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Research paper

Determination of the Parameters Controlling Mixed Convection in Double Lid Driven Shallow Rectangular Cavity Uniformly Heated

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Abstract

Mixed convection, in double-lid driven horizontal rectangular cavity filled with a Newtonian fluid and subjected to uniform heat flux along the vertical walls, has been studied numerically, via the finite volume method, and analytically, by using the parallel flow concept. A good agreement has been found between the results of the two approaches. The frontiers of transition from one regime to another of convection (forced-mixed and mixed-natural) were determined by a modified Richardson Number. Moreover, it has been found that the parameter which specifies the natural and the forced convection regime is Gr/Re3.

Keywords: *finite volume method, heat transfer, mixed convection, lids-driven cavity, parallel flow*

1. Introduction

The mixed convection in lid-driven cavities has received considerable attention from researchers. This phenomenon is commonly encountered in many engineering applications including cooling of electronic devices, food processing, float glass production, thermal hydraulics of nuclear reactors [1], dynamics of lakes, crystal growth, flow and heat transfer in solar ponds [2], and lubrication technologies [3].It will be interesting to help to elaborate some proceeding in order to lead to best climate conditions of habitat building and smart-cities.

In the closed cavities, the mixed convection flow is induced by both the shear force caused by the movement of the wall and the buoyancy force produced by thermal non-homogeneity of the cavity boundaries where the forced convection as well as the natural convection effects are the comparable magnitudes. Thus, mixed convection occurs if the effect of buoyancy forces on a forced flow or the effect of forced flow on a buoyancy flow is significant.

On the other hand, most of the investigations, concerning such phenomenon in fluid filed closed square cavities, were carried out. These works were mainly focused on the study of mixed convection flow in square cavities in which one or two walls are driven. Comparatively, few works are concerned with the mixed convection in rectangular driven cavities, as in the present case, which may reveal something different as reported in the study conducted in the same way by Lamsaadi et al. [4-5], where all the walls are motionless.

Otherwise, the majority of investigations concerning rectangular driven cavities dealt with Dirichlet boundary conditions on temperature. Thus, as we know, the problem of mixed convection heat transfer of Newtonian fluids in a lead-driven enclosure subjected to thermal boundary conditions of Neumann type (i.e., heat fluxes imposed to the boundaries) is not yet examined. So, in order to know more about the effect of the boundary conditions kind on flow and heat transfer, the present paper deals with such a problem within a double lid driven horizontal rectangular cavity filled with a Newtonian fluid. The cavity is submitted to constant fluxes of heat from its short vertical sides, while its long horizontal boundaries are insulated and moving in opposite directions. In what follows, a numerical solution of the full governing equations is obtained, for a wide range of the governing parameters. In addition, an analytical solution, valid for stratified flows in slender enclosures, is derived on the basis of the parallel flow concept. Useful correlating relations between Grashof (Gr) and Reynolds (Re) numbers to realize the contribution of mixed convection to heat transfer are also proposed.

2. Mathematical Formulation

The geometry of the problem herein investigated is depicted in Fig. 1. The system is made of a Shallow rectangular cavity of height H’ and length L’, filled with a Newtonian fluid and submitted to a uniform density of heat flux, q', from its short vertical sides, while its long horizontal boundaries are insulated. The upper and lower walls are assumed to slide in the opposite directions, with constant speed u_0' .

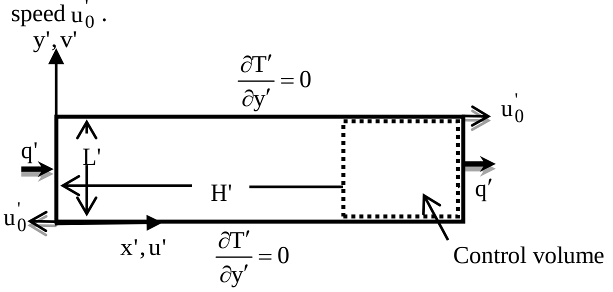


Fig. 1: Schematic view of the geometry and coordinates system

In this study, all the physical properties of the fluid are considered constant except the density in the buoyancy term, which obeys the Boussinesq approximation. It is assumed that the third dimension of the cavity is large enough that the flow and heat transfer are two-dimensional. It is also assumed that the flow is laminar, the fluid is Newtonian and incompressible and that the viscous dissipation and the radiation heat transfer are negligible. Based on the above assumptions and using the characteristic scales H', $\rho u_0^{'2}$, $H'/u_0^{'}$, $u_0^{'}$ and $q'H'/\lambda$, corresponding to length, pressure, time, velocity, and temperature, respectively, the dimensionless governing equations and the corresponding boundary conditions, written in terms of velocity components (u, v), pressure (p) and temperature (T) are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+\frac{Gr}{Re^2}T \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Re\,Pr}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{4}$$

$$u=v=0 \text{ and } \frac{\partial T}{\partial x}=-1 \text{ for } x=0 \text{ and } x=A \tag{5}$$

$$u+1=v=0 \text{ and } \frac{\partial T}{\partial y}=0 \text{ for } y=0 \tag{6}$$

$$u-1=v=0 \text{ and } \frac{\partial T}{\partial y}=0 \text{ for } y=1 \tag{7}$$

