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Research Paper

Optimal attitude determination method in presence of noise and bias on different star sensors

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Abstract

There are different attitude determination methods which have been used in satellite and spacecraft by star tracker. Each of these methods have its own advantages and disadvantages depending on their application, stochastic characteristic of noise on sensors (bias or noise), and weight of noise falling on different sensors. The present study has thus explored the major methods from two perspectives: the effect of input noise or bias on each star sensor and the corresponding weight of each noise or bias falling on each sensor. These aspects are compared in each method and the optimal method according to each condition is introduced. N Vector, Triad, Quest, Q method and least square method are the methods studied and simulated in this article. Finally, a comparison is made between the methods and the optimal method is introduced theoretically and practically.

Keywords: *Attitude Determination, Celestial Navigation, Triad, Quest, Least Square, Satellite.*

1. Introduction

Today, modern spacecraft's take advantage of many devices to accurately determine their position. Magnetometers are used for measuring the Earth's magnetic field. Sun sensors can be used for attitude determination, yet they depend on the visibility of the sun. The problem is that sun sensors and magnetometers can only achieve an accuracy of 0.1 degree [1, 2]. Using star tracker could offer a solution to this accuracy problem.

Star trackers are the most accurate devices used for determining a spacecraft's position. The star tracker is essentially a camera for the sole purpose of observing star patterns as observed on the celestial sphere [3]. The star tracker is attached to the satellite onboard computer as part of the ADCS (Attitude Determination and Control Subsystem) [3]. The star tracker operates automatically, getting images of star patterns within its Field of Vision (FOV). The stars observed by the camera can then be identified and the orientation of the spacecraft can be calculated. There are various star catalogues, of which sky2000 is the most commonly used catalogue which is applied in space missions. This catalogue consists of more than 300000 stars with information about right ascension, declination, magnitude and position of each star in celestial sphere [4].

Different algorithms have been introduced for star identification as pyramid, triangle and etcetera. These methods lead to position vector identification of stars in catalogues or inertial frame. By mapping star vector from body frame to inertial frame, direction cosine matrix or attitude will be determined in i-frame.

There are currently many different types of attitude determination algorithms used by star trackers, but a commonly-used algorithm is a class that estimates the four Euler symmetric parameters that form the quaternion [5]. The quaternion also provides an attitude matrix, which is quadratic in the parameters and is also free of transcendental trigonometric functions. The optimal estimator of the quaternion can be used to solve the constrained least-squares Wahba problem.

Other algorithms used to solve Wahba's problem by obtaining the quaternion is the TRIAD algorithm as well as the Quaternion Estimator (QUEST) algorithm. The TRIAD and QUEST algorithms each provide quaternions as well as the direction cosine matrix of the satellite. The TRIAD algorithm is fairly simple, without requiring any inversion of matrices, while the QUEST algorithm requires fairly complex eigenvalue calculations [5], [8]. Also, using horizon sensor, star trackers could determine attitude in n-frame which may be corrupted by mixing noise of horizon sensor and star tracker, with the effect of horizon sensor noise being greater than that of star tracker.

In continue, all algorithms are introduced mathematically and their advantages and disadvantages are studied through simulation.

2. Attitude determination algorithms:

In this section, the optical system specification of star tracker is initially introduced.

FOV specification would be as follows:

$$\begin{aligned}
&FOV = 8^0 \times 8^0 \\
&f = 107mm \\
&Pixel\ No = 1024 \\
&Pixel\ Size = 1.5 \times 10^{-5} \\
&FOV = 2 \times \tan^{-1}\left(\frac{W}{2 \times f}\right) \\
&Sample\ time = 0.1\,s = 10\ hz
\end{aligned} \tag{1}$$

Considering x and y axes of boresight coordinate being equal to zero in CCD frame, boresight coordinate in body frame would be as follows [9]:

$$\begin{aligned}
&X_{SCF_{star}} = \frac{X_{mm_{star}}}{\sqrt{X^2_{mm_{star}} + Y^2_{mm_{start}} + f^2}} \\
&Y_{SCF_{start}} = \frac{Y_{mm_{star}}}{\sqrt{X^2_{mm_{start}} + Y^2_{mm_{start}} + f^2}} \\
&Z_{SCF_{star}} = \frac{f}{\sqrt{X^2_{mm_{star}} + Y^2_{mm_{star}} + f^2}} \\
&R_{SCF_{star}} = \begin{bmatrix} X_{SCF_{star}} \\ Y_{SCF_{star}} \\ Z_{SCF_{star}} \end{bmatrix} \\
&\theta_{star_{inertial}} = \cos^{-1}(S_{1_{inertial}}, S_{2_{inertial}}) \\
&\theta_{star_{Body}} = \cos^{-1}(S_{1_{Body}}, S_{2_{Body}}) \\
&\theta_{star_{inertial}} \cong \theta_{star_{Body}}
\end{aligned} \tag{2}$$

Also assuming that star identification has been done in the pattern recognition phase and their ID is known, the unit vector of all stars in star catalog would be fairly straightforward as follows [10]:

$$S = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{CRF} = \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} \tag{3}$$

FOV dimension is $8^0 \times 8^0$ and so the angle of stars and bore sight in CCD coordinate could not be greater than 4 degree. Therefore, the angle of bore sight vector in inertial frame and all-star vectors in catalog would be calculated at any time and stars in each frame would be recognized as follows:

$$\cos^{-1}(S_i \,.\, R_{CRF_{boresight}}) < 4^\circ \quad \Rightarrow S_i \ is\ in\ Frame \tag{4}$$

