International Journal of Engineering & Technology, 7 (3.24) (2018) 419-421

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

An Enhanced Critical Path Method to solve Time-Cost Trade-Off using Type-2 Pentagonal Fuzzy numbers

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Abstract

Critical path method is widely used for analyzing and managing task sequences in complex projects. Crashing is a schedule compression technique used to reduce the project duration.An approach has been made to decide the optimal crash plan to complete the project in minimal duration.Type-2 Pentagonal fuzzy numbers are used in the time-cost trade-off problem which includes ranking process.The project crashing problem is solved using deterministic solution technique.

Keywords: *Crashing, Fuzzysets, Ranking, ProjectNetwork, Type-2PentagonalFuzzynumber*

1. Introduction

Project management is the discipline of using established principles,procedures and policies to manage a project from conception through completion.The primary challenge of project management is to achieve all of the project goals within the given constraints. The primary constraints are scope,time,quality and budget.The secondary and most ambitious challenge is to optimize the allocation of necessary inputs and apply them to meet pre-defined objectives. Project managers mainly focus on finding the most effective way to complete the project within a specified completion time.
Crashing is reducing the completion time of a project by sharply increasing manpower and or other expences. Shortening the duration of an activity will normally increase its direct cost.Therefore there is some optimum project duration-a balance between excessive direct cost for shortening the project and excessive indirect cost for lengthening the project.Crashing is widely used by the project managers in complex projects

2. Preliminaries

2.1. Fuzzy Set

A fuzzy set A in X is characterized by a membership function $f_A(x)$ which associates with each point in X which is a real number defined in the interval [0,1],with the values of $f_A(x)$ at x representing the grade of membership of x in A. Thus the nearer the value of $f_A(x)$ to unity,the higher the grade of membership of x in A.

2.2. Pentagonal Fuzzy Number

A pentagonal fuzzy number $\bar{A}=(a_1,a_2,a_3,a_4,a_5)$ have the following properties

(1)$\mu_{\bar{A}}(x)$is a function which is continuous in the interval [0,1]

(2) $\mu_{\bar{A}}(x)$is strictly increasing and continuous function on $[a_1,a_2]$ *and* $[a_2,a_3]$

(3)$\mu_{\bar{A}}(x)$ must be continuous and strictly decreasing on $[a_3,a_4]$ *and* $[a_4,a_5]$

2.3. Type-2 Pentagonal Fuzzy Number

A type-2 pentagonal fuzzy number $\bar{P}$ on R should be $\bar{P}=\{x,\mu_p^1(x), \mu_p^2(x), \mu_p^3(x), \mu_p^4(x), \mu_p^5(x));x\varepsilon\ R\}$ and $\mu_p^1(x)<= \mu_p^2(x)<= \mu_p^3(x)<= \mu_p^4(x)<= \mu_p^5(x)$ for all $x\varepsilon R$.

Denote $\bar{P}=(P_1,P_2,P_3,P_4,P_5)$ where $P_{1=}(p_1^I, p_1^J, p_1^K, p_1^L, p_1^M)$, $P_{2=}(p_2^I, p_2^J, p_2^K, p_2^L, p_2^M)$, $P_{3=}(p_3^I, p_3^J, p_3^K, p_3^L, p_3^M)$, $P_{4=}(p_4^I, p_4^J, p_4^K, p_4^L, p_4^M)$, $P_{5=}(p_5^I, p_5^J, p_5^K, p_5^L, p_5^M)$.

3. Arithmetic Operations

Let $\bar{P}=(P_1, P_2,P_3, P_4, P_5) = ((p_1^I, p_1^J, p_1^K, p_1^L, p_1^M,), (p_2^I, p_2^J, p_2^K, p_2^L, p_2^M,), (p_3^I, p_3^J, p_3^K, p_3^L, p_3^M,), (p_4^I, p_4^J, p_4^K, p_4^L, p_4^M,), (p_5^I, p_5^J, p_5^K, p_5^L, p_5^M,))$ and

$\bar{Q}=(Q_1, Q_2,Q_3, Q_4, Q_5,) = ((q_1^I, q_1^J, q_1^K, q_1^L, q_1^M,), (q_2^I, q_2^J, q_2^K, q_2^L, q_2^M,), (q_3^I, q_3^J, q_3^K, q_3^L, q_3^M,), (q_4^I, q_4^J, q_4^K, q_4^L, q_4^M,), (q_5^I, q_5^J, q_5^K, q_5^L, q_5^M,))$ be two type-2 pentagonal fuzzy number.Then we define,

