



# The Extended Monod Model for Microalgae Growth and Nutrient Uptake in Different Wastewaters

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## Abstract

Water pollution is a serious issue which always being concerned by public. Microalgae for wastewater treatment is an effective way to solve the problem due to its eco-friendly and apparently low cost. This research aims to investigate the efficiency of the mathematical model to estimate the microalgae growth and nutrient uptake by microalgae in wastewaters. The extended Monod model is applied in the Verhulst model to describe the microalgae growth and nutrient uptake by microalgae whereas microalgae *Botryococcus sp.* is the species of microalgae used in this research. The microalgae *Botryococcus sp.* growth and nutrient uptake in domestic, agricultural and industrial wastewater are estimated and the results reveal that the extended Monod model is suitable for the estimation of microalgae growth and nutrient uptake by microalgae. In addition, microalgae *Botryococcus sp.* is promising for treating domestic, agricultural and industrial wastewater.

**Keywords:** Extended Monod Model; Microalgae Growth; Nutrient Uptake; Verhulst Model; Wastewater Treatment.

## 1. Introduction

Wastewater pollution is one type of water pollutions and it is the direct result from human activities which involving in agricultural, industrial and domestic sectors. These activities resulted in eutrophication due to excessive nutrient such as nitrogen and phosphorus into the nearby water [1]. An efficient and cost saving way for wastewater treatment is phycoremediation [2]. Phycoremediation which was first introduced by Oswald [3] in 1950s refers to the capability of microalgae to consume nutrients in the wastewater such as nitrogen, phosphorus and total carbon [4]. Microalgae wastewater treatment to improve water quality has been gained importance in recent years [5]. Microalgae are unicellular species which exist individually, or in chains or groups and their sizes can range from a few micrometers ( $\mu\text{m}$ ) to a few hundreds of micrometers depend on their species [6]. Microalgae can be found in most habitats on the earth ecosystems, not just aquatic but also terrestrial which they specially adapted to an environment dominated by viscous forces. There exist more than 50,000 microalgae species, but only around 30,000 species of microalgae have been studied and analysed [7]. *Euglenophyta*, *Pyrrophyta*, *Chlorophyta*, and *Botryococcus sp.* are some examples of microalgae species. In Malaysia, microalgae *Botryococcus sp.* is one of the species that can be easily found in lakes and reservoirs and this species have been used in wastewater treatment [8]. Microalgae demand various nutrients to support growth which nitrogen and phosphorus are the most important nutrient [9]. Microalgae wastewater treatment offers an elegant solution to improve water quality due to the ability of microalgae to consume nutrients such as nitrogen and phosphorus for their growth which would not lead to secondary pollution [10]. Microalgae are able to convert nitrogen and phosphorus from wastewater into biomass and bio-products at a short hydraulic

retention time to improve the sustainability of wastewater treatment [11]. Besides, human use microalgae as bio-filters to remove nutrients or other pollutants from wastewater to obtain a good level of water quality [12]. Large scale of wastewater can be treated with microalgae and resulted in less sludge production compared to conventional wastewater treatment [13]. Since the cultivation of microalgae has a great contribution for wastewater treatment, thus it is important to study about the microalgae growth system. The growth of microalgae is high when there exists high concentration of nutrients [14]. Most of the microalgae growth models are based on modified Monod kinetics which often involve many parameters to be identified [15]. The factors that affect microalgae growth are nutrient availability, light intensity, temperature and water pH [16]. Monod equation is one of the best models that has been used to describe the direct relationship between specific growth rate and essential substrate concentration [17]. Monod model presented considers all these factors which have the influences on the microalgae growth and are used to determine the microalgae growth. Moreover, Verhulst logistic kinetic model is also a simple tool used in estimate the microalgae growth and nutrient uptake by microalgae. The Verhulst logistic kinetic model was used to model the evolution of the experimental biomass concentration in the reactors [18]. The study of [19] proved the result of microalgae biomass accumulation corresponds to Verhulst model for population's limited growth. Therefore, this research aims to investigate the growth of microalgae *Botryococcus sp.* and their effectiveness in treating domestic, agricultural and industrial wastewater by implementing the extended Monod model in the Verhulst model.