Then, for star mapping on CCD frame, star coordinate would be calculated n millimeters as follows:

$$\begin{bmatrix} x_{mm} \\ y_{mm} \end{bmatrix} = \begin{bmatrix} \dfrac{x_{SCF}}{z_{SCF}} f \\ \dfrac{y_{SCF}}{z_{SCF}} f \end{bmatrix} \tag{5}$$

Also, the star place in CCD according to φ and λ would be as follows:

$$\begin{aligned} \alpha &= \tan^{-1} \frac{y}{\sqrt{x^2 + y^2}} \\ \beta &= \tan^{-1} \frac{x}{z} \\ x_{mm} &= \tan \beta \times f \\ y_{mm} &= \tan \alpha \times \frac{f}{\cos \beta} \end{aligned} \tag{6}$$

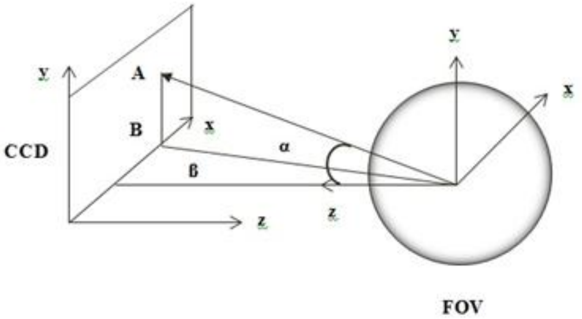

Fig. 1: Star Imaging in CCD

Six star frames in 100th of second, 400th of second, 700th of second, 1000th of second, 1300th of second and 1500th of second between 15000 frames by 0.1s sample time for example would be:

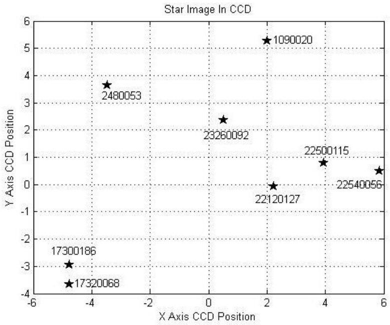

Fig. 2: Star Imaging in 100th of Second

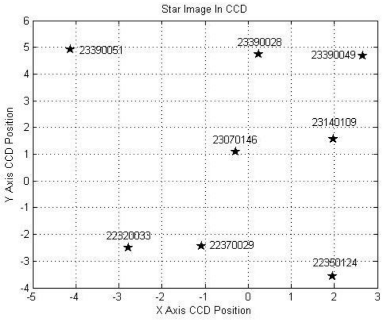

Fig. 3: Star Imaging in 400th of Second

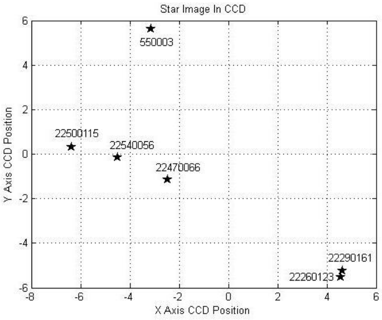

Fig. 4: Star Imaging in 700th of Second

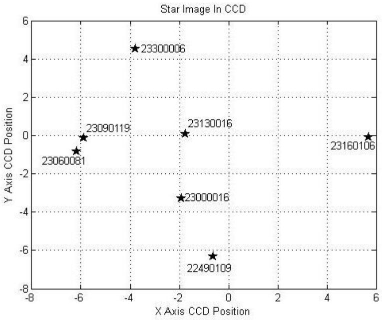

Fig. 5: Star Imaging in 1000th of Second

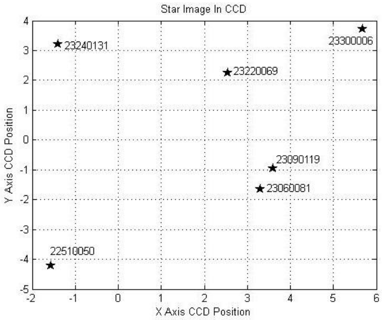

Fig. 6: Star Imaging in 1300th of Second

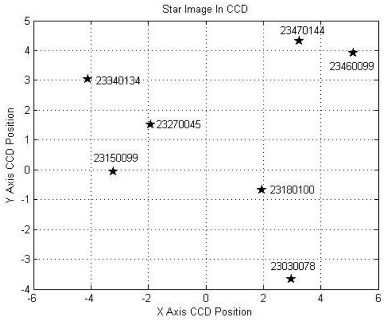

Fig. 7: Star Imaging in 1500th of Second

2.1. N vector method

This method is based on observations of three separate stars. In other words, at least three stars are needed for attitude determination in this method. Two matrixes that consist of star vectors in body and inertial frame are constructed and the cosine matrix is calculated through the following relations [11], [13]:

$$R^b = \left[r_1^b \quad r_2^b \quad \dots \quad r_N^b \right]$$
$$R^i = \left[r_1^i \quad r_2^i \quad \dots \quad r_N^i \right] \tag{7}$$

$$C_b^i = R^i (R^b)^T \left[R^b (R^b)^T \right]^{-1} \tag{8}$$
$$C_b^i = R^i (R^b)^T$$

The first formula is used when the number of observations is more than three and so the observation matrix is non-squared. In this case, pseudo inverse is applied for attitude determination.

The weight of each observation vector of different sensors in attitude determination is equal. Thus if the stochastic characteristics of input bias or noise in mean and variance are different on each sensor, there would be no difference in using any of sensor measurement.