Addition

$\bar{P}+\bar{Q} = ((p_1^I+q_1^I, p_1^J+q_1^J, p_1^K+q_1^K, p_1^L+q_1^L, p_1^M+q_1^M), (p_2^I+q_2^I, p_2^J+q_2^J, p_2^K+q_2^K, p_2^L+q_2^L, p_2^M+q_2^M), (p_3^I+q_3^I, p_3^J+q_3^J, p_3^K+q_3^K, p_3^L+q_3^L, p_3^M+q_3^M), (p_4^I+q_4^I, p_4^J+q_4^J, p_4^K+q_4^K, p_4^L+q_4^L, p_4^M+q_4^M), (p_5^I+5^I, p_5^J+q_5^J, p_5^K+q_5^K, p_5^L+q_5^L, p_5^M+q_5^M))$

Subtraction

$\bar{P}-\bar{Q} =((p_1^I-q_5^M , p_1^J-q_5^L , p_1^K-q_5^K , p_1^L-q_5^J , p_1^M-q_5^I),(p_2^I-q_4^M , p_2^J-q_4^L , p_2^K-q_4^K , p_2^L-q_4^J , p_2^M-q_4^I),(p_3^I-q_3^M , p_3^J-q_3^L , p_3^K-q_3^K , p_3^L-q_3^J ,$

$p_3^{M}-q_3^{I}$), ($p_4^{I}-q_2^{M}$ $p_4^{J}-q_2^{L}$, $p_4^{K}-q_2^{K}$, $p_4^{L}-q_2^{J}$, $p_4^{M}-q_2^{I}$), ($p_5^{I}-q_1^{M}$, $p_5^{J}-q_1^{L}$, $p_5^{K}-q_1^{K}$, $p_5^{L}-q_1^{J}$, $p_5^{M}-q_1^{I}$))

4. Ranking Process

Let F(R) be the set of type-2 Pentagonal fuzzy number

Assume $\bar{P}=(P_1,P_2,P_3,P_4,P_5)=((p_1^{I}, p_1^{J}, p_1^{K}, p_1^{L}, p_1^{M}), (p_2^{I}, p_2^{J}, p_2^{K}, p_2^{L}, p_2^{M}), (p_3^{I}, p_3^{J}, p_3^{K}, p_3^{L}, p_3^{M}), (p_4^{I},p_4^{J}, p_4^{K}, p_4^{L}, p_4^{M}), (p_5^{I},p_5^{J},p_5^{K},p_5^{L},p_5^{M}))$ then we define

$\bar{R}(\bar{P})=(p_1^{I}+p_1^{J}+p_1^{K}+p_1^{L}+p_1^{M}+p_2^{I}+p_2^{J}+p_2^{K}+p_2^{L}+p_2^{M}+p_3^{I}+p_3^{J}+p_3^{K}+p_3^{L}+p_3^{M}+p_4^{I}+p_4^{J}+p_4^{K}+p_4^{L}+p_4^{M}+p_5^{I}+ p_5^{J}+p_5^{K}+p_5^{L}+p_5^{M})/25.$

Orders of F(R) is given by

$$\hat{R}(\bar{P})\geq \hat{R}(\bar{Q}) \text{ iff } \bar{P}\geq \bar{Q},$$
$$\hat{R}(\bar{P})\leq \hat{R}(\bar{Q}) \text{ iff } \bar{P}\leq \bar{Q}$$
$$\text{and } \hat{R}(\bar{P})= \hat{R}(\bar{Q}) \text{ iff } \bar{P}= \bar{Q}$$

5. Algorithm

A type used here is to examine the optimal solution of fuzzy for Time-Cost Trade-off by applying Type-2 pentagonal fuzzy numbers.

Step-1: Produce the network diagram.

Step-2: First examine the critical path and the normal duration.Then the critical activities is also found .

Step-3: Now total normal cost and the normal duration of the project is determined.

Step-4: Apply the formula

Cost of the Project = Direct cost of the project+(Indirect cost of the project * Project duration).

Step-5: Calculate cost slope by

$$\text{Cost Slope} = \frac{Crash\ cost-Normal\ cost}{Normal\ time-Crash\ time}$$

Step-6: Arrange the activities of cost slope in ascending using ranking process.

Step-7: Calculate the cost slope by finding the Crash time and Crash cost for all the activity .

Step-8: Now find the cost of the project using,

Cost of the Project = ((Direct cost of the project + crashed activity crashing cost + (Indirect cost of the project * Project Duration))

Step-9: Meanwhile crash all the project activities.

Step-10: Identify the critical path and noncritical paths and then trace the critical activities.

