making (MCGDM). The proposed method utilizes IAA to model the preferences (word) given by decision maker in the initial step of TOPSIS-MCGDM. This initial step will handle two type uncertainties in different membership function. The first uncertainty is intra-expert uncertainty which variation may involve in the opinions of a particular participant and the second uncertainty is inter-expert uncertainty which variation may exist amongst the individual opinions of a group of participants. These two types of uncertainty will occur when subject/expert unsure about picking a specific point on the scale due to vagueness in the meaning of words/preferences. Thus, it is better to use interval-based to provide broad selections as a guideline to decision maker/experts. According to [10], preferences can be considered as consequence of our judgement and way of reasoning. Thus, it can be seen as a preliminary step towards making up our minds and giving a decision. In [28], they showed that there is direct relationship between the variation in decision making and the uncertainty in linguistic term used. This showed that how important the chosen linguistic term to be implemented in any decision making process since it can caused bias decision.

The paper is organized as follows: Section 2 describes the background of the study. Section 3 demonstrates the proposed model. Section 4 provides application example whereas Section 5 discusses about the comparison of our model with the existing one. Finally, Section 5 provides the final thoughts and conclusions.

2. Background

In this section, we briefly review Fuzzy Multi Criteria Decision Making [29], Fuzzy TOPSIS [30] and Interval Agreement Approach (IAA) method [7].

2.1. Fuzzy Multi Criteria Decision Making (MCDM)

Fuzzy MCDM [29] were initially introduced as fuzzy decision, based on two basic principles which are fuzzy goals and fuzzy constraints. Then, the fuzzy decision can be viewed as intersection between these two principles which higher membership function at this point indicates a maximizing decision.

Fuzzy TOPSIS

Fuzzy Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was introduced by [30]. This paper aims to extend ordinary TOPSIS into uncertain environment where the ratings of each alternative and the weight of each criterion are described by linguistic term which can be expressed as triangular fuzzy numbers (TFNs). By adapting the original concept in TOPSIS, the final ranking is determined by calculating the distance between each alternative with both the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS) simultaneously. The fuzzy TOPSIS method [30] involves nine steps which can be illustrated as follows:

1) Select a few persons as a committee of decision-makers (DMs) - identify the evaluation criteria.
2) DMs choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to criteria.
3) Aggregate (combine) the weight of criteria to get the aggregated fuzzy weight of criterion, and pool the decision makers' opinions to get the aggregated fuzzy rating of alternative under each criterion.
4) Construct the fuzzy decision matrix and the normalized fuzzy decision matrix – to ensure that the ranges of TFNs belong to [0, 1] and to standardize the different unit measure.
5) Construct the weighted normalized fuzzy decision matrix by multiplying the weight of criteria with the preference values.
6) Determine Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) – the value of FPIS is the closest value to 1 and the value of FNIS is the closest value to 0.
7) Calculate the distance of each alternative from FPIS and FNIS, respectively – A Euclidean distance formula is used as a distance measurement.
8) Calculate the closeness coefficient of each alternative – to determine a ranking order of all alternatives.
9) According to the closeness coefficient (CC), the ranking order of all alternatives can be determined – an alternative is said to be close to FPIS and farther from FNIS as CC value approaches 1.

In [30], the Fuzzy TOPSIS method is demonstrated with a MCDM problem to choose the best candidates in job application. Since then, most of researches used this basic theory as a platform to solve a decision making problem such as in [31-33]. We focus on the Fuzzy TOPSIS method as a basis comparison in this study as both Fuzzy TOPSIS and our proposed method, Fuzzy TOPSIS – IAA, has similar aim to determine the final ranking from a set of alternatives.

