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Research paper

A Software Tool for Multiple Model Adaptive Control Design in Active Suspension System Applications

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Abstract

The multiple model adaptive control (MMAC) method has the possibility to keep up control performance over an extensive variety of parameter variations. A key part in the design of a MMAC system is the choice of candidate models. A computer package tool has been developed to optimize and determine these candidate models. The method exploits the relationship between Grobner bases and polynomial spectral factorization through the notion of sum of roots. The symbolic solution to the Algebraic Riccati Equation, and hence the H2 optimal control problem, is obtained using computer algebra technique. The MMAC system was implemented on a quarter car active suspension with changing sprung mass. The symbolic solution giving the relationship between the H2 cost function and varying sprung mass values was obtained. The user inputs the number of candidate models to be used, and the package selects suitable intervals between candidate cost values. From the analytical solution obtained, sprung mass values corresponding to each selected candidate cost value could be found. The software functions as a tool to optimally choose sprung mass values for each candidate model in the MMAC active suspension system.

Keywords: *MMAC; sum of roots; Grobner bases; spectral factorization; active suspension*

1. Introduction

The ride and handling capabilities of a ground vehicle very much depends on its suspension characteristics. During operation, vehicle suspensions would be subjected to different and discontinuous dynamics. This might be because of variations in the sprung mass, which relies upon the number of passengers and the load being carried and may likewise be because of the loss of vehicle body parts due to an accident or wear and tear. Another type of variation may originate from the road surface and climate conditions, which could change suddenly. Suspension systems need to give satisfactory ride and handling performance under such varying and uncertain situations, such that safety and passenger comfort are constantly guaranteed. In designing vehicle suspension systems, the uncertainty in the sprung mass, such as its loading conditions, must be taken into account to meet the vehicle performance criteria. Active suspensions aim to improve vehicle ride comfort and handling characteristics using feedback control.

Conventional control methods are not designed to deal with widely varying uncertainties and in this manner, do not sufficiently guarantee robustness [1]. Adaptive control methods based on switching of fixed controller parameters have, as of late, received the attention of the control community. Early work on such control systems was directed for the F-8C aircraft [2]. The concept has since generated interest in control problems where a single linear model is insufficient, for example, in fault tolerant control problems where sudden changes happen in the system dynamics caused by sensor and actuator faults and in stabilizing linear systems with unknown parameters [3]. The utilization of parameter-dependent controllers has been proposed for quarter-vehicle suspensions with sprung mass variations [4, 5]. In [4,5,6], polytopic parameter uncertainties were utilized to demonstrate the differing sprung and unsprung masses of vehicles.

Multiple model adaptive control (MMAC) is a concept that extends the use of established linear control design methods so as to be applicable to systems having wider regions of uncertainty. When the real parameter of uncertainty is much larger, the range of parameter values can be divided into sets of smaller ranges, where each set is associated with a specific plant model. Therefore, in each set, the uncertainty of the parameters is decreased and therefore satisfactory performance can always be achieved within the uncertainty bounds in each set.

In the MMAC framework, a good choice of candidate models will determine its successful implementation. To date, this choice is made heuristically. In [7,8], a method for optimizing the candidate model has been proposed. The method makes use of computer algebra techniques to formulate the analytical solution to the optimal control problem. The solution is in the form of an algebraic expression with the parameters of interest denoted by symbolic variables. Following on from there, in the current work, a software package has been developed in which candidate models can be automatically selected such that the effects of parameter variations can be reduced in the most effective way.

2. MMAC system design

Figure 1 showed the principal idea behind MMAC, which is to have a set of candidate models encompassing the whole operating region of the plant. For every candidate model, a controller that creates the required plant performance is determined a priori. By using some form of cost criterion during system operation, the candidate model that best matches the plant is decided, and the corresponding controller is activated into the control loop. In the event that a huge change in plant dynamics happens, an alternate candidate model may match the new plant parameters better and therefore, the activated controller would switch accordingly. A steady acceptance of such systems might be accomplished either by guaranteeing that a typical Lyapunov function exists for the Youla parameters or by resetting the conditions of the time-varying system at switching instances [9,10].