In addition, to analysis the flow structure, the stream function, ψ, related to the velocity components via $u=\frac{\partial \psi}{\partial y}$ and $v=-\frac{\partial \psi}{\partial x}$ (ψ = 0 on all boundaries) is used.

The above equations give rise to some dimensionless parameters that govern the problem, namely, the aspect ratio of the enclosure, A, the Prandlt, Pr, Reynolds, Re, and Grashof, Gr, numbers. For the last four, the expressions are

$$A=\frac{L'}{H'},\ Pr=\frac{\nu}{\alpha},\ Re=\frac{u_0^{'}H'}{\nu},\ Gr=\frac{g\beta q' H^{'4}}{\nu^2\lambda} \tag{8}$$

where ρ, ν, α, g, β and λ are the density of fluid, the kinematic viscosity, the thermal diffusivity, the gravitational acceleration, the thermal expansion coefficient and the thermal conductivity, respectively.

The mean heat transfer, through the fluid layer filling the cavity, can be expressed in term of the average Nusselt number defined as

$$\overline{Nu}=\int_0^1 \frac{1}{(\partial T/\partial x)_{x=A/2}}dy \tag{9}$$

3. Numerical Approach

The governing equations of the problem (1)–(4) associated with (5)–(7), were solved numerically, using a finite volume method and SIMPLER algorithm in a staggered uniform grid system [6]. The convergence has been considered as reached when $\sum_{i,j}\left|f_{i,j}^{k+1}-f_{i,j}^{k}\right|<10^{-5}\sum_{i,j}\left|f_{i,j}^{k+1}\right|$, where $f_{i,j}^{k}$ stands for the value of u, v, p, or T at the k^{th} iteration level and grid location (i, j) in the plane (x, y). In the limit of the values selected for A, Pr, Gr and Re, a uniform grid of 381×121 has been judged sufficient to model accurately the flow and temperature fields within a cavity of A = 24.

Typical numerical results, in terms of streamlines and isotherms, are presented in Figure 2, obtained, for A = 24, Gr = 10^3 and various values of Re.

As appears, from this figure, the flow is parallel to the horizontal boundaries and the temperature is linearly stratified in the x-direction of the core region. The approximate analytical solution, developed in the next section, relies on these observations.

4. Parallel Flow Approach

On the basis of Figure 2, the following simplifications, in the central part of the cavity, can be made:

$u(x,y)=u(y)$, $v(x,y)=0$, $\psi(x,y)=\psi(y)$ and

$$T(x,y)=C(x-A/2)+\theta(y) \tag{10}$$

where θ(y) represents the transverse evolution of the temperature for a given abscissa *x* in the parallel flow region and C is unknown constant temperature gradient in x-direction (see for instance Refs. [4-5] and [7-8] for natural and mixed convection respectively).

According to those approximations, the system (1-4), associated with the boundary conditions (5-7), becomes:

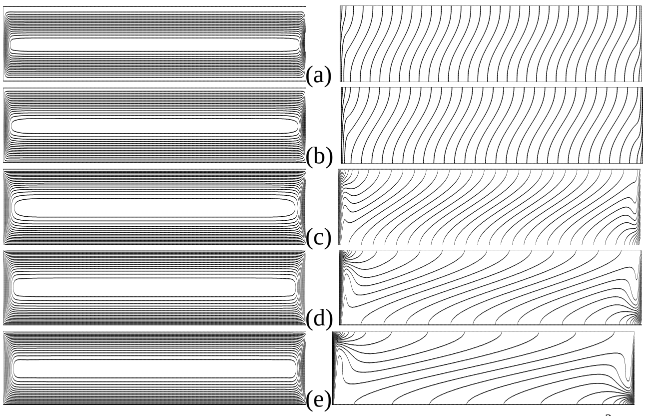


Fig.2: Streamlines (left) isotherms (right) for A = 24, Pr = 7, Gr = 10^3 and various values of Re ((a) Re = 0.1, (b) Re = 1, (c) Re = 5, (d) Re = 10 and (e) Re = 15) (scale is not respected)