## 2. Mathematical modelling and kinetic expression for microalgae growth

There are many mathematical models that have been used to predict the microalgae growth which consider different parameters [20, 21]. In this research, we focus on two types of substrate concentration which are nitrogen and phosphorus, and the light intensity. Monod model is one of the kinetic growth models which shows the relationship between growth and the essential substrate concentration and is defined as [22]

$$\mu = \mu_{\max} \left[ \frac{S_i}{K_i + S_i} \right], \quad (1)$$

where  $\mu$  represents the specific growth rate,  $S_i$  is the substrate concentration,  $\mu_{\max}$  is the maximum specific growth rate and  $K_i$  is the saturation constant.

Since we only considered two types of substrate and light intensity, the specific growth rate,  $\mu$  can be expressed as

$$\mu = \mu_{\max} \left[ \frac{S_N}{K_N + S_N} \right] \left[ \frac{S_P}{K_P + S_P} \right] \left[ \frac{I}{K_I + I} \right], \quad (2)$$

where  $S_N$ ,  $S_P$  are concentrations of nitrogen and phosphorus in the medium and  $I$  is the light intensity.  $K_N$ ,  $K_P$  and  $K_I$  are half saturation constants for nitrogen, phosphorus and light intensity, respectively. The specific growth rate is the extended model of Monod model that is applied in this research.

The Verhulst model is expressed as [10]

$$X(t) = \frac{X_m X_0 e^{\mu t}}{X_m - X_0 + X_0 e^{\mu t}}, \quad (3)$$

where  $X$  represents the biomass concentration of microalgae,  $X_m$  is maximum cell concentration that the system can reach in batch,  $X_0$  is initial concentration of microorganism,  $t$  represents time and  $\mu$  is the specific growth rate of the microalgae. The specific growth rate in the extended Monod model (2) is substituted into the Verhulst model (3) to estimate the microalgae growth.

## 3. Mathematical modelling and kinetic expression for nutrient uptake by microalgae

For the nutrient uptake by microalgae, the exact solution of Verhulst model can be expressed as [18]

$$S = \frac{\left( \frac{X_0}{Y} + S_0 \right) (S_0 - S_{na}) - S_{na} \left( S_0 - \left( \frac{X_0}{Y} + S_0 \right) \right)}{(S_0 - S_{na}) - \left( S_0 - \left( \frac{X_0}{Y} + S_0 \right) \right) e^{pt}}, \quad (4)$$

where  $S$  is the total nutrient concentration,  $S_0$  is initial amount substrate concentration in the culture medium,  $S_{na}$  is unassimilated substrate concentration,  $p$  represents the specific growth rate of microalgae and  $Y$  is microalgae yield coefficient for the ratio of biomass produced per mass of substrate

incorporated as organic or structural which can be calculated using the equation as stated as below

$$Y = \left[ \frac{(X_m - X_0)}{(S_b - S_{na})} \right]. \quad (5)$$

By some transformations to (4) as the details are provided in [18], the equation can be produced as similar as (3)

$$X(t) = \frac{X_m X_0 e^{pt}}{X_m - X_0 + X_0 e^{pt}}. \quad (6)$$

Hence, we can conclude that  $p$  represents the specific growth rate of microalgae,  $\mu$  and it has the same relation to estimate microalgae growth and nutrient uptake by using Verhulst model. Therefore, the specific growth rate,  $\mu$  of extended Monod model in (2) will be substituted into  $p$  for nutrient uptake by microalgae.

## 4. Results

### 4.1. Microalgae *Botryococcus sp.* growth

The growths of microalgae *Botryococcus sp.* in different wastewaters which are domestic, agricultural and industrial wastewater are estimated by implementing the extended Monod model in Verhulst model. The experimental data used for domestic, agricultural and industrial wastewater were provided by the researchers from Universiti Tun Hussein Onn [23, 24]. Parameters used to estimate the microalgae growth are shown in Table 1 for three types of wastewater.