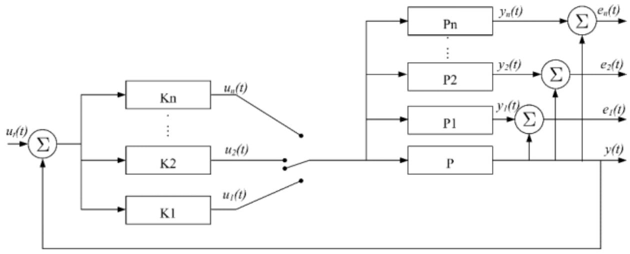

Fig. 1: Multiple model adaptive control.

Relatively an unexplored area, few studies regarding application of MMAC on active suspension systems can be found in the literature. In [11], the utilization of MMAC with linear quadratic regulator (LQR) controllers to improve ride and handling properties was studied. Stability was guaranteed by the presence of a common quadratic Lyapunov function (CQLF). Zhong et al. [12] proposed a Linear quadratic Gaussian (LQG) tuned MMAC system to improve vehicle ride and handling, subject to dynamic reliability constraints. In our work, the candidate controllers $\{K_1, K_2, \ldots, K_n\}$ are all H_2 Optimal Controllers. The developed software package would automatically determine the candidate controller parameters based on design parameters which are input by the user.

3. H_2 optimal control

Consider the state feedback control problem as illustrated in Figure 2. Here, P represents the plant to be controlled and K is the feedback gain.

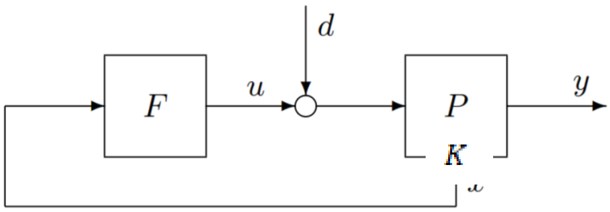

Fig. 2: State feedback design.

Here, x, u, y and d are respectively, the plant states, the control input, the plant output and disturbance input. The H_2 control problem is to reduce the impact of the disturbance, d, on the output, y subject to constraints due to physical limitations on the control input u. This may be achieved by minimizing the H_2-norm of the transfer function matrix from d to $[y\ u]^T$ with an optimal state feedback gain, K. The optimal K can be found from the quadratic cost function given by

$$E(P,K) = \int_0^\infty (\|y(t)\|_2^2 + \|u(t)\|_2^2)dt \qquad (1)$$

where $\|\cdot\|_2$ is the (Euclidean) 2-norm. Then, the greatest attainable efficiency is given by

$$E^*(P) = \inf_{K_{stabilizing}} E(P,K) \qquad (2)$$

This is commonly found by obtaining the solution to the Algebraic Riccati Equation (ARE).

A plant P of degree n with m inputs and p outputs in the state-space model representation is given as

$$P = \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \qquad (3)$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. Under state feedback control, the control input would be given by

$$u = (A + BK)x \qquad (4)$$

Suppose (A, B) and (C, A) are stabilizable and detectable. Then, the H_2 optimal controller may be obtained as follows. Let $X = X^T \geq 0$ be the solution of the ARE given by

$$A^T X + XA - XBB^T X + C^T C = 0 \qquad (5)$$

Then, the H_2 optimal state feedback gain is given by

$$K_{opt} = -B^T X \qquad (6)$$

The best obtainable control performance may be obtained from the cost function

$$E^*(P) = tr\{B^T XB\} \qquad (7)$$

X will vary according to the value of the plant parameters given by A, B and C. If the analytical solution to the ARE could be found in terms of symbolic variables, the relationship between X (and hence K) and the varying (uncertain) parameters can be established. Thus, the candidate models can be chosen based on this relationship to give the optimal result.

4. Analytical solution of the are

There are several numerical methods to solve the ARE through spectral factorization, but these methods are incapable of performing computation with symbolic parameters. More recently, it has been discovered that there exists an interesting connection between the sum of roots (SoR) and polynomial spectral factorization through the concept of Grobner Bases [13].