A summary of N Vector method simulation results in various inputs will be as follows:

Table 1: Comparing Different Input Noises and Biases in the N Vector Attitude Determination Method

		N Vector method						INS
Frame	Attitude	No bias	Equal bias	Bias in Sensor 1	Bias in Sensor 2	Bias in Sensor 3	Noise in Sensors	Noise in Gyros
i-Frame	Roll (rad)	0.54×10^{-4}	9.75×10^{-4}	0.001	7.93×10^{-4}	7.42×10^{-4}	0.001	0.0404
	Pitch (rad)	0.0157	0.0153	0.015	0.015	0.015	0.015	0.027
	Yaw (rad)	9.2×10^{-5}	1.11×10^{-4}	0.004	0.004	0.004	0.002	0.0401
n-Frame	Roll (rad)	6.87×10^{-4}	0.0011	0.0011	0.0011	0.0012	0.001	0.027
	Pitch (rad)	0.0157	0.0153	0.0153	0.0153	0.0153	0.015	0.023
	Yaw (rad)	2.37×10^{-4}	1.80×10^{-4}	0.004	0.004	0.003	0.002	0.0408

2.2. Triad method:

The TRIAD algorithm is a deterministic solution that generates a direction cosine matrix between two coordinate systems when two vectors are given in each of the particular coordinate systems [7]. Applying this algorithm to the attitude determination problem is fairly straightforward. The star tracker needs only to see two stars within its FOV to determine two unit vectors. These are referred to as the observed vectors. The other two unit vectors, or reference vectors, are found using the angle, planar triangles, or spherical triangles algorithms defined previously. Using the TRIAD algorithm, the two non-parallel unit vectors to stars in the inertial frame as well as the two non-parallel unit vectors in the star tracker frame are obtained.

These vectors are identified as R_1^i and R_2^i for inertial stars with two body frame vectors from the star tracker as R_1^b and R_2^b.

The algorithm then finds an orthogonal matrix *A,* which becomes the attitude matrix for the satellite, finds the orientation difference between the two systems [7]. The equations that the algorithm must satisfy are:

$$R_1^i = C_b^i R_1^b$$
$$R_2^i = C_b^i R_2^b \tag{9}$$

The algorithm then requires computation of the following column matrices or triads:

$$
\begin{aligned}
&r_1 = R_1^i && s_1 = R_1^b && \\
&r_2 = \frac{R_1^i \times R_2^i}{\left|R_1^i \times R_2^i\right|} && s_2 = \frac{R_1^b \times R_2^b}{\left|R_1^b \times R_2^b\right|} && Ar^i = s^i \quad i = 1,2,3 \\
&r_3 = \frac{R_1^i \times (R_1^i \times R_2^i)}{\left|R_1^i \times R_2^i\right|} && s_3 = \frac{R_1^b \times (R_1^b \times R_2^b)}{\left|R_1^b \times R_2^b\right|} && A = C_b^{iT} = \sum_{i=1}^{3} s_i r_i^T
\end{aligned} \tag{10}
$$

However, there are problems in using Triad algorithm. The first vector has more prominence in determination of A. Also, some of the information in the second vector is discarded [8]. It is, therefore, necessary to use the most accurate instrument to find the first vector of each set, in this case R_1^b and R_1^i. Accordingly, the first anchor (anchor vector) may be obtained by the star tracker, while the second vector comes from the magnetometer.

Summarize of Triad method simulation results in various input will be as follows:

Table 2: Provides A Summary of Triad Simulation Results in Various Inputs.

Frame	Attitude	Triad method No bias	Equal bias	Bias in Sensor 1	Bias in Sensor 2	Bias in Sensor 3	Noise in Sensors	INS Noise in Gyros
i-Frame	Roll (rad)	5.43×10^{-5}	9.31×10^{-4}	0.0178	0.0109	9.31×10^{-4}	0.004	0.0404
	Pitch (rad)	0.0157	0.0151	0.0176	0.0159	0.0151	0.0166	0.027
	Yaw (rad)	9.21×10^{-5}	1.18×10^{-4}	0.0815	0.047	1.18×10^{-4}	0.0216	0.0401
n-Frame	Roll (rad)	6.88×10^{-4}	0.001	0.0173	0.0109	0.001	0.0048	0.027
	Pitch (rad)	0.0157	0.015	0.0179	0.0159	0.015	0.0160	0.023
	Yaw (rad)	2.33×10^{-4}	1.68×10^{-4}	0.0808	0.048	1.68×10^{-4}	0.0214	0.0408

2.3. Q-method and quest method:

The main idea in Q-method and quest algorithm is prompted by the Wahba's problem. Due to errors and corruption in both star tracker measurements and the inertial vectors, there is no exact solution for A. Therefore, an approach is needed to be taken as to select an A that matches R_1^b and R_1^i. This is known as Wahba's Problem [14].