**Table 1:** Parameters used in three types of wastewater

Parameters	Domestic	Agricultural	Industrial
$\mu_{\max}$	3.454	4.588	4.218
$K_N$	20.433	328.200	53.970
$K_P$	0.007	2.413	2.819
$K_I$	120.000	120.000	120.000
$S_N$	15.787	1122.333	43.570
$S_P$	3.267	267.000	164.700
$I$	300.000	300.000	300.000
$X_0$	24.899	24.899	24.899
$X_m$	101.979	1391.011	299.874

Figure 1 shows the growth of microalgae *Botryococcus sp.* in domestic, agricultural and industrial wastewater for 20 days. Continuous plot line represents the estimated result by the extended Monod model and Verhulst model while the discrete plot represents the experimental data. The microalgae growth is considered decreases once the stationary phase was nearly reached due to the depletion of nutrients. In Figure 1, the growth of microalgae in three types of wastewater have a positive growth and reach the maximum cell concentration which are 101.9789 mg/L, 1391.011 mg/L and 299.8737 mg/L in domestic, agricultural and industrial wastewater, respectively. The similar result between estimated and experimental data indicates the substitution of extended Monod model into Verhulst model is suitable for microalgae *Botryococcus sp.* growth.

### 4.2. Nutrient uptake by microalgae *Botryococcus sp.*

There are two types of nutrients considered which are nitrogen and phosphorus. The nitrogen and phosphorus are investigated by using (4) with the substitution of specific growth of extended Monod model in domestic, agricultural and industrial wastewater. The experimental data was taken from researchers in Universiti Tun Hussein Onn [23, 24]. Parameters used for nitrogen and

phosphorus uptake in three types of wastewater are provided in Table 2 and Table 3.

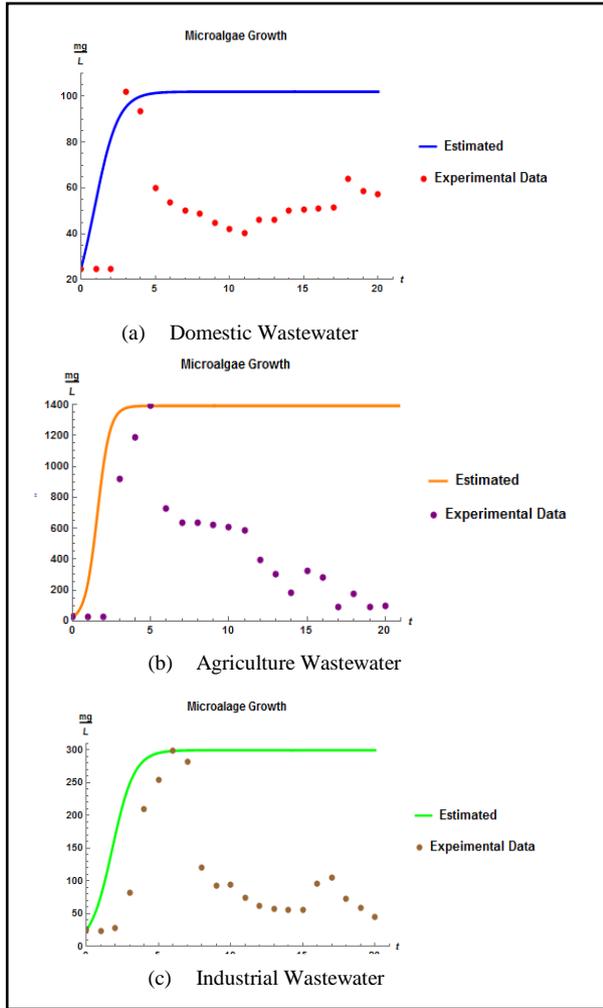


Fig 1: Microalgae *Botryococcus sp.* growth in (a) Domestic wastewater, (b) Agriculture wastewater and (c) Industrial wastewater for 20 days

Table 2: Parameters of nitrogen uptake by microalgae *Botryococcus sp.*

Parameter	Domestic	Agricultural	Industrial
$S_0$	15.787	1122.333	43.570
$S_{na}$	20.080	261.800	20.820
$X_0$	24.899	24.899	24.899
$\mu$	1.073	2.513	1.323
$Y$	-17.955	5.188	12.087