The ARE may be written in the Hamiltonian matrix form, which is of degree $2n \times 2n$ and composed in the form of

$$H = \begin{bmatrix} A & R \\ -Q & A^T \end{bmatrix} \qquad (8)$$

where, $Q, R \in \mathbb{R}^{n\times n}$ are symmetric weighting matrices

Then, the spectral factor of H is found [14]. Assume that the eigenvalues in the open left half plane are $\lambda_i, i = 1,2, \ldots, n$. By seeking a basis for $\mathcal{V}$ wherein $\mathcal{V}$ is the invariant subspace of H according to the $i's$, one obtains X_1, X_2 such that

$$\mathcal{V} = Range \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, X_1, X_2 \in \mathbb{C}^{n\times n}. \qquad (9)$$

Then, the solution takes the form of

$$X = X_2 \times X_1^{-1} \in \mathbb{R}^{n\times n} \qquad (10)$$

Having obtained the analytical solution to the ARE, from (7), we now have the relationship between the cost function $E^*(P)$ and the sprung mass m_1. We can now choose the candidate models based on sprung mass values such that $E^*(P)$ is equally spaced between candidate models. This would ensure that the bounds on uncertainty for each candidate controller are equal, and hence, no candidate controller would have to handle a larger range of uncertainty compared to the others.

5. Implementation on an active suspension system with varying sprung mass

Figure 3 shows the quarter car suspension model used in this study. This is the basic model where m_1 and m_2 are the sprung mass and unsprung mass respectively and k_t is the tyre stiffness. The shock absorber damping and spring forces, and controller actuator force are lumped together and represented by u.

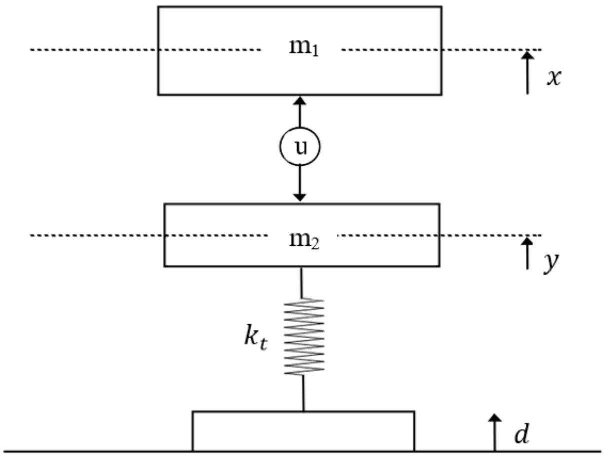

Fig.3: Quarter car model.

The dynamic equations representing this model is then

$$\begin{aligned} m_1\ddot{x}(t) &= u(t) \\ m_2\ddot{y}(t) &= -k_t[-d(t)+y(t)] - u(t) \end{aligned} \tag{11}$$

This can be represented in the matrix-vector state space form (3) as

$$\begin{bmatrix}\dot{y}\\ \dot{x}\\ \ddot{y}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -k_t/m_2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}y\\ x\\ \dot{y}\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ -1/m_2 & k_t/m_2\\ 1/m_1 & 0\end{bmatrix}\begin{bmatrix}u\\ d\end{bmatrix} \tag{12}$$

$$Output = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}y\\ x\\ \dot{y}\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}u\\ d\end{bmatrix}$$

This can be written in the augmented state space form as

$$P = \begin{bmatrix}0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1/m_1 & 0\\ 0 & -k_t/m_2 & 0 & 0 & -1/m_2 & -k_t/m_2\\ 1 & 1 & 0 & 0 & 0 & 0\end{bmatrix} \tag{13}$$

Suppose $k_t = 155900$, $m_2 = 20$ kg and m_1 is the design parameter. When the software tool is run, the user is required to enter the value of k in the dialogue box as shown in Figure 4.

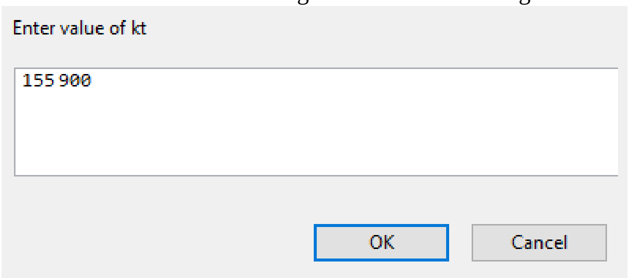

Fig. 4: Input of k_t.

The program then computes the analytical solution for the cost function in terms of m_1, which in this case, is given as

$$E(\rho) = \sqrt{2}\sqrt{-3118 + \sqrt{\frac{24304810001}{2500} + \frac{311802}{25}\sqrt{1559}\sqrt{\frac{1}{m_1}} + 4\sqrt{1559}\left(\frac{1}{m_1}\right)^{3/2} + \frac{1}{m_1^2} + \frac{311801}{25m_1} - 2\sqrt{1559}\sqrt{\frac{1}{m_1}}}} \tag{14}$$

Suppose also that the sprung mass, m_1 may vary in the range of 5 to 500 kg. The user inputs this in the respective dialogue boxes as shown in Figures 5 and 6. Equation (13) is then solved for the range of m_1 from 5 to 500 kg.