2.2. Interval Agreement Approach (IAA)

Interval agreement approach (IAA) was introduced by [7] based on previous work in [9, 34-35]. The main purpose of IAA is to accurately model information captured through the collection of crisp or uncertain intervals. Before this approach had been published, there is one more approach that has been introduced with similar aim and objective which called as Interval Approach (IA) [8]. However, in [7], they showed that IAA can outperform the IA performance which IAA present better model in capturing uncertainties and involve less calculation. Even a few attempts were made to improve IA method such as in [26-27], and called as Enhanced Interval Approach (EIA), they still required more calculations and need to make a few assumptions before getting to the final decision. By considering those facts, we chose IAA as one of basis theory in this proposed integration method. Specifically, we propose an integration of Fuzzy TOPSIS method with IAA method in order to provide better decision making process. The IAA will act to captured information and generating fuzzy sets from interval preferences. There are 2 possible stages in capturing uncertainty, namely:

a) Capturing the information from one or more intervals (with crisp or uncertain interval endpoints) collected from single source (e.g., single Decision Maker over one or multiple surveys)
b) Capturing the information from one or more intervals (with crisp or uncertain interval endpoints) collected from multiple sources (e.g., multi Decision Makers responses over one or multiple surveys).

There are four types of modelling information in IAA:

1) Modelling of interval-based data items such as survey responses (rather than just crisp points);
2) Modelling of uncertainty about the endpoints of those intervals (i.e., the modelling of uncertain intervals)
3) Modelling uncertainty about the above from a single source over one or more data capture cycles (e.g. surveys)
4) Modelling uncertainty arising from responses of multiple sources over one or more data capture cycles / surveys.

One of the strength in IAA during the modelling process is it distinguished the two types of uncertainties (intra- and inter-) in different degree of freedom. For example, the intra- uncertainty commonly captured by primary memberships $(y \in [0,1])$ and inter-uncertainty commonly captured by secondary memberships of zGeneral Type 2 Fuzzy Sets (ZGT2 FSs) [36], $(z \in [0,1])$. In this section, we briefly describe the process of modelling interval-based data from a single source. For the other type of modelling data, please see [7] for the details of approaches.

$$\mu(A) = y_1 \Big/ \bigcup_{i_1}^{N} \bar{A}_{i_1} + y_2 \Big/ \left(\bigcup_{i_1}^{N-1} \bigcup_{i_2=i_1+1}^{N-1} (\bar{A}_{i_1} \cap \bar{A}_{i_2}) \right) + y_3 \Big/ \left(\bigcup_{i_1}^{N-2} \bigcup_{i_2=i_1+1}^{N-1} \bigcup_{i_3=i_2+1}^{N-1} (\bar{A}_{i_1} \cap \bar{A}_{i_2} \cap \bar{A}_{i_3}) \right) + \dots + y_N \Big/ \left(\bigcup_{i_1=1}^{1} \dots \bigcup_{i_N=N}^{N} (\bar{A}_{i_1} \cap \bar{A}_{i_2} \cap \dots \cap \bar{A}_{i_N}) \right), \quad y_i = i/N. \tag{1}$$

2.3. Modelling one or more crisp intervals from a single source

For a given information source, (e.g., an expert), a T1 FS is created on the basis of the provided crisp interval(s). The resulting T1 FS encodes the intra-expert uncertainty present in the survey data over multiple survey iterations. The degree of membership y of the set over the survey domain x captures the number of intervals that are overlapping (are in agreement) at a particular point.

Consider a series of intervals $\bar{A}_n$, $n \in \{1, \dots, N\}$, where N is the number of intervals/survey iterations. In order to combine the intervals $\bar{A}_n$, T1 FS A was created by defining MF $\mu(A)$ as described in (1). The detail algorithm for generating T1 FS model can be find in [7].

3. Methodology

3.1. Implementation of IAA method

In [30] proposed extension of TOPSIS for group decision making under uncertain environment. One possible implication of this is that the use of linguistics term which expressed in TFN and given to experts in initial step as predefined scale to choose the appropriate importance ratings for criteria. Since then, most of researchers use this predefined scale which act as a guideline for experts to choose the importance rating. While previous approaches to capture opinion from experts using predefined scale such as in [30] and [37], have shown great promise, they require expert to have at least basic knowledge to express their judgement including preferences. In classical MCDM approach, the ratings and the weights of the criteria are given as exact values. However, in reality, by considering many conditions, exact data are inadequate to model real life situations. For example, preferences are often vague and cannot estimate the real meaning of preferences with an exact numerical data. The data also may be having some structures such as bounded data, ordinal data, interval data, and fuzzy data. In this paper, by considering the fact that, in some cases, determining precisely the exact meaning of preferences is difficult and the values are in interval based data, therefore, we extended the concept of Fuzzy TOPSIS to develop a methodology for solving MCDM interval-based data. Our aim is to precisely capture and model uncertainty in interval-based preferences given by experts that might exist while the decision making process was made. An overview of integration IAA, TOPSIS and MCGDM can be summarized in **Error! Reference source not found.**.