Table 3: Parameters of phosphorus uptake by microalgae *Botryococcus sp.*

Parameter	Domestic	Agricultural	Industrial
$S_0$	3.267	267.000	164.700
$S_{na}$	0.000	2.148	2.562
$X_0$	24.899	24.899	24.899
$\mu$	1.073	2.513	1.323
$Y$	23.593	5.158	1.696

Figure 2 shows the nitrogen and phosphorus uptake by microalgae *Botryococcus sp.* in domestic wastewater for 20 days. The estimated results are showed in continuous plot line while discrete plot represents the experimental data [23]. According to the result, the initial nitrogen concentration in domestic wastewater increases from 15.787 mg/L to 20.080 mg/L at the end of treatment process. This shows the nitrogen in domestic wastewater did not consumed by microalgae *Botryococcus sp.* and can be concluded that nitrogen in domestic wastewater did not remove successfully by microalgae *Botryococcus sp.* In addition, the initial substrate concentration of phosphorus for both are 3.267 mg/L and final substrate concentration was 0 mg/L. The phosphorus decreases drastically from day 0 until day 5 and this is due to the reason of

microalgae *Botryococcus sp.* are efficient in removing the phosphorus from wastewater.

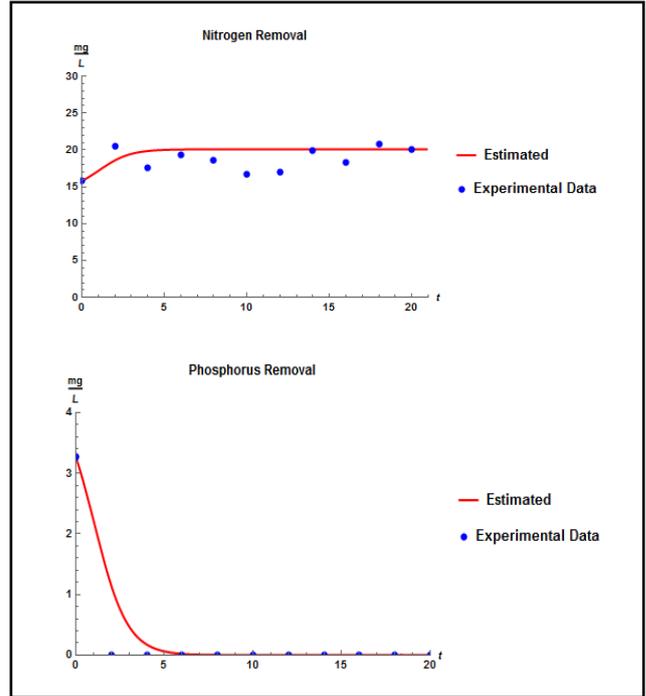


Fig 2: Nitrogen and phosphorus uptake by microalgae *Botryococcus sp.* from domestic wastewater for 20 days

Furthermore, Figure 3 shows the nitrogen and phosphorus uptake by microalgae *Botryococcus sp.* in agricultural wastewater for 20 days. The estimated results are showed in continuous plot line while the discrete plot represents the experimental data [23]. Initial nitrogen concentration which is 1122.333 mg/L and decreases dramatically in day 3 to deplete all the way to 261.80 mg/L in day 20. This illustrates that total nitrogen consumed by microalgae *Botryococcus sp.* during phycoremediation is about 860.533 mg/L in agricultural wastewater. The pattern of the graph shows phosphorus decreases from the initial reading which is 267 mg/L to 2.148 mg/L in day 20. It means that phosphorus is one of the nutrients that necessarily needed by microalgae *Botryococcus sp.* for its growth. As conclusion, the nutrients in agricultural wastewater are consumed by microalgae *Botryococcus sp.* successfully to improve the water quality.