$$\frac{d^3u(y)}{dy^3}=\frac{Gr}{Re}C \text{ and } \frac{d^2\theta(y)}{dy^2}=C\,Re\,Pr\,u(y) \tag{11}$$

with

$$u(0)+1=\left.\frac{d\theta\theta(y}{dy}\right)_0=0,\ u(1)-1=\left.\frac{d\theta(y)}{dy}\right)_1=0 \tag{12}$$

$$\int_0^1 u(y)dy=0 \text{ and } \int_0^1 \theta(y)dy=0 \tag{13}$$

as boundary, return flow and mean temperature conditions, respectively.

Using such an approach, the solutions of (11), satisfying (12-13) are:

$$u(y)=\frac{GrC}{12Re}\left(2y^3-3y^2+y\right)+\left(3y^2-2y\right)-\left(3y^2-4y+1\right) \quad (14)$$

$$\theta(y)=\frac{GrPrC^2}{1440}\left(12y^5-30y^4+20y^3-1\right)+\frac{RePrC}{60}\left[\left(15y^4-20y^3+2\right)-\left(15y^4-40y^3+30y^2-3\right)\right] \quad (15)$$

On the other hand, according to Bejan [9], the energy balance in x-direction gives

$$-C+Pe\int_0^1 u\theta dy=1 \quad (16)$$

which, when substituted to Eqs. (14) and (15), gives the following transcendental equation :

$$\frac{Gr^2Pr^2}{362880}C^3-\frac{GrPr^2Re}{1680}C^2+\left[\frac{Pr^2Re^2}{30}+1\right]C+1=0 \quad (17)$$

whose solution, via Newton-Raphson method, for given Re and Gr, leads to C.

The mean Nusselt number can be expressed as

$$\overline{Nu}=-\frac{1}{C} \quad (18)$$

5. Results and Discussion

The results presented here are obtained for A = 24, $1\leq Gr\leq 10^7$, $0.1\leq Re\leq 200$ and Pr = 7 (water-based solutions), which means that the only governing parameters are Gr and Re.
Moreover, according to the Table 1, the results of the two approaches adopted in this study agree perfectly, at least for the governing parameters selected values, which validates the parallel flow assumption used in section 4 and justifies the choice of A = 24 as a large aspect ratio approximation value. Another confirmation of this is given by figure 3, where the perfect agreement between such approaches is obtained.

Table 1: Values of $\overline{Nu}$ for Gr = 10^4.

Re	1	10	20
Numerical solution	28.9081	181.227	664.892
Analytical solution	28.8971	180.872	663.160

Several geometric configurations have been treated in the past and are reported in the literature. It is found that the value of the group Gr/Re^n (modified Richardson number or the mixed convection parameter) was systematically used to delimit the convective regimes, where the exponent n depends on the geometry, thermal boundary condition and the fluid. For example, in confined mediums, Turki et al [10] observed that, for the flows in a square cavity filled with Newtonian fluid, the mixed convection regime is defined by the criterion $24.4 < Gr/Re^{1.2} < 293.4$ for Pr = 6.97.
It is of practical interest in the computation of heat transfer to distinguish the conditions under which a given convection may be regarded as pure (either natural or forced) from those under which it must be regarded as mixed. The importance of this matter stems from the fact that frequently, only results for the pure convections are available. In this work, on the basic of the work conducted by Sparrow et al. [11], a convection will be considered to be effectively pure (either natural or forced) if the heat transfer deviates by no more than 5 percent from the value associated with the completely pure convection. This 5% criterion is used to determine the parameters characterizing mixed convection. Thus, in order to differentiate the three convective regimes (forced, natural and mixed convection), the following relative variations are introduced:

$$\varepsilon_f=\frac{\left|\overline{Nu}-\overline{Nu_f}\right|}{\overline{Nu_f}} \text{ and } \varepsilon_n=\frac{\left|\overline{Nu}-\overline{Nu_n}\right|}{\overline{Nu_n}} \quad (19)$$

where $\overline{Nu_f}$ and $\overline{Nu_n}$ are the mean Nusselt numbers correspond to pure forced and natural convection, respectively. It is assumed that the forced and natural convections are predominant when $\varepsilon_f\leq 5\%$ and $\varepsilon_n\leq 5\%$, respectively. When $\varepsilon_f>5\%$ or $\varepsilon_n>5\%$, the convective regime will be qualified mixed. On the basis of Eq. (19), the calculations performed for Re and Gr, varying in their ranges, allowed to construct the diagram shown in Figure 3. The places of the points (log(Gr), log(Re)), obtained analytically (depicted by solid lines) and numerically (represented by symbols) and correspond approximately to ε_f = 5%and ε_n = 5%, are almost the parallel straight lines. These results are correlated in terms of log(Gr) versus log(Re) in the following mathematical form :