Wahba's problem is the estimation of a satellite's attitude by using direction cosines .Given two sets of points, in this case $r_1^b K K\ r_n^b$ and $r_1^i K K\ r_n^i$. n ≥ 2, find a rotation matrix A which aligns the first set of vectors into the best least squares coincidence with the second set of vectors. Mathematically, a matrix A minimizes:

$$
J(C_i^b) = \frac{1}{2}\sum_{j=1}^{n} w_j \left|r_j^b - C_i^b r_j^i\right|^2 \tag{11}
$$

Subject to a constraint:

$$
AA^T = I_{3\times3}
$$

$$
\sum_{j=1}^{n} w_j = 1 \qquad w_j = \frac{\sigma_{tot}^2}{\sigma_i^2} \qquad \sigma_{tot}^2 = \left(\sum_{i=1}^{n} (\sigma_i^2)^{-1}\right)^{-1} \tag{12}
$$

The quadratic loss function in the attitude matrix can be transformed into a quadratic loss function in the corresponding quaternion (Shuster & Oh, 1981). Wahba presents a least squares criterion to define the best estimate for an orthogonal matrix A that minimizes the cost function represented by the above Equation.

$$
\begin{aligned}
J(C_i^b) &= \frac{1}{2}\sum_{j=1}^{n} w_j (r_j^b - C_i^b r_j^i)^T (r_j^b - C_i^b r_j^i) = \frac{1}{2}\sum_{j=1}^{n} w_j \left(\left|r_j^b\right|^2 + \left|r_j^i\right|^2 - (r_j^i)^T C_b^i r_j^b - (r_j^b)^T C_i^b r_j^i\right) \\
&= \frac{1}{2}\sum_{j=1}^{n} w_j \left(\left|r_j^b\right|^2 + \left|r_j^i\right|^2\right) - \underbrace{\sum_{j=1}^{n} w_j \left((r_j^b)^T C_i^b r_j^i\right)}_{G(C_i^b)}
\end{aligned} \tag{13}
$$

In this equation, the loss function is minimized when an optimal matrix A_{opt} is determined, however, we can also maximize a gain, G, which also solves the same equation

$$B \overset{\Delta}{=} \begin{bmatrix} r_1^b & r_2^b & \dots & r_n^b \end{bmatrix}, \quad I \overset{\Delta}{=} \begin{bmatrix} r_1^i & r_2^i & \dots & r_n^i \end{bmatrix}, \quad W \overset{\Delta}{=} diag(w_1 \quad w_1 \quad \dots \quad w_n)$$

$$G(C_i^b) = trace(C_i^b \times I \times W \times B^T)$$

$$S \overset{\Delta}{=} B \times W \times I^T = \sum_{j=1}^{n} w_j \; r_j^b (r_j^i)^T \tag{14}$$

$$G(C_i^b) = trace(C_i^b \times S^T)$$

As it is an orthogonal matrix, so there exists a unit quaternion as $\bar{q} = \begin{bmatrix} q_4 \\ q \end{bmatrix}, \; q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$ for defining this matrix.

$$C_i^b(\bar{q}) = (q_4^2 - q^T q)I + 2qq^T - 2q_4[q\times] = I + 2[q\times]^2 - 2q_4[q\times] \tag{15}$$

Here the Wahba's problem is changed to find the optimal quaternion that maximizes cost function in the above equation.

$$G(\bar{q}) = \begin{bmatrix} q^T & q_4 \end{bmatrix} \begin{bmatrix} (S^T + S - trace\{S\} I & Z \\ Z^T & trace\{S\} \end{bmatrix} \tag{16}$$

$$G(\bar{q}) = \bar{q}^T K \bar{q}$$

$$K \overset{\Delta}{=} \begin{bmatrix} (S^T + S - trace\{S\} I & Z \\ Z^T & trace\{S\} \end{bmatrix} \tag{17}$$

$$z \overset{\Delta}{=} \sum_{j=1}^{n} w_j \; (r_j^b \times r_j^i)$$

$$S = \sum_{j=1}^{n} w_j \; r_j^b (r_j^i)^T$$

$$\bar{S} = S + S^T$$

Because of the unity of quaternion norm, the following constraint is added to the relation as follows:

$$G'(\bar{q}) = \bar{q}^T K \bar{q} + \lambda(\bar{q}^T \bar{q} - 1)$$

$$\frac{dG'}{dq} = 2K\bar{q} - 2\lambda\bar{q} = 0 \tag{18}$$

Solving the above equation leads to the relation below:

$$K\bar{q} = \lambda\bar{q} \tag{19}$$

By solving Characteristic equation, the Eigen vector which is correspond to maximum Eigen value and minimize cost function is gained. This is optimal quaternion.

Quest algorithm supposes simpler method to solve Wahba's problem. This algorithm is based on minimizing cost function similar to Q-method but mathematically different in using Rodrigues parameters. The relation of Quest is defined as follows:

$$\begin{bmatrix} \bar{S} - \sigma I & Z \\ Z^T & \sigma \end{bmatrix} \begin{bmatrix} q \\ q_4 \end{bmatrix} = \lambda \begin{bmatrix} q \\ q_4 \end{bmatrix}$$

$$\sigma = trace(S) \tag{20}$$

$$Z^T q + \sigma q_4 = \lambda q_4$$

$$\lambda = Z^T e + \sigma = \frac{1}{\gamma} Z^T \left[\alpha I + \beta \bar{S} + \bar{S}^2 \right] Z + \sigma$$

$$\gamma = (\lambda + \sigma)\alpha - d \quad \text{و} \quad \beta = \lambda - \sigma \quad \text{و} \quad \alpha = \lambda^2 + k - \sigma^2 \tag{21}$$

Optimal quaternion is found by solving below equation:

$$P(\lambda) = \lambda^4 - (a+b)\lambda^2 - c\lambda + (ab + c\sigma - e) = 0 \tag{22}$$

$$e = Z^T \bar{S}^2 Z \quad , \quad c = d + Z^T \bar{S} Z \quad , \quad b = \sigma^2 + Z^T Z \quad , \quad a = \sigma^2 + k$$

Summary of the Quest simulation results is provided in Table 3.

