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Research paper

Boundary Layer of a Dusty Fluid Flow Over A Stretching Sheet

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Abstract

A numerical analysis has been carried out to investigate the effects of suction parameter and fluid-particle interaction parameter on boundary layer of dusty fluid towards the stretching sheet. The governing equations of boundary layer were transformed into system of coupled non-linear ordinary differential equations with the help of similarity transformation. The transformed equations then solved numerically using bvp4c solver of MATLAB software. The effects of physical parameters on velocity profile of fluid phase and dust particle phase were obtained and analysed through several plots. Useful discussion were carried out with the help of plotted graphs and tables. The numerical results obtain were compared and found to be in good agreement with the previous study. It is observed that the presence of suction increase the velocity of fluid meanwhile opposite with velocity of particle. Besides that, the fluid-particle interaction parameter increase the velocity of particle. These findings will be used for future studies involving nanofluid.

Keywords: *Boundary layer; Bvp4c; Dusty fluids; Stretching sheet*

1. Introduction

The study of boundary layer of dusty fluid over stretching sheet has attracted interest many researchers due to its various applications to engineering and industrial disciplines. These applications include aerodynamic industries of automobile and airplanes, soil erosion by natural winds and cooling liquid. Hence, the study of boundary layer had begun since 1904 by German engineer, Prandtl. The earliest researcher is Sakiadis [1], presented about boundary flow problem generated by continuous solid surface moving with constant velocity. Then, Gireesha et al. [2] continue the research on study of heat transfer characteristics of an incompressible dusty fluid past a vertical stretching sheet. The boundary layer is highly influenced by the Prandtl number. For further investigation, Gireesha et al. [3] discussed on steady boundary layer flow and heat transfer of a dusty fluid over a stretching sheet with non-uniform heat sources/sink. They considered two types of heating processes namely prescribed surface temperature and surface heat flux. The behaviour of two-phase flow had took interest in wide range of technical problems especially dust or solid particles that distributed in fluid, such as environmental pollution, sedimentation and blood rheology [2]. Datta et al. [4] investigated dusty fluid in boundary layer flow over semi-infinite flat plate. Then, Agranat [5] discussed on the effect of pressure gradient on friction and heat transfer in dusty boundary layer. Pavithra and Gireesha [6] also further the studied by investigated the effects of heat absorbtion and generation on dusty fluid flow over an exponentially stretching sheet. The suction parameter reduces the velocity and the temperature profiles.
Motivated by previous study, we extend the work of Vajravelu and Nayfeh [7]. In the present paper, we study the effect of suction parameter and particle loading parameter on boundary layer of dusty fluid towards the stretching sheet. The governing equations of boundary layer are reduced into system of coupled non-linear ordinary differential equations with help of similarity transformation. The transformed equations then solved numerically using bvp4c solver of MATLAB software.

2. Problem Formulation

We are considering steady state laminar boundary layer of an incompressible viscous dusty fluid with constant speed parallel on horizontal stretching sheet. The flow is generated along the x-axis and y-axis by two equal and opposites forces which being normal to the flow. The sheet being stretched with velocity along the x-axis, keeping the origin fixed. Both fluid and dust particles supposed to be static at the beginning.

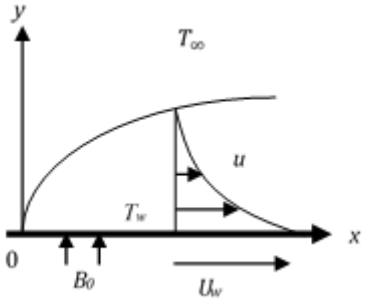

Fig. 1: Physical diagram for the problem.

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Under these assumptions, the governing equations of boundary layer are given as,

$$\rho\left(u\frac{\partial u}{dx}+v\frac{\partial u}{dy}\right)=\mu\frac{\partial^2 u}{dy^2} \tag{1}$$

$$\frac{\partial u}{dx}+\frac{\partial u}{dy}=0 \tag{2}$$

$$u_p\frac{\partial u_p}{dx}+v_p\frac{\partial u_p}{dy}=\frac{1}{\tau}\left(u-u_p\right) \tag{3}$$

$$u_p\frac{\partial v_p}{dx}+v_p\frac{\partial v_p}{dy}=\frac{1}{\tau}\left(v-v_p\right) \tag{4}$$

$$\frac{\partial}{dx}\left(\rho_p u_p\right)+\frac{\partial}{dy}\left(\rho_p v_p\right)=0 \tag{5}$$