Step-11: Continue the same procedure to find the next least cost slope till the critical path is fully crashed

Step-12: In this stage all the activities are crashed to its maximum.Non critical activities which is not crashed do not affect the project duration.

6. Numerical Example

Consider the following example where the normal time,crash time,normal cost ,crash cost are all type-2 pentagonal fuzzy numbers which is given in Table 1 and Table 2.

Table-1: Duration of the Project

Activity	Normal Time(N_T)	Crash Time(C_T)
1→2	(9,7,5,4,3),(6,8,4,5,10),(8,11,15,10,5),(12,9,8,6,5),(5,21,9,8,7)	(4,3,3,2,1),(3,5,1,2,5),(5,6,9,5,2),(5,4,4,2,3),(3,11,5,4,3)
1→3	(5,6,9,10,3),(6,5,2,3,7),(11,2,3,4,),(10,3,5,2,11),(15,12,3,8,2)	(2,3,3,5,1),(3,4,1,1,4),(6,1,1,2,2),(5,1,2,1,6),(8,7,1,4,1)
1→4	(15,6,3,6,9),(7,11,13,5,9),(17,3,6,11,8),(16,15,13,5,9),(11,6,3,10,8)	(9,3,2,2,3),(4,6,8,3,3),(9,1,3,6,5),(11,9,9,4,3),(6,3,1,6,6)
2→5	(6,5,3,4,3),(6,7,3,4,5),(6,8,4,3,5),(8,9,6,4,3),(5,6,4,5,3)	(4,3,1,4,2),(4,5,1,3,2),(4,5,3,1,3),(5,4,3,3,1),(3,4,3,3,1)
3→6	(15,8,2,12,9),(8,5,3,7,8),(12,3,9,2,5),(19,3,5,2,3),(6,7,11,3,8)	(6,4,1,5,4),(3,2,1,3,2),(5,1,4,1,2),(7,1,3,1,2),(3,4,5,1,4)
4→6	(12,8,7,9,3),(10,4,7,2,6),(15,13,6,9,8),(10,8,14,4,15),(9,3,8,6,4)	(6,4,3,4,2),(6,2,3,1,3),(8,7,3,4,4),(6,4,7,2,6),(5,1,4,3,2,)
5→6	(2,4,4,3,2),(4,2,6,2,5),(2,5,3,6,2),(2,7,9,2,4),(5,7,7,2,3)	(1,2,3,2,1),(2,1,2,1,3),(1,3,2,2,1),(1,4,3,1,2),(2,3,4,1,2)

Table 2: Project costs

Activity	Normal Cost(N_C)	Crash Cost(C_C)
1→2	(100,120,130,110,140),(120,110,130,120,100),(130,100,110,140,120),(160,140,110,100,120),(140,100,120,100,130)	(250,150,280,290,300),(300,220,240,280,280),(350,300,320,380,320),(370,300,320,300,280),(340,320,350,320,340)
1→3	(250,140,180,130,170),(220,200,230,250,200),(160,140,200,280,200),(180,230,240,200,140),(160,150,170,160,120)	(320,340,390,300,310),(480,360,500,400,340),(370,260,360,340,330),(400,390,420,480,470),(340,280,360,320,340)
1→4	(220,280,290,230,210),(180,150,200,200,210),(160,240,190,220,200),(160,180,160,190,210),(190,160,200,180,190)	(480,510,530,400,480),(380,420,270,350,380),(400,290,540,450,400),(380,400,360,320,420),(380,360,420,360,420)
2→5	(100,120,150,120,110),(110,120,140,130,110),(150,110,100,120,130),(100,120,130,140,100),(120,140,120,110,100)	(200,230,280,230,210),(220,230,270,250,210),(280,220,200,210,250),(200,230,250,270,180),(240,280,240,220,200)
3→6	(140,240,120,180,130),(150,100,200,210,240),(120,150,200,100,120),(150,160,120,160,190),(280,100,250,120,160)	(280,420,440,320,250),(300,220,420,420,480),(300,320,400,310,230),(300,320,240,340,400),(520,280,500,340,350)
4→6	(210,180,100,270,120),(150,160,210,140,190),(210,200,160,200,130),(140,160,130,120,100),(120,160,190,280,270)	(420,320,250,500,240),(310,350,420,280,400),(430,400,280,400,260),(280,310,260,240,200),(270,290,350,520,570)
5→6	(150,120,180,140,150),(120,150,170,140,130),(150,160,200,170,130),(180,150,120,180,110),(150,160,130,140,170)	(280,250,290,240,220),(240,270,280,270,280),(260,240,200,290,270),(280,270,230,210,200),(210,230,260,200,250)