$$\log(Gr)=3\log(Re)+b \quad (20)$$

where b is the coefficient which set equal to 5.64948 and 0.68494 for pure natural and forced convection, respectively.
The results obtained with Eq. (20) are presented in Figure 3 with dashed lines. They are seen to be in good agreement with both the parallel flow solution and the numerical results. In addition, the straight lines defined by Eq. (20) separate the quarter plane (Gr, Re) into three zones. A first zone, located below the straight line (1), where the heat transfer is dominated by forced convection. A second zone, located above the straight line (2), wherein the natural convection is predominant. Finally, the third zone delimited by the two straight lines, where natural and forced convection similarly contribute to heat transfer. This clearly shows that the transition from one regime to another, for the same value of Re, requires a high buoyancy force.

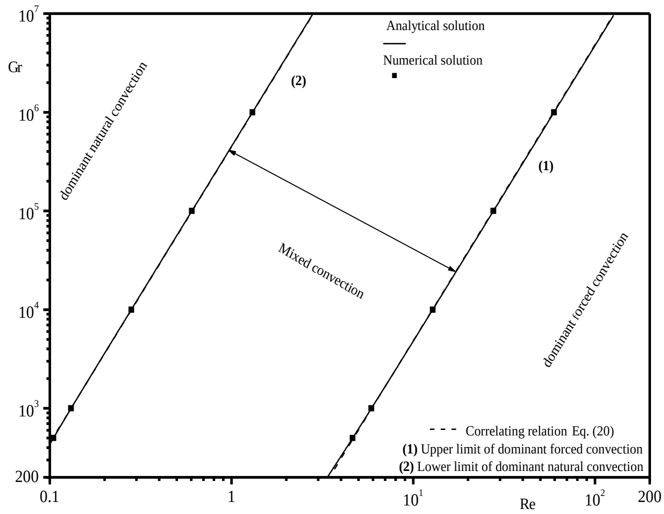


Fig.3: Diagram characterizing the different convective regimes

In summary, the mixed convection regime is defined by the criteria:

$$4.841<\frac{Gr}{Re^3}<446149.08 \quad (21)$$

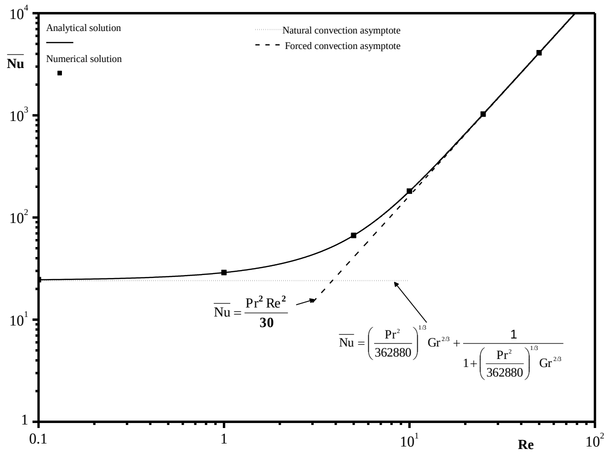
For further analysis of the problem, the effect of Re and Gr on heat transfer rate, has been studied, by varying Gr at constant Re or vice versa (figures 4 and 5).
On Fig. 4, we plot the dimensionless heat transfer as function of Re for Gr = 10^4. For small values of Re, the flow is close to pure natural convection; while for large values of this parameter, the flow is approaching pure forced convection, where the buoyancy effect is negligible. With this in mind, there are included lines

representing the results for the pure natural and forced convection flows (dashed lines). The equations of these limiting lines are expressed in the form of the following correlations:

$$\overline{Nu} = \left(\frac{Pr^2}{362880}\right)^{\frac{1}{3}} Gr^{\frac{2}{3}} + \frac{1}{1+\left(\frac{Pr^2}{362880}\right)^{\frac{1}{3}} Gr^{\frac{2}{3}}} \text{ pure natural convection,} \quad (22)$$

$$\overline{Nu} = \frac{Pr^2 Re^2}{30} \text{ pure forced convection.,} \quad (23)$$

The interesting feature of this plot is that the heat transfer results for mixed convection show a surprisingly small deviation from the envelope formed by the two limiting lines. In fact, the heat transfer prediction based on the envelope lines would be in error somewhat unexpected finding has important practical implications.