3.2. Fuzzy TOPSIS method with interval-based preferences data

In some cases, by considering the fact that determining precisely the exact value of the meaning of preferences is difficult and as a result of this, their values are considered as interval-based data, therefore, now we try to capture and model these interval-based preferences in Fuzzy TOPSIS method. In the first study of Fuzzy TOPSIS to solve MGCDM [30], the predefined scale for rating the importance of criteria is given to the experts as a guideline for them to give their own rating. However, this procedure will make an expert has less freedom to express their exact preferences based on real experience. For example, if DMs have an uncertainty to give an importance rating about a criterion, they may face less certain in giving the rating. Thus, they has less freedom to indicate their uncertainty level. In this study, we proposed to overcome this situation by giving the experts more freedom to choose their rating without using any predefined scale that associated with the linguistic term. This can be done by let the DMs express their uncertainty by specifying an interval, rather than specifying or choosing predefined crisp point associated with linguistic term such as on Likert scale. The width of the interval represents the expert's certainty in their answer, a slight interval is used when they are certain where on the scale the answer lies, and a broader one where they are less certain.

Assume that a decision group has K persons. Suppose $A_1, A_2, \dots, A_m$ are m possible alternatives which decision makers have to choose, $C_1, C_2, \dots, C_n$ are criteria, x_{ij} is the ratings of alternative A_i with respect to each criterion C_j and not known exactly and only know $x_{ij} \in [x_{ij}^L, x_{ij}^U]$, w_j is an importance weight of the Kth decision maker.

In classical method as in [30], x_{ij} and w_j can be aggregate as:

$$x_{ij} = \frac{1}{K} \left[x_{ij}^1 (+) x_{ij}^2 (+) \dots x_{ij}^K \right], \tag{2}$$

$$w_j = \frac{1}{K} \left[w_j^1 (+) w_j^2 (+) \dots w_j^K \right] \tag{3}$$

Table 1: Fuzzy Interval Decision Matrix

	C_1	C_2	…	C_n
A_1	$[x_{11}^L, x_{11}^U]$	$[x_{12}^L, x_{12}^U]$	…	$[x_{1n}^L, x_{1n}^U]$
A_2	$[x_{21}^L, x_{21}^U]$	$[x_{22}^L, x_{22}^U]$	…	$[x_{2n}^L, x_{2n}^U]$
A_m	$[x_{m1}^L, x_{m1}^U]$	$[x_{m2}^L, x_{m2}^U]$	…	$[x_{mn}^L, x_{mn}^U]$

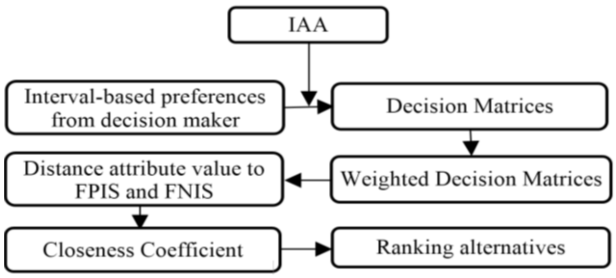

Fig. 1: An overview on the proposed IAA-TOPSIS-MCGDM

However, in this study, we proposed to implement IAA method [7] to capture uncertainty and at the same time aggregate all the preferences made by DMs and give a result of T1 FS. This method will implement IAA to extract the meaning of their preferences from interval-based data. By doing this, we can model and capture if there any uncertainty while the DMs give their preferences based on their own knowledge and experience. A step by step approach to implement IAA in Fuzzy TOPSIS method is described in this section. We focus on demonstrating to model individual crisp intervals for a single survey from multiple sources (DMs), which assuming that no uncertainty about the endpoints of the interval-based data. For further details on modelling other than this type of interval-based data, please see [7].