Where the coefficients of viscosity of the fluid, density of fluid, density of particle phase and the relaxation time of particles represents by μ, ρ, ρ_p and τ respectively. Furthermore, (u,v) and (u_p, v_p) denote the velocity components of the fluid and dust particle phases along x and y direction, respectively. In deriving these equations, the small magnetic Reynolds number flow will be neglected by the Stokesian drag force which considered for the interaction between the fluid and particle. We assume that the external electric field is zero and the electric field due to polarization of charges is neglect. The boundary conditions applicable towards the problem are,

$$\begin{aligned} &u=cx, \quad v=-v_0, \quad \text{at} \quad y=0, \\ &u\to 0, \quad u_p\to 0, \quad v_p\to v, \quad \rho_p\to k\rho, \quad \text{as} \quad y\to\infty. \end{aligned} \tag{6}$$

where the constant variables are c and $v_0>0$ known as prescribed constants meanwhile k the density ratio also constants. To convert the governing equation into a set of similarity equations, we introduced new variables to obtain non-linear ordinary differential equation. The new variables are,

$$\begin{aligned} &u=cxf'(\eta), \quad v=-\sqrt{vc}\,f(\eta), \\ &\eta=\sqrt{\frac{c}{v}}y, \quad u_p=cxF(\eta), \\ &v_p=\sqrt{vc}\,G(\eta), \quad \rho_r=H(\eta) \end{aligned} \tag{7}$$

and substituting (7) into (1) – (6) gives,

$$f'''-\left(f'\right)^2-ff''=0 \tag{8}$$

$$F^2+GF'+\beta\left(F-f'\right)=0 \tag{9}$$

$$GG'+\beta\left(f+G\right)=0 \tag{10}$$

$$HF+H'G+G'H=0 \tag{11}$$

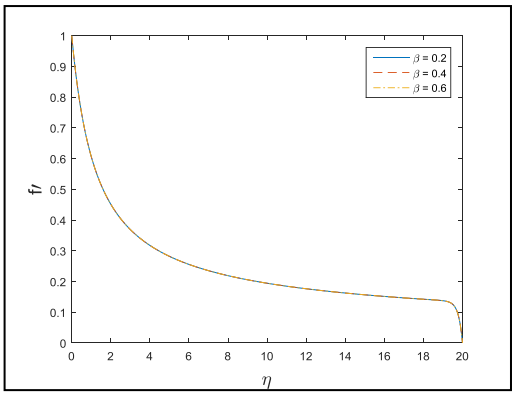
and boundary condition in (6) become

$$\begin{aligned} &f'(\eta)=1, \quad f(\eta)=R \quad \text{at} \quad \eta=0 \\ &f'(\eta)=0, F(\eta)=0, G(\eta)=-f(\eta), H(\eta)=k \quad \text{at} \quad \eta\to\infty. \end{aligned} \tag{12}$$

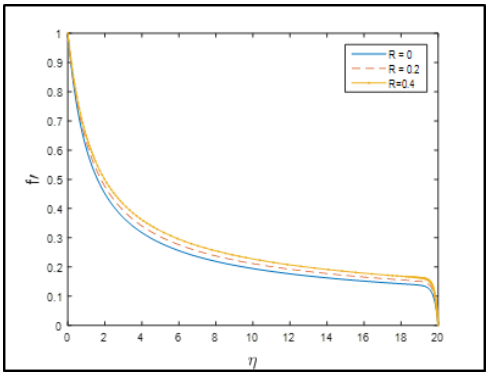
where a prime denotes differentiation with respect to η, $\rho_\tau=\frac{\rho_p}{\rho}$ is relative density, $R=\frac{v_0}{\sqrt{v_c}}$ is the suction parameter and $\beta=\frac{1}{c\tau}$ is the fluid-particle parameter. While skin friction, τ^* defined by $\tau^*=\frac{\tau'}{\mu c\sqrt{vc}x}=\left(\frac{d^2 f}{d\eta^2}\right)_{\eta=0}$.

3. Results and Discussion

The set of ordinary differential equations (8) – (11) with boundary conditions (12) are solved using MATLAB software with bvp4c solverfunction. The function is used to solve the equations due to its effectiveness in solving the boundary value problems [8]. For several sets of values of the physical parameters R, k and β, numerical results were obtained for the velocity profile component of fluid phase (f', f), the particle phase F, the particle density, H and skin friction, f''. Thus, the numerical results obtained are presented in Figure 2 – 4 and Table 1, and discussed.