Table 3: Comparing Different Input Noises and Biases in the Quest Attitude Determination Method

		Quest method						INS
Frame	Attitude	No bias	Equal bias	Bias in Sensor 1	Bias in Sensor 2	Bias in Sensor 3	Noise in Sensors	Noise in Gyros
i-Frame	Roll (rad)	---	8.89×10^{-4}	8.69×10^{-4}	8.62×10^{-4}	8.72×10^{-4}	8.96×10^{-4}	0.0404
	Pitch (rad)	---	0.0148	0.0148	0.0148	0.0148	0.0151	0.027
	Yaw (rad)	---	4.71×10^{-4}	0.0011	0.0012	0.0011	0.0027	0.0401
n-Frame	Roll (rad)	---	0.017	0.017	0.017	0.017	0.0178	0.027
	Pitch (rad)	---	0.089	0.089	0.089	0.089	0.0905	0.023
	Yaw (rad)	---	0.006	0.006	0.006	0.006	0.0078	0.0408

2.4. Least square method

Because observed star vectors are noisy in body frame and also uncertainty of star catalogs, cosine matrix is calculated by least square determination as follows [14]:

$$\tilde{b}_i = A\tilde{r}_i + \tilde{e}_i$$

$$\tilde{b}_i = \begin{bmatrix} \tilde{b}_1 & \tilde{b}_2 & \cdots & \tilde{b}_n \end{bmatrix}^T$$

(23)

$$\tilde{e}_i = \begin{bmatrix} \tilde{e}_1 & \tilde{e}_2 & \cdots & \tilde{e}_n \end{bmatrix}^T$$

$$\tilde{r}_i = \begin{bmatrix} \tilde{r}_1 & \tilde{r}_2 & \cdots & \tilde{r}_n \end{bmatrix}^T$$

Error between estimated and measured star vector in body frame will be as follows:

$$\tilde{e}_i = A\tilde{r}_i - \tilde{b}_i \tag{24}$$

And so cost function of error that should be minimized to obtain optimal value for a matrix will formed as follows:

$$J = \frac{1}{2}\tilde{e}^T\tilde{e} \qquad \Rightarrow J = \frac{1}{2}\left[\tilde{b}^T\tilde{b} - 2\tilde{b}^T A\tilde{r} + \tilde{r}^T A^T A\tilde{r}\right]$$

$$\nabla_r J = \begin{bmatrix} \dfrac{\partial J}{\partial \tilde{r}_1} \\ \vdots \\ \dfrac{\partial J}{\partial \tilde{r}_n} \end{bmatrix} = A^T A\tilde{r} - A^T\tilde{b} = 0 \tag{25}$$

$$\nabla^2{}_r J = \frac{\partial^2 J}{\partial r \partial r^T} = A^T A > 0 \quad \Rightarrow P.d.f$$

$$\tilde{r} = (A^T A)^{-1} A^T \tilde{b} \tag{26}$$

Assuming equal noise of mean and variance in all sensors, the above relation would be rewritten as follows:

$$R_{3n\times1}=r_{3n\times9}a_{9\times1}+v$$

$$\begin{bmatrix}R_x\\R_y\\R_z\end{bmatrix}_b=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\times\begin{bmatrix}r_x\\r_y\\r_z\end{bmatrix}_i$$

$$\begin{bmatrix}R_x\\R_y\\R_z\end{bmatrix}_{3\times1}=\begin{bmatrix}r_x&r_y&r_z&0&0&0&0&0&0\\0&0&0&r_x&r_y&r_z&0&0&0\\0&0&0&0&0&0&r_x&r_y&r_z\end{bmatrix}\times\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{21}&a_{22}&a_{23}&a_{31}&a_{32}&a_{33}\end{bmatrix}^T \tag{27}$$

$$a_{9\times1}=(r^Tr)^{-1}_{9\times9}r^T_{9\times3n}\times R_{3n\times1}$$

Finally, all parameters of transformation matrix A are determined.

Table 4 gives a summary of least square simulation results in various inputs.

Table 4: Comparing Different Input Noises and Biases in the Least Square Attitude Determination Method

Frame	Attitude	Least Square method						INS
		No bias	Equal bias	Bias in Sensor 1	Bias in Sensor 2	Bias in Sensor 3	Noise in Sensors	Noise in Gyros
i-Frame	Roll (rad)	---	8.58×10^{-4}	0.0015	0.0015	0.0014	9.29×10^{-4}	0.0404
	Pitch (rad)	---	0.0148	0.0146	0.0149	0.0148	0.0152	0.027
	Yaw (rad)	---	1.16×10^{-4}	0.0070	0.0040	0.0036	0.0031	0.0401
n-Frame	Roll (rad)	---	0.0014	0.0018	0.0017	0.0017	0.0012	0.027
	Pitch (rad)	---	0.0148	0.0146	0.0149	0.0148	0.0152	0.023
	Yaw (rad)	---	1.16×10^{-4}	0.006	0.0039	0.0034	0.0030	0.0408

The authors emphasize that attitude in I frame is directly determined by star tracker while to determine attitude in n frame, star tracker attitude information should be combined with horizon sensor measurements. Horizon sensor specify transformation matrix of n-frame to e-frame. By given cosine matrix of e frame to i-frame which is clear and related to angular rate of earth rotation, and also obtained attitude matrix of body frame to i-frame by star tracker, attitude of body frame to n-frame will be calculated.

Three sources of error generate effect on attitude determination. The first one concerns star catalog uncertainty. The second one refers to Horizon sensor error and last one is related to star sensor error.