$$\mu(A) = z_1 \Big/ \cup_{i_1}^{N} \bar{A}_{i_1} + z_2/\left(\cup_{i_1}^{N-1} \cup_{i_2=i_1+1}^{N-1} (\bar{A}_{i_1} \cap \bar{A}_{i_2})\right) + z_3/\left(\cup_{i_1}^{N-2} \cup_{i_2=i_1+1}^{N-1} \cup_{i_3=i_2+1}^{N-1} (\bar{A}_{i_1} \cap \bar{A}_{i_2} \cap \bar{A}_{i_3})\right) + \dots + z_N/\left(\cup_{i_1=1}^{1} \dots \cup_{i_N=N}^{N} (\bar{A}_{i_1} \cap \bar{A}_{i_2} \cap \dots \cap \bar{A}_{i_N})\right),\ z_i = i/N. \quad (4)$$

3.3. The proposed implemented IAA with Fuzzy TOPSIS method

As already mentioned above, we assume the data do not include uncertainty information about the interval endpoints, i.e., interval are crisp. Then, as each expert has only ask for the preferences once, only a single interval is available per expert, avoiding the potential to model intra-expert uncertainty. A step by step approach will show how IAA can be implemented to model individual crisp intervals from multiple sources.

Step 1
The DMs (experts) were ask to rate an importance weight of each criteria in the particular decision-making problem. Each expert gives a single response to rate each criterion in the form of an interval contained in [0, 1]. Intervals are in the range 0 (Very Low) to 1 (Very High). The importance weight of each criterion can be express in matrix format as

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

$$W = [w_1, w_2, \dots, w_n]$$

where $x_{ij} \in [x_{ij}^L, x_{ij}^U]$, $i, j = 1, 2, \dots, n$ are in interval-based data. This step aims to generate T1 FS for the preference weight importance intervals given by each of expert. Consider a series of intervals $\bar{A}_n$, $n \in \{1, \dots, N\}$, where N is the number of intervals / survey collection iterations, as shown in Table 1. In order to aggregate intervals $\bar{A}_n$, we create T1 FS defined by MF $\mu(A)$ as described in (1).

Step 2
After generating T1 FS models for each expert, then proceed to aggregate this T1 FS to produce zGT2 FS that represents the inter-expert uncertainty. The set is computed in (4), where N is the number of sources/sets/experts. Combine all the individual zSlices into the complete zGT2 FS $\tilde{Z}$ as follow:

$$\tilde{Z} = Z_1 \cup Z_2 \cup \dots \cup Z_N$$

Step 3
Then, a MCDM problem can be concisely expressed in matrix format which the value in matrix is resulted from IAA.

$W_j = [w_1, w_2, \dots, w_n]$, where w_j is the weight of criterion C_j.

Step 4
Construct interval weighted normalized decision matrix using formulas

$$v_{ij}^L = w_i n_{ij}^L \ , j = 1, \dots, m,\ i = 1, \dots, n, \quad (5)$$

$$v_{ij}^U = w_i n_{ij}^U \ , j = 1, \dots, m,\ i = 1, \dots, n, \quad (6)$$

Step 5
Define the fuzzy positive-ideal solution (FPIS, A^+) and fuzzy negative-ideal solution (FNIS, A^-) as:

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+) = \left\{\left(\max_j v_{ij}^U \mid i \in I\right), \left(\min_j v_{ij}^L \mid i \in J\right)\right\} \quad (7)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) = \left\{\left(\min_j v_{ij}^L \mid i \in I\right), \left(\max_j v_{ij}^U \mid i \in J\right)\right\} \quad (8)$$

where I is associated with benefit criteria and J is associated with cost criteria.

Step 6
The distance of each alternative from positive ideal solution, A^+ can be calculated as

$$d_j^+ = \sum_{i \in I} \left(v_{ij}^L - \tilde{v}_i^+\right)^2 + \sum_{i \in J} \left(v_{ij}^U - \tilde{v}_i^+\right)^2,\ j = 1, 2, \dots, m, \quad (9)$$

Similarly, the distance of each alternative from negative ideal solution, A^- can be calculated as

$$d_j^- = \sum_{i \in I} \left(v_{ij}^U - \tilde{v}_i^-\right)^2 + \sum_{i \in J} \left(v_{ij}^L - \tilde{v}_i^-\right)^2,\ j = 1, 2, \dots, m \quad (10)$$

Step 7
A closeness coefficient is calculated to determine the ranking order of all alternatives using d_j^+ and d_j^- of each alternative A_j. The closeness coefficient of the alternative A_j with respect to A^+ is defined as

$$CC_j = \frac{d_j^-}{d_j^- + d_j^+} \quad (11)$$

Then, we can observe that alternative A_j is closer to the A^+ and farther from A^- as CC_j approaches 1. Therefore, based on CC_j value, we can determine the ranking order of all alternatives.