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g.4: Heat transfer for $Gr = 10^4$, $Pr = 7$ and various values of Re In order to examine the influence of Gr on heat transfer rate, the quantity $\overline{Nu}$ was plotted against Gr, for various Re = 5, in Figure 5. On this graph, small values of Gr correspond to predominantly forced convection flows; while large values of Gr correspond to nearly natural convection flows, where the shear effect is negligible. The limiting lines (dashed lines) corresponding to pure forced and natural convection are also shown, and their equations are given as follows:

$$\overline{Nu} = 1 + \frac{Pr^2 Re^2}{30} \text{ pure forced convection,} \quad (24)$$

$$\overline{Nu} = \left(\frac{Pr^2}{362880}\right)^{\frac{1}{3}} Gr^{\frac{2}{3}} \text{ pure natural convection} \quad (25)$$

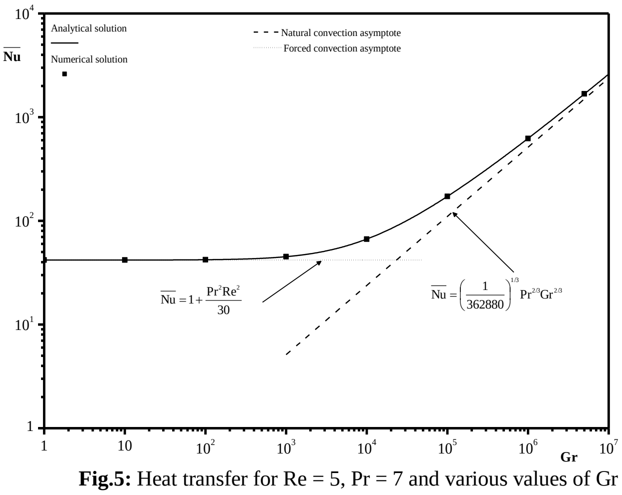

Fig.5: Heat transfer for Re = 5, Pr = 7 and various values of Gr

In addition to that, the figures 4 and 5 show that, for fixed Gr/(Re), the heat transfer is almost constant at the low value of Re/(Gr), then increases relatively slowly and, finally increases very rapidly. The value of Re/(Gr) corresponding to the beginning of the increase of $\overline{Nu}$ depends on Gr/(Re) : it is smaller when Gr/(Re) is small. This value is approximately expressed as $(Gr/446149.08)^{1/3}/(4.841\,Re^3)$. When Re/(Gr) is getting close to $(Gr/4.841)^{1/3}/(446149.08\,Re^3)$, the forced convection is predominant/(natural convection is predominant).

These figures highlight three regimes: at the extreme values of Re/(Gr), natural or forced convection is predominate; while at intermediate ones, the two transfer modes have a similar importance. In this mixed convection zone, the flux transferred is always less than the sum of those transferred by natural and forced convection.

Finally, the general remarks about figure 4 apply as well to figure 5 and the evolutions of $\overline{Nu}$ with Re or Gr confirm well the criteria obtained with Eq. (21) for the mixed convection regime. The tendencies of the $\overline{Nu}$ curves are similar to those obtained by Turki et al [10], for flows in a square cavity filled with Newtonian fluid with Pr = 6.97.

6. Conclusion

Mixed convection in a two-dimensional horizontal shallow enclosure, filled with Newtonian fluid with Pr = 7, has been studied, by both numerical and analytical ways, in the case where both short vertical sides are submitted to uniform heat fluxes while the long horizontal ones are assumed to be insulated and uniformly moving in opposite directions. The full partial differential equations, governing the problem, have been solved numerically using a finite volume method. The computations, which have been limited to water-based solutions, with Pr = 7, have been carried out with governing parameters, Re, and Gr, varying, respectively, in the ranges $0.1 \le Re \le 200$ and , $1 \le Gr \le 10^7$. Analytical solution is derived on the basis of a parallel flow assumption in the core region of the enclosure. It has been found that:

- In the limit of the selected values of the governing parameters, analytical results, agree perfectly with the numerical ones.
- The parameter which specifies the natural and the forced convection regime is Gr/Re^3
- The mixed convection regime is defined by the criteria $4.841 < Gr/Re^3 < 446149.08$.
- Flow and temperature fields strongly depend on the group Gr/Re^3, measuring the relative importance of both lid and buoyancy-driven effects.
- Increasing the (Gr/Re^3) is, in general, associated with the decrease of heat transfer rate, due to shear flow, and the increase of that due to buoyancy-driven flow.
- The elaborated correlations can help to design thermal system of obvious practical interest.

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