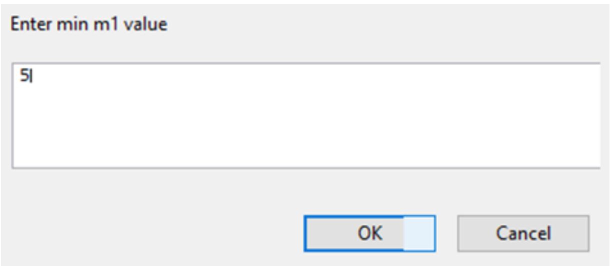

Fig. 5:. Input of minimum m_1 value.

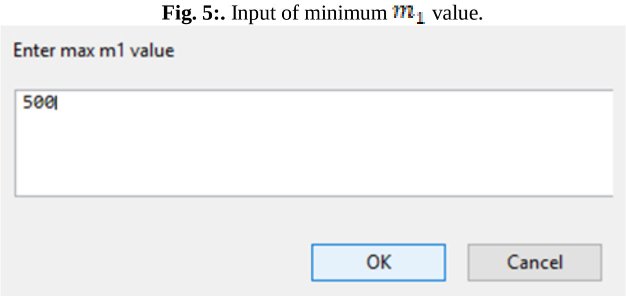

Fig. 6: Input of maximum m_1 value.

Then, in the next dialogue box (Figure 7), the user is required to enter the number of candidate models to be used. Suppose we wish to create an MMAC system with 8 candidate models.

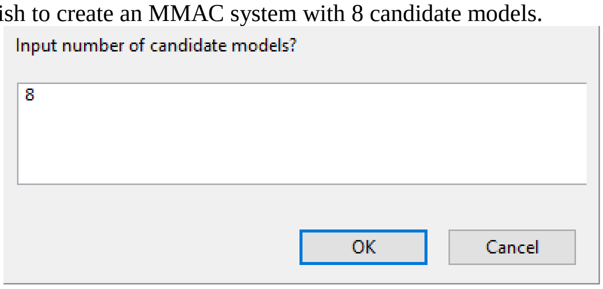

Fig. 7: Input of candidate models.

Upon entering 8 through the dialogue box, the tool returns the result as

$m_1 = \{204.8, 69.2, 34.4, 20.5, 13.6, 9.7, 7.2, 5.6\}$

These are the values of m_1 that correspond to the range of $E(\rho)$ being divided equally into 8 regions. Graphically, this result can be visualised as shown in Figure 8.

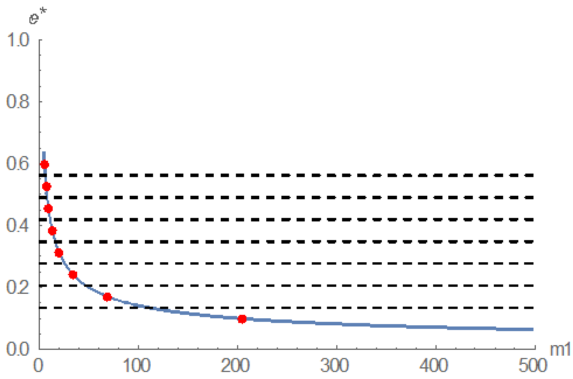

Fig. 8: Graph of $E(\rho)$ against m_1.

Subsequently the 8 candidate state space models are formed as

$$\dot{x} = A_i x + B_i u$$
$$y = C_i x + D_i u(t) \tag{15}$$
$$i = \{1,2,\ldots,8\}$$

The corresponding H_2 optimal controllers, K_i can then be found from (4). The resulting candidate controllers are shown in Table 1. The effectiveness of this approach to MMAC design for quarter car active suspensions has been demonstrated through simulation tests, which have been presented in [8]. This software package now enables the design process to be automated for any user determined value of unsprung mass, tyre stiffness, number of candidate models and range of allowed sprung mass.

6. Conclusions

Through the use of computer algebra, the analytical solution to the ARE in a H_2 optimal control problem for a quarter car active suspension system can be obtained. This gives the connection between the cost function and the sprung mass, providing an insight on how the sprung mass affects the performance of the control system. Consequently, the selection of the MMAC candidate models can be made optimally, based on this observed relationship.