4. Application

This section illustrates the proposed method for decision-making problems with interval-based data. The case study here is based on case study presented in [30]. Suppose that a software company desires to hire a system analysis engineer. After preliminary screening, 3 candidates, A_1, A_2 and A_3 remain for further evaluation. A committee of 3 decision-makers, D_1 D_2 and D_3 has been formed to conduct the interview and to select the most suitable candidate. 5 benefit criteria are considered:

C_1: emotional steadiness
C_2: oral communication skill
C_3: personality
C_4: past experience
C_5: self-confidence

Step 1
Each expert gives their preferences on importance weight for each criterion. In classical method such as in [30], experts used predefined scale which gave them a guideline to make any decision. Example of the most widely used scale is as follow:

Table 2: An example of commonly used scale in TFNs parameter

Very Low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)

Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0)

This method will make experts less freedom to show their uncertainty if exist while choosing the weight of each criterion. However, in this study, we proposed that experts give their preferences in interval which range from 0 (Very Low) to 1 (Very High) without specified any linguistic variable associated with that value. By doing this, more uncertainty can be captured as experts can show the level of certain about the weight of each criterion by showing the width of interval. A slight interval is used when they are sure where on the scales answer lies and a broader one where they are less certain. For this case study, the experts give the following interval as their weight preferences for each criterion. The expert preferences are summarized as **Error! Reference source not found.**.

Table 3: Weight of criteria in interval-valued number

	DM_1	DM_2	DM_3
C_1	[0.7,1.0]	[0.5,1.0]	[0.5,0.9]
C_2	[0.9,1.0]	[0.9,1.0]	[0.9,1.0]
C_3	[0.9,1.0]	[0.7,1.0]	[0.5,1.0]
C_4	[0.9,1.0]	[0.7,1.0]	[0.5,1.0]
C_5	[0.3,0.7]	[0.5,0.9]	[0.6,0.9

We propose to aggregate all experts preferences using IAA [7]. In this case, there is no intra-expert uncertainty since each expert has only been surveyed once, only a single interval is available per expert and per criterion. As we mentioned above, there is no need to model the intra-expert uncertainty which the computing of agreement over multiple intervals collected from the same expert for the same criterion. The generated T1 Fuzzy Sets for each experts' interval using IAA can be done in (1). Figure 2 shows the generated T1 FS for Criterion 1 (C1) for each expert.
After generating T1 FS models for each expert, then we proposed to aggregate these T1 sets to produce zGT2 FS that represents inter-expert uncertainty. As described in Section 3 (Background of Study), in order to create a model capturing the outputs of all 3 experts, the secondary membership domain is divided into 3 levels, one for each level of agreement between the 3 experts, giving secondary membership degrees of $z_1 = {}^1/_3$, $z_2 = {}^2/_3$, $z_3 = 1$.