From the given tables the Type-2 Pentagonal Fuzzy Numbers are converted to crisp number**.**

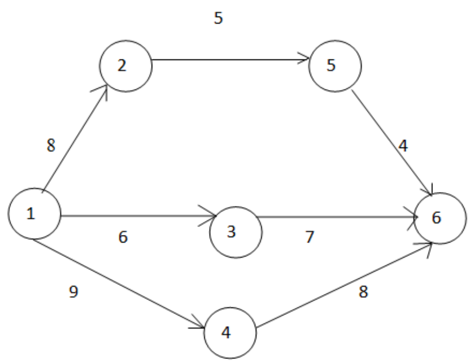


Fig.1: Network diagram

Critical paths = 1→2→5→6

1→4→6

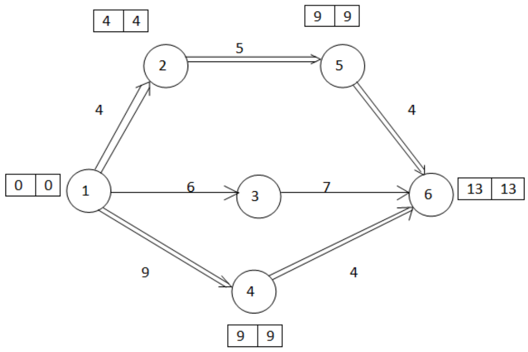
Project Duration = { 8+5+4 ; 9+8 } = 17

Table 3: Calculation of slope cost

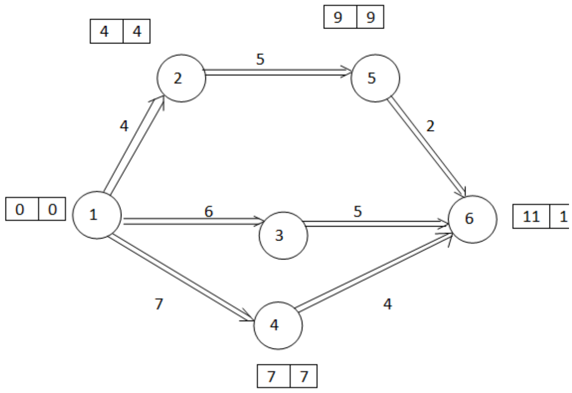
Activity	N_T	C_T	N_C	C_C	$\Delta_T = N_T - C_T$	$\Delta_C = N_C - C_C$	$\frac{\Delta C}{\Delta T}$
1→2	8	4	120	300	4	180	45
1→3	6	3	188	368	3	180	60
1→4	9	5	200	404	4	204	51
2→5	5	3	120	232	2	112	56
3→6	7	3	160	348	4	188	47
4→6	8	4	172	342	4	170	42
5→6	4	2	150	252	2	102	51
			1110				

Direct Cost = Rs. 1110
Indirect Cost = Rs. 0
Total Cost = Rs.1110

STEP 1:



STEP 2:



STEP 3:

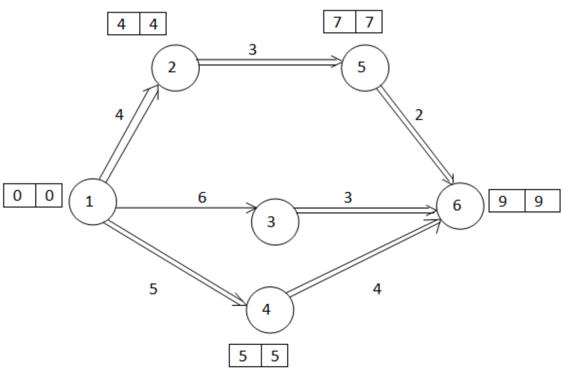


Table 4: Details of Total Cost

Step	Critical Path	Project Duration	Total Cost
Step 1	1→2→5→6 1→4→6	13	1197
Step 2	1→2→5→6 1→3→6 1→4→6	11	1297
Step 3	1→2→5→6 1→4→6	9	1402

7. Conclusion

In this paper some Arithmetic Operations and Ranking method of Type-2 Pentagonal fuzzy numbers are defined.Further investigates Time-Cost Trade off Problem using Type-2 Pentagonal Fuzzy number, we conclude that the Project can be completed in shortest duration, which is faster and easier method.

Acknowledgements

Firstly, I would like to express my sincere gratitude to my professor for her continuous support, for my research. Finally I thank my family who encouraged and supported me throughout the project.

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