The following is the block diagram of star tracker and horizon sensor data:

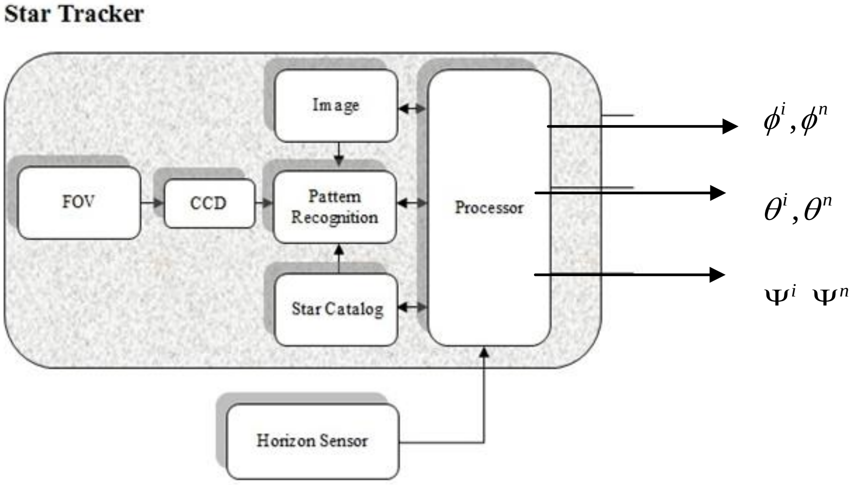

Fig. 8: Star Tracker and Horizon Sensor Data Fusion

3. Simulation results

3.1. Attitude determination in No-bias phase

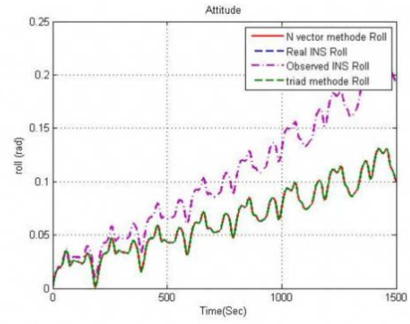

Fig. 9: Comparison of INS Roll and Triad and N Vector Roll in I-Frame

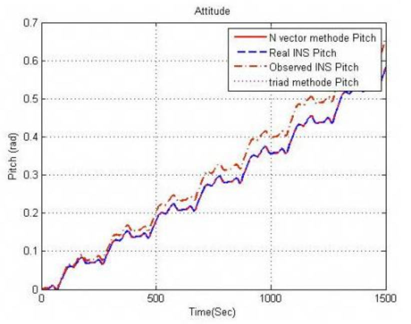

Fig. 10: Comparison of INS Pitch and Traid and N Vector Pitch in I-Frame

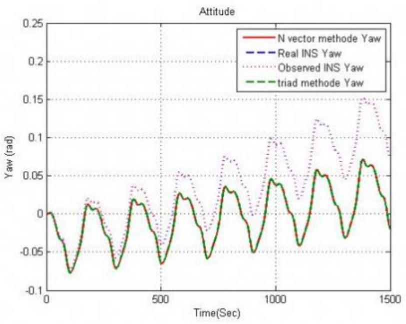

Fig. 11: Comparison of INS Yaw and Traid and N Vector Yaw in I-Frame

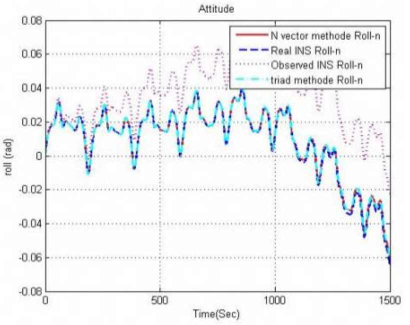

Fig. 12: Comparison of INS Roll and Triad and N Vector Roll in N-Frame

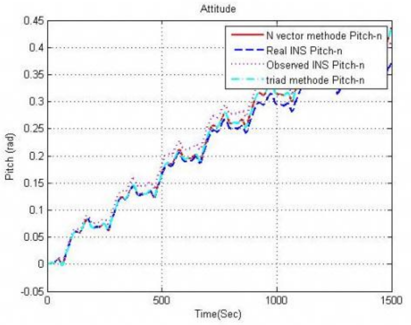

Fig. 13: Comparison of INS Pitch and Triad and N Vector Pitch in N-Frame

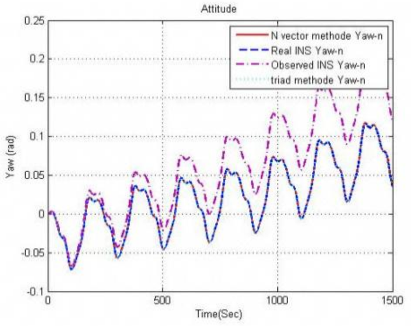

Fig. 14: Comparison of INS Yaw and Triad and N Vector Yaw in N-Frame

3.2. Attitude determination in equal-bias phase

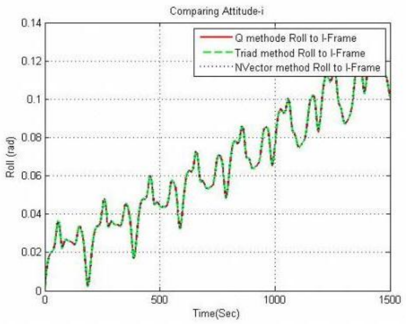

Fig. 15: Comparison of Q-Method Roll, Triad Roll and N Vector Roll in I-Frame

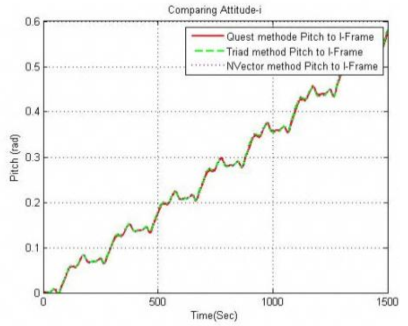

Fig. 16: Comparison of Quest Method Pitch, Triad Pitch and N Vector Pitch in I-Frame