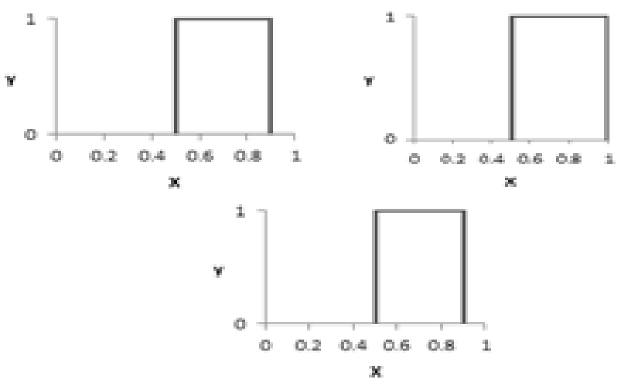

Fig. 2: Example of T1 FSs for each individual expert using IAA

To calculate the actual degree of secondary membership at given x, in (12) which can be computed using the number of T1 sets which intersect at that particular point in the domain of x and y. In this numerical example, the GT2 FS $\tilde{\tilde{Z}}$ consists of three zSlices, $\tilde{\tilde{Z}}_1$,$\tilde{\tilde{Z}}_2$ and $\tilde{\tilde{Z}}_3$. In (12), it shows the combination of the individual zSlices into the complete zGT2 FS $\tilde{\tilde{Z}}$. To obtain final defuzzified outputs, y_c (crisp number which is weight of criterion), in (11) was used [36].

$$\tilde{\tilde{Z}} = \tilde{\tilde{Z}}_1 \cup \tilde{\tilde{Z}}_2 \cup \tilde{\tilde{Z}}_3 \quad (12)$$

$$y_c = \frac{(z_1((yl_1+yr_1)/2)+z_2((yl_1+yr_1)/2)+\cdots+z_Iy_I)}{(z_1+z_2+\cdots+z_I)} \quad (13)$$

where yl_1 is the left value in interval and yr_1 is the right value of the interval. **Error! Reference source not found.** provides numeric details of all of the zSlices, as well as the centroids of each zSlice and the defuzzified value of the overall zGT2 set $\tilde{\tilde{Z}}$.

Table 4: zSlice Parameters with intervals associated primary and secondary membership

	$z_1 = 0.33$	$z_2 = 0.67$	$z_3 = 1$
$y = 1$	[0.5,1.0]	[0.5,1.0]	[0.7,0.9]
Centroid	0.33/0.75	0.67/0.75	1/0.8
Defuzzified	0.775		

The defuzzified value in **Error! Reference source not found.** is the importance weight for Criterion 1(C1) which aggregated using IAA technique. The rest of the importance weight (C2 – C5) were obtained with similar step as C1 and can be summarized in Table 5.

Table 5: The importance weight of criteria

	C1	C2	C3	C4	C5
Weight	0.7750	0.9500	0.8835	0.8835	0.6585

Step 2
In this step, Decision maker also need to give their preference rating of alternative with respect to each criterion. In classical method [30], the DM will use a predefined scale of linguistic variables as in

Table 2 to guide them. However, in this study, we avoid using that scale in order to capture more uncertainty that might exist in each DM while making preferences rating of alternatives. Again, experts give their preferences in interval which range from 0 (Very Poor) to 1 (Very Good) without specified any linguistic variable associated with that value. We can see the level of certain in each DM by observing preferences for example slight interval indicates they are sure where on the scales answer lies and a broader one where they are less certain. This gives the DM more freedom to express their preferences without hiding any uncertainties that might exist among them. In this numerical example, the collected preferences interval can be summarized in Table 6.

Table 6: The ratings of alternatives by DMs under all criteria

		Decision Maker		
		DM_1	DM_2	DM_3
C_1	A_1	[0.5,1.0]	[0.7,1.0]	[0.5,0.9]
	A_2	[0.5,1.0]	[0.7,1.0]	[0.5,0.9]
	A_3	[0.5,1.0]	[0.7,1.0]	[0.3,0.7]
C_2	A_1	[0.5,1.0]	[0.5,0.9]	[0.3,0.7]
	A_2	[0.7,1.0]	[0.7,1.0]	[0.9,1.0]
	A_3	[0.5,0.9]	[0.7,1.0]	[0.9,1.0]
C_3	A_1	[0.2,0.7]	[0.7,1.0]	[0.7,1.0]
	A_2	[0.7,1.0]	[0.9,1.0]	[0.7,1.0]
	A_3	[0.5,1.0]	[0.5,0.9]	[0.9,1.0]
C_4	A_1	[0.5,1.0]	[0.7,1.0]	[0.9,1.0]
	A_2	[0.7,1.0]	[0.9,1.0]	[0.9,1.0]
	A_3	[0.7,0.8]	[0.7,1.0]	[0.5,0.9]
C_5	A_1	[0.6,0.7]	[0.3,0.7]	[0.3,0.7]
	A_2	[0.5,1.0]	[0.5,0.9]	[0.7,1.0]
	A_3	[0.5,1.0]	[0.7,1.0]	[0.5,0.9]

Then, to get the aggregated value for all the preferences here, use similar technique as in Step 1. This step will aggregate all the decision maker preferences and generated the final ratings for each alternative using IAA technique. Then, calculate the centroid to get the defuzzification value for A_1 by considering C_1. **Error! Reference source not found.** provides numeric details of all of the zSlices, as well as the centroids of each zSlice and the defuzzified value of the overall zGT2 set $\tilde{\tilde{Z}}$.