Figure 4 is plotted to estimate the efficiency of nutrient uptake by microalgae *Botryococcus sp.* for industrial wastewater. The experiment is undergone for 20 days and discrete plot shows the experimental data which taken from researcher [24] while the continuous plot line represents the estimated result. Based on Figure 4, the initial nitrogen concentration starts to decrease on day 1 to day 5 which from 43.57 mg/L to 21.18 mg/L. The phosphorus concentration is also reduced significantly from day 0 which is 164.70 mg/L to day 5 and deplete all the way to 2.562 mg/L in day 20. This reveals that the microalgae *Botryococcus sp.* consume a large amount of phosphorus in order to accomplish the process of phycoremediation. Hence, the phycoremediation process in industrial wastewater is considered as successful due to the phosphorus and nitrogen had been consumed efficiently by microalgae *Botryococcus sp.*

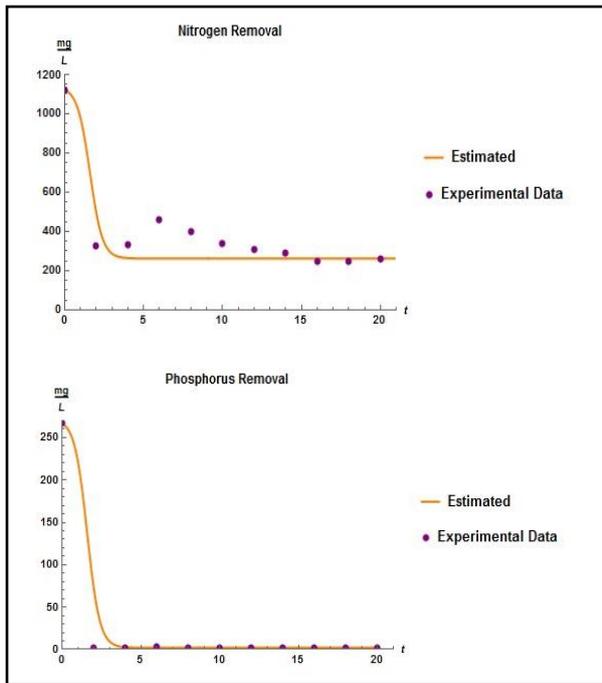


Fig 3: Nitrogen and phosphorus uptake by microalgae *Botryococcus sp.* from agriculture wastewater for 20 days

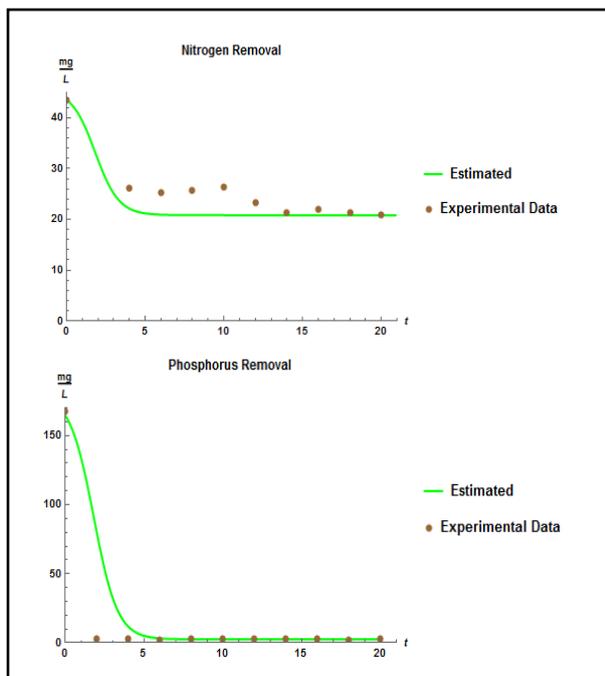


Fig 4: Nitrogen and phosphorus uptake by microalgae *Botryococcus sp.* from industrial wastewater for 20 days

### 4.3. Comparison of Microalgae *Botryococcus sp.* Growth and Nutrient Uptake in Three Types of Wastewater

The comparison of microalgae growth and nutrient uptake in domestic, agricultural and industrial wastewater are discussed in this section.