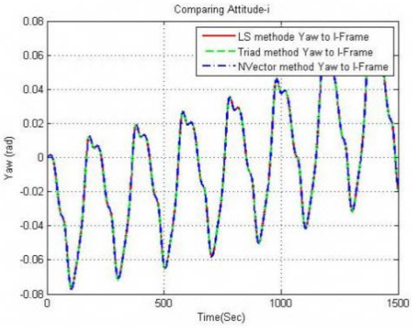

Fig. 17: Comparison of Least Square Yaw, Triad Yaw and N Vector Yaw in I-Frame

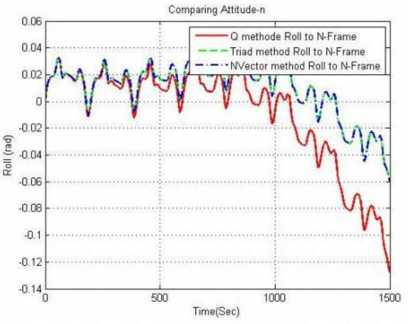

Fig. 18: Comparison of Q-Method Roll, Triad Roll and N Vector Roll in N-Frame

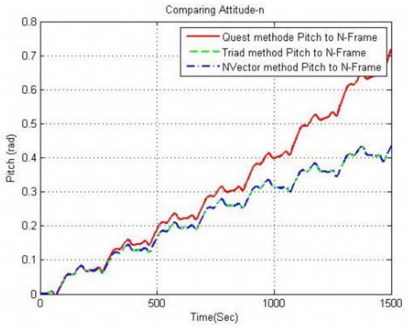

Fig. 19: Comparison of Quest Method Pitch, Triad Pitch and N Vector Pitch in N-Frame

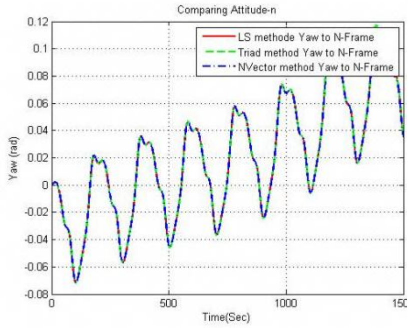

Fig. 20: Comparison of Least Square Yaw, Triad Yaw and N Vector Yaw in N-Frame

3.3. Attitude determination in bias-sensor 1 phase

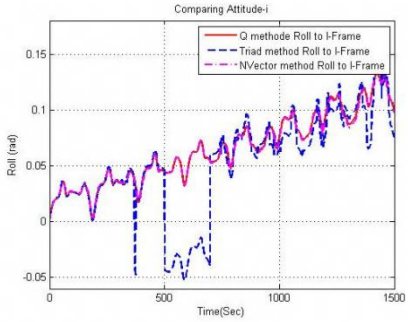

Fig. 21: Comparison of Q-Method Roll, Triad Roll and N Vector Roll in I-Frame

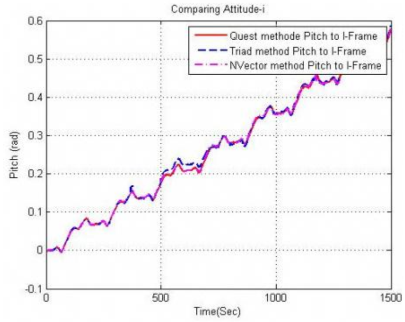

Fig. 22: Comparison of Quest Method Pitch, Triad Pitch and N Vector Pitch in I-Frame

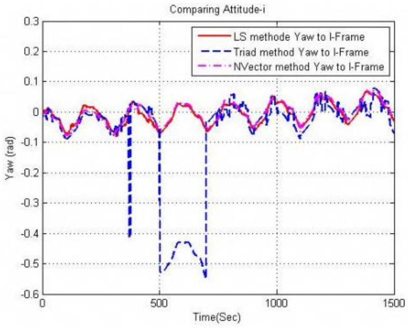

Fig. 23: Comparison of Least Square Yaw, Triad Yaw and N Vector Yaw in I-Frame

3.4. Attitude determination in bias-sensor 2 phase

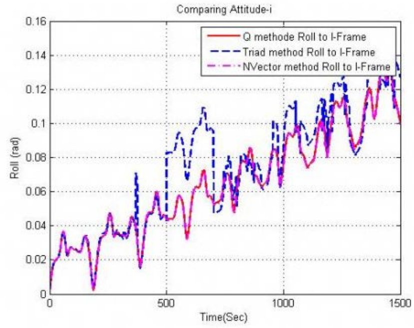

Fig. 24: Comparison of Q-Method Roll, Triad Roll and N Vector Roll in I-Frame

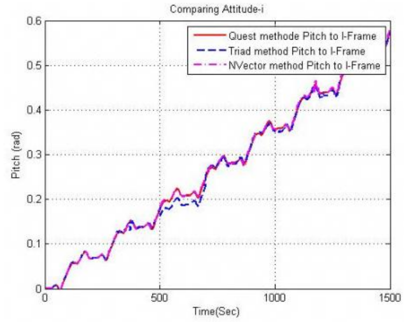

Fig. 25: Comparison of Quest Method Pitch, Triad Pitch and N Vector Pitch in I-Frame