(a)

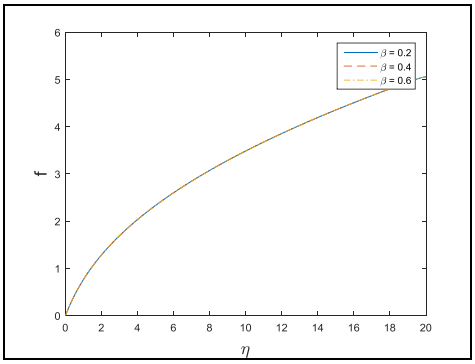


(b)

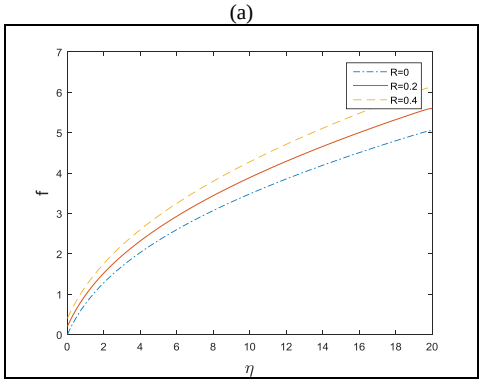
Fig. 2: Velocity profile of fluid phase $f'(\eta)$. (a): Several values of β with $R=0$ and $k=0.2$. (b): Several values of R with $\beta=0.1$ and $k=0.2$.

Figures 2 and 3 show the effects of β and R on fluid horizontal and transverse velocity respectively. It is observed that the horizontal and transverse velocity remain unchanged despite the increasing value of β. This phenomenon is expected as β is not included in momentum equation (8) and boundary conditions (12).
On the other hand, the increasing R causes both horizontal and transverse velocity to increase. As R increases, the surface gradients for both horizontal and transverse velocity decrease which results in thicker boundary layer for higher R. Hence, the velocity increases.

In Figures 4(a) and 4(b), the effects of β and R on velocity of dust particles are shown. Higher fluid-particle interaction β increases dust particle velocity. This may happen due to the fact that the interaction between the fluid and particle phase is high then the particle phase develops the opposite force to the fluid phase until the particle velocity reaches the fluid velocity [9]. Meanwhile, velocity of dust particle decreases due to increasing R.

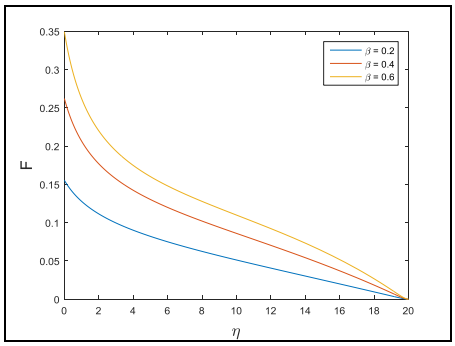


(a)

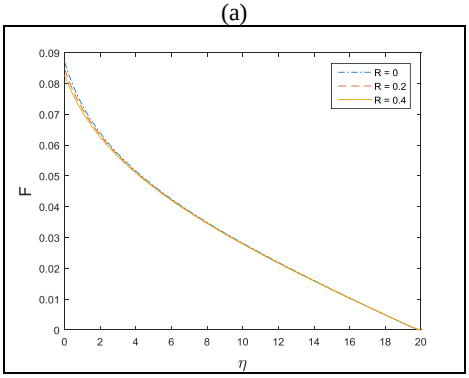


(b)

Fig. 3: Transverse velocity profile of fluid phase $f(\eta)$. (a): Several values of β with $R = 0$ and $k = 0.2$. (b): Several values of R with $\beta = 0.1$ and $k = 0.2$.



(a)



(b)

Fig. 4: Velocity profile of dust particle phase $F(\eta)$. (a): Several values of β with $R = 0$ and $k = 0.2$. (b): Several values of R with $\beta = 0.1$ and $k = 0.2$.

The influence of physical parameters towards the values of skin friction, $f''(\eta)$ and particle density, $H(\eta)$ were presented in Table 1.

Table 1: Values of $f''(\eta)$ and $H(\eta)$ with different values of physical parameters.

R	β	k	$-f''(0)$	$H(0)$
0	0.1	0.2	0.687714395	0.169197110
0.2			0.633074811	0.174346145
0.4			0.584880284	0.178082132
0	0.2	0.2	0.687714395	0.143224423
	0.4		0.687714395	0.102363612
	0.6		0.687714393	0.072465316

4. Conclusion

This paper is focusing on boundary layer flow of dusty fluid over a stretching sheet. The set of governing equation were transform from partial differential equation into ordinary differential equation with the help on new variables alongside the boundary conditions by similarity transformation. In order to solve on the boundary value problem, solver called bvp4c in MATLAB software been used to gain on numerical solution. The influence of physical parameters, which are fluid-particle parameter, β and suction parameter, R towards the velocity profile have been investigated and shown graphically in Figure 2 – 4. Some of the important findings are listed below:

- The presence of suction parameter increase the velocity distribution of fluid. Therefore, R effect on decreasing the velocity of dust particle.
- The interaction of particle and fluid will increase as due to influence of velocity of particle phase.
- Skin friction show decreasing pattern due to effect of suction and fluid-particle parameter.
- The effect on particle density differ on β and R. Increases due to R but decrease on β.

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