Figure 5 indicates the microalgae *Botryococcus sp.* growth in three types of wastewater which are domestic, industrial and agricultural wastewater while Figure 6 illustrates the nitrogen and phosphorus removal by microalgae *Botryococcus sp.* in the same types of wastewater. Figure 5 shows that microalgae *Botryococcus sp.* have the highest growth in agricultural wastewater followed by industrial wastewater and domestic wastewater. This is due to the agricultural wastewater contain of high turbidity and nutrient

concentrations and high concentration of insoluble organic compounds which favoured growth of microalgae [25].

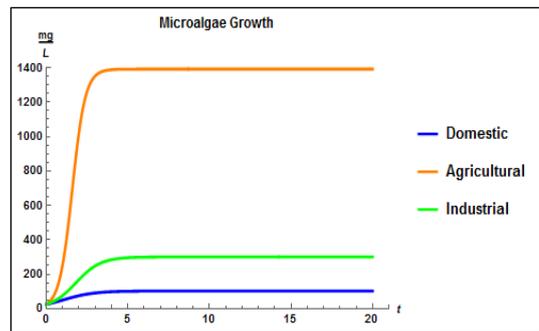


Fig 5: Microalgae *Botryococcus sp.* growth in three types of wastewater

Besides that, the nutrients content in agricultural wastewater is the highest and followed by industrial wastewater and domestic wastewater based on Figure 6. Domestic wastewater came from surrounding area of school contain low concentration of nutrients compared to industrial wastewater. Nitrogen did not assimilate by microalgae *Botryococcus sp.* in domestic wastewater is due to the oxygen content was low in domestic wastewater and it caused the anaerobic denitrifying bacteria fail to carry out nitrification that favour consumed by microalgae which resulted as Figure 6 [26,27]. However, the phosphorus consumed by microalgae *Botryococcus sp.* has resulted a positive growth as shown in Figure 5 for domestic wastewater. Thus, we can conclude that the nutrient concentrations have the direct impact on microalgae *Botryococcus sp.* growth. Therefore, the microalgae *Botryococcus sp.* is suitable for treating wastewaters.

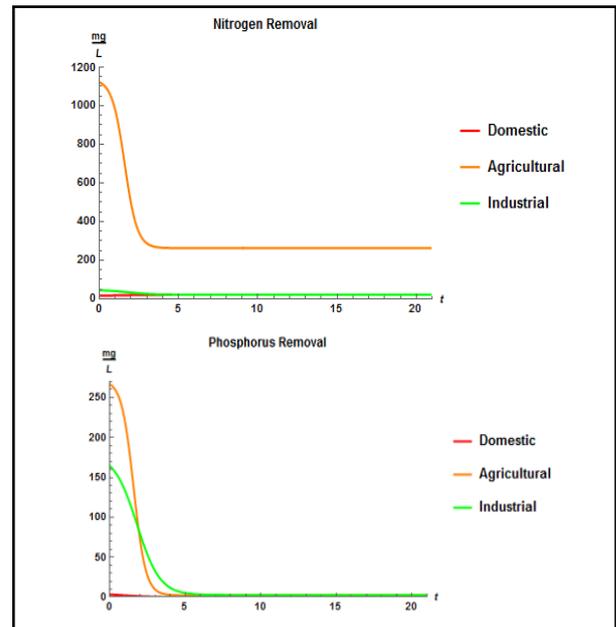


Fig 6: Nutrients removal by microalgae *Botryococcus sp.* in three types of wastewater

## 5. Conclusion

In conclusion, the estimation for microalgae *Botryococcus sp.* and nutrient uptake by microalgae *Botryococcus sp.* were obtained by substituting the specific growth rate of extended Monod model into Verhulst model in domestic, agricultural and industrial wastewater. The results were described the microalgae growth and nutrient uptake by microalgae for 20 days. The microalgae *Botryococcus sp.* growth in agricultural wastewater is the highest and followed by industrial and domestic wastewater which is

depend on the nutrients removal. Besides, the estimated results shown in this research are giving similar pattern with the experimental data. Hence, the extended Monod model together with the Verhulst model are suitable for estimation of microalgae growth and nutrient uptake by microalgae. Moreover, the microalgae species of *Botryococcus sp.* is promising for treating wastewater in different types of wastewater.

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