In the present work, through the use of computer algebra methods, a software package has been developed to automatically identify the optimal candidate models and the corresponding candidate controllers. In future, the software could be further developed for uncertainties on other parameters such as the shock absorber damping coefficient and spring constant.

Table 1: Corresponding candidate controller K_i for given sprung mass.

Candidate Model *i*	**Sprung Mass (kg)**	**Candidate Controller K_i**
1	204.8	$K_1 = [-0.0057 \quad -0.0001 \quad 0.0003 \quad -0.1557]$
2	69.2	$K_2 = [-0.0057 \quad -0.0002 \quad 0.0003 \quad -0.4608]$
3	34.4	$K_3 = [-0.0057 \quad -0.0005 \quad 0.0003 \quad -0.9269]$
4	20.5	$K_4 = [-0.0057 \quad -0.0008 \quad 0.0003 \quad -1.5554]$
5	13.6	$K_5 = [-0.0057 \quad -0.0012 \quad 0.0003 \quad -2.3445]$
6	9.7	$K_6 = [-0.0057 \quad -0.0017 \quad 0.0003 \quad -3.2871]$
7	7.2	$K_7 = [-0.0057 \quad -0.0023 \quad 0.0003 \quad -4.4285]$
8	5.6	$K_8 = [-0.0057 \quad -0.0030 \quad 0.0003 \quad -5.6938]$

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References

[1] Gaspar, P., Szaszi, I and Bokor, J. 2003. Design of robust controllers for active vehicle suspension using the mixed µ-synthesis, Vehicle System Dynamics, 40(4), pp. 193–228.

[2] Athans, M., Dunn, K.-p., Greene, C.S., Lee, W.H., Sandell, N.R., Segall, I. and Willsky, A.S. 1975. The stochastic control of the F-8C aircraft using the multiple model adaptive control (MMAC) method, 1975 IEEE Conference on Decision and Control including

the 14th Symposium on Adaptive Processes, Houston, TX, USA, 217-228.

[3] Chen W. and Anderson, B.D.O. 2009. Multiple model adaptive control (MMAC) for nonlinear systems with nonlinear parameterization, Proceedings of the 48th IEEE Conference on Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference, CDC/CCC 2009, 7669 - 7674.

[4] Du, H., Zhang, N. and Lam, J. 2008. Parameter-dependent input delayed control of uncertain vehicle suspensions, Journal of Sound and Vibration, 317:537-556.

[5] Gao, G. and Yang, S. 2006. Semi-active control performance of railway vehicle suspension featuring magnetorheological dampers, Proc. of the 1ST IEEE Conference on in Industrial Electronics and Applications, 1-5.

[6] Du, H. and Zhang, N. 2007. H∞ control of active vehicle suspensions with actuator time delay, Journal of Sound and Sibration, 301, pp. 236-252, 2007.

[7] Haris, S.M., Shavalipour, A., and Nopiah, Z.M. 2015. International Review on Modelling and Simulations (I.RE.MO.S.), 8(3):377-385.

[8] Haris, S.M., Shavalipour, A., Nopiah, Z.M. and Yaacob, Y. 2016. Multiple model adaptive control design: A computer algebra approach, International Journal of Advancements in Mechanical and Aeronautical Engineering, 3(1):131-136.

[9] Hespanha, J.P. and Morse, A.S., 2002. Switching between stabilizing controllers, Automatica, 38:1905–1917.

[10] Hespanha, J.P., Liberzon, D. and Morse, A.S. 2003. Overcoming the limitations of adaptive control by means of logic-based switching, Systems and Control Letters, 49:49–65.

[11] Koch, G. 2011. Adaptive Control of Mechatronic Vehicle Suspension, Ph.D Thesis, Technical University of Munich, Institute of Automatic Control.

[12] Zhong, X., Ichchou, M., Gillot, F. and Saidi, A. 2010. A dynamic-reliable multiple model adaptive controller for active vehicle suspension under uncertainties. Smart Materials and Structures, 19(4), 045007.

[13] Anai, H., Hara, S., Kanno, M. and Yokoyama, K. 2009. Parametric polynomial spectral factorization using the sum of roots and its application to a control design problem‖, Journal of Symbolic Computation, 44:703-725.

[14] Kanno, M. and Hara, S. 2012. Symbolic-numeric hybrid optimization for plant/controller integrated design in H∞ loop-shaping design. Journal of Math-for-Industry (JMI), 4:135-140.