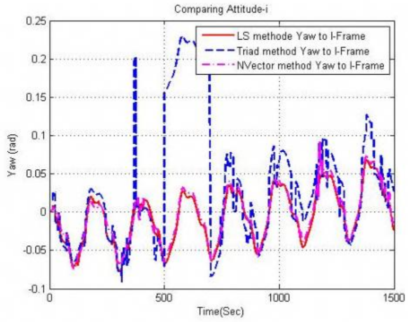

Fig. 26: Caparison of Last Square Yaw Triad Yaw and N Vector Yaw in I-Frame

3.5. Attitude determination in bias-sensor 3 phase

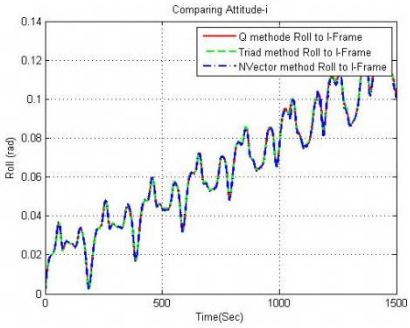

Fig. 27: Comparison of Q-Method Roll, Triad Roll and N Vector Roll in I-Frame

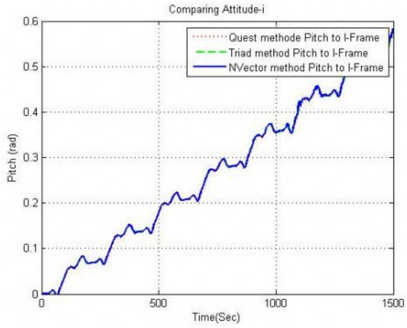

Fig. 28: Comparison of Quest Method Pitch, Triad Pitch and N Vector Pitch in I-Frame

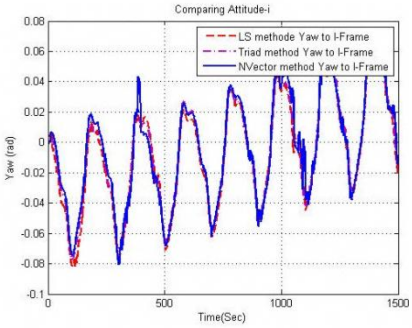

Fig. 29: Comparison of Least Square Yaw, Triad Yaw and N Vector Yaw in I-Frame

3.6. Attitude determination in noisy-sensor phase

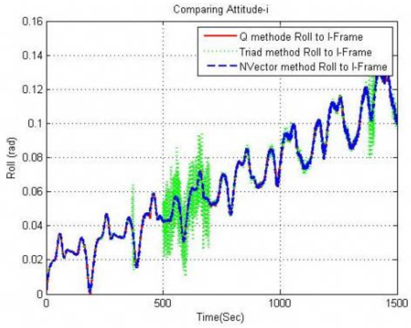

Fig. 30: Comparison of Q-Method Roll, Triad Roll and N Vector Roll in I-Frame

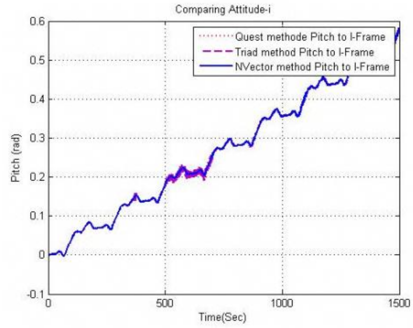

Fig. 31: Comparison of Quest Method Pitch, Triad Pitch and N Vector Pitch in I-Frame

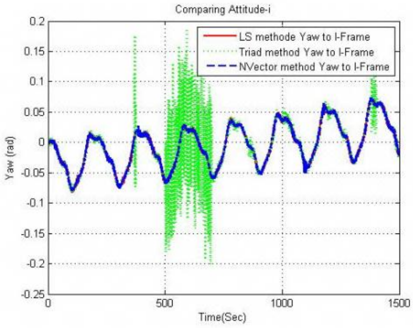

Fig. 32: Comparison of Least Square Yaw, Triad Yaw and N Vector Yaw in I-Frame

4. Conclusion

Different attitude determination methods were studied in this article. In the case of no bias and noise in star sensors, there is no difference between various methods. However attitude determination by star tracker in i-frame or in n-frame using horizon sensor is more accurate than attitude obtained by INS in either frame. In the case of bias in one of the three sensors, there is a difference between N Vector, Triad, Quest and least square method. Triad has the ability to provide three-axis attitude determinations, without costly computations, making it ideal for onboard attitude determination. Yet if bias of sensor 1 or 2 has more weight than that of other sensors, attitude would be determined inaccurately in this method.

This problem also exists in case of noisy star sensors. Quest is stochastic algorithm which is efficient in noisy cases but the computation of which is an expensive case has better performance in comparison with other methods. Each sensor measurements has equal share in calculating attitude in the N Vector method. So there is no difference in attitude determination in various cases of noisy inputs. Finally, the least square proved to be best algorithm which is both user-friendly and optimal in stochastic cases. Yet this method has also computational problem in calculating $(r^T r)^{-1}$ particularly when in computing pseudo inverse.

To sum up, it should be stressed that each method may be efficient in specific cases. The N Vector method in deterministic case and the least square or quest method in stochastic case are the most efficient methods for determining attitude on satellites or spacecraft's.

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