Step 3
Construct fuzzy weighted decision matrix by multiplying the IAA weighted value in Table 5 with the IAA ratings.

Step 4
Fuzzy positive-ideal solution (FPIS, A^+) and fuzzy negative-ideal solution (FNIS,A^-) defined as:

$$A^+ = [0.6006, 0.8550, 0.7952, 0.8247, 0.5103]$$

$A^{-} = [0.5107, 0.6099, 0.6483, 0.6775, 0.3786]$

where A^{+} and A^{-} are closest from (1, 1, 1, 1, 1) and (0, 0, 0, 0, 0) respectively among the criteria ratings value.

Step 5
Calculate the distance of each alternative from A^{+} and A^{-}. The distance of each alternative to A^{+} and A^{-} is calculated using the Euclidean distance formula.

Step 6
Calculate closeness coefficient (CC_i) of each alternatives.

Step 7
According to closeness coefficient, rank the alternatives based on CC_i value.

$A_2 > A_3 > A_1$

The best selection in this case study is candidate A2.

5. Comparison

In general, multi criteria problems usually involve uncertain and imprecise data, and fuzzy set theory is an ideal solution for this kind of problem. In this study, we avoid to use a predefine scale which associated with the linguistic variable as proposed by [30]. However, we proposed to collect an interval form preferences and aggregate those interval using the new IAA technique proposed by [7]. We use the same case study in [30] to illustrate our proposed method with a few modification in data form. In our study, we use interval-based data which originally given by decision maker and we aggregate those interval preferences using IAA. The decision of this proposed method is same as in [30] which the best selection is Candidate A2, then followed by candidate A3 and A1.
However, the closeness coefficient (CC_i) values in our proposed study are higher for all the alternatives which make the decision can be made in clear and obviously stage. For example, in [30], the CC_1 value is 0.62, CC_2 is 0.77 and CC_3 is 0.71. But, in our proposed method, the CC_1 value is 0.3010, CC_2 is 0.998 and CC_3 is 0.5765. So, the decision can be made easily with the significant value different among those alternatives. The ranking are similar for both method, but the CC_i values are different. In our proposed method, decision maker have their own freedom to express their preferences using interval form based on range given. Compared to [30] method, this step will capture more uncertainty (if exist) in the preferences given by each decision makers. Furthermore, there are no assumptions were made on the initial step of this process. The interval preferences collected from decision maker were used originally to generate T1 FSs without any modification. Hence, this method will minimises any loss of information during the decision making process.
Also, it can be noted that in the proposed method, the different degrees of freedom are used to represent different kind of uncertainties. For capturing intra-expert uncertainty, it is commonly use primary membership while for inter-expert uncertainty, it is commonly use secondary membership. For this case study, it can be noted that in the resulting set the primary degree of membership is 1 over the whole support (see Figure 2), indicating that there is no intra-expert uncertainty.

6. Conclusion

This paper presents a proposed method to integrate IAA with Technique for order performance by similarity to ideal solution (TOPSIS) for Multi-Criteria Group Decision Making (MCGDM). The proposed method utilizes IAA to model the preferences (word) given by decision maker in the initial step of TOPSIS-MCGDM. To date, there is none other MCGDM method utilized IAA to accurately model preferences. By doing this, the proposed method is able to capture the information from one or more intervals (with crisp or uncertain interval endpoints) associated with preferences given by decision maker. IAA is capable in minimizing the number and scope of assumptions during the word/preferences model. This integration made MCGDM process more precise and flexible since decision maker do not depends on predefined scale to select as a rating for importance weight of criterion and rating for alternatives under all criteria. For future works, an attempt is needed to implement it with the real-world decision-making problem which is complex to solve.

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