International Journal of Engineering & Technology, 7 (4) (2018) 3697-3701

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i4.16885
Research paper

Compensation method of geodynamic trend in the systems of geoelectric control

Gleb S. Vasilyev [1] *, Oleg R. Kuzichkin, Anastasia V. Grecheneva [1], Nikolay V. Dorofeev [2]

[1] Belgorod National Research University, Russia
[2] Murom Institute of Vladimir State University, Russia
**Corresponding author E-mail: vasilievgleb@yandex.ru*

Abstract

To ensure the safety of important national economic objects in the conditions of technogenic, biogenic and anthropogenic interference, it is advisable to carry out automated electromagnetic control of geodynamic processes on the basis of multipolar sensing systems. The paper substantiates the use of methods of compensation of geodynamic variations to increase the sensitivity of multipolar geoelectric control sys-tems. The developed method involves the implementation of compensation of changes in the components of electric field in the control points caused by the natural geodynamic mismatch of the probing system from the object under study, using additional sources. Selection of sources parameters is based on the solution of an approximation problem of the variations of the transverse and longitudinal field compo-nent caused by the geodynamics of the controlled medium. Mathematical modeling has confirmed high accuracy of compensation of real geoelectric fields by the field of additional sources in the conditions of both narrow and wide area of location of the receiving electrodes of the probing installation, which determines an efficiency of the method in the conditions of complex construction, typical for most geotech-nical objects.

Keywords: *Engineering-Technical Objects; Geodynamic Control; Geoelectric Control; Mathematical Modeling; Multi-Pole Electro Location.*

1. Introduction

The use of geodynamic monitoring systems based on geoelectric methods of control of the geological environment allows to ensure the safety of operation of important national economic objects in the conditions of technogenesis and activation of exogenous hydrogeological processes. In particular, to increase the geodynamic sensitivity of geoelectric control, equipotential methods are widely used [1-2]. The major drawbacks limiting the application of these methods are the difficulty of accurate installation of measuring electrodes on equipotential lines, which is necessary to ensure high sensitivity of the control system to small geodynamic changes in geological objects [3]. In addition, to preserve the metrological properties of measurement systems in the process of their long-term operation, it is necessary to carry out a time-consuming procedure for resetting the measuring electrodes for positioning them on the equipotential lines arising from the geodynamic imbalance of the system.

One of the ways to solve the problem is the use of multi-pole electrical installations with additional compensation sources [4-5]. Their use allows selection of the currents values of additional sources to eliminate an influence of additive noise caused by the initial inaccurate installation of measuring electrodes or changes in the field structure caused by the geodynamics of the controlled medium during the system operation. Thus, it is possible to increase the sensitivity of the system to small geodynamic variations.

One of the requirements for multipolar systems of geoelectric monitoring of geodynamic processes is the possibility of algorithmic current adjustment of the system to the expected change in the conditions of geodynamic control and arbitrary spatial location of a multipolar electrical installation [6]. For the implementation of the adjustment flexible control of additional sources currents to create a field, reverse the geodynamic change of the field of the controlled environment, is required. The choice of sources parameters requires the solution of the approximation problem of the variations of the transverse and longitudinal field component caused by geodynamics of the controlled medium, near the typical simplest inhomogeneities.

The aim of the work is to study the method of approximation of geodynamic changes in the electric field by the field of additional sources to compensate for the mismatch of the geoelectric monitoring system due to geodynamic variations of the parameters of the controlled area of the geological environment.

2. Compensation method of geodynamic changes by the field of additional sources

The conditions of application of geoelectric systems of geodynamic control are quite diverse and are primarily determined by the controlled object and its geodynamic parameters. In small-depth geophysics, standard models are used to describe near-surface inhomogeneities and their geodynamic variations [7-9]. Usually, when using geoelectric methods at small distances and depths, typical for the tasks of geotechnical monitoring of objects, two media with a flat boundary can be selected as the model of the geoelectric section [10]. The model is illustrated in figure 1.

Copyright © 2018 Gleb S. Vasilyev et. al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

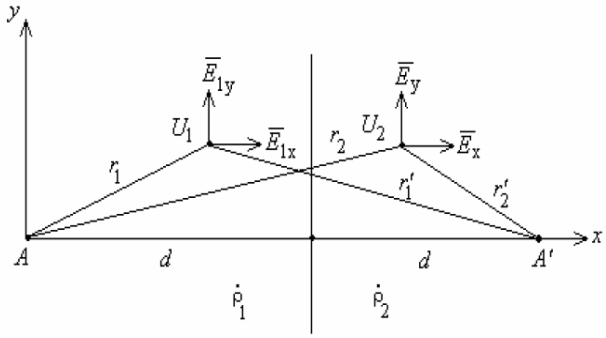


Fig. 1: Model of Two Media Boundary Using One Point Source of Geoelectric Field.

Here, the point source with current I0 is located on the surface of the half-space at the origin (point A), the axis z is directed to the depth. The distance from the source A to the media boundary is equal to d, so the mirror image of the source has coordinates $(2d;\ 0;\ 0)$. Taking into account the boundary conditions, the field potentials at the ground-air boundary at the point (x, y) with coordinates of the first medium U_1 and the second medium U_2 are determined by the expressions:

$$\dot{U}_1(x,y,d)=\frac{\dot{I}_0\dot{\rho}_1}{2\pi}\left(\frac{1}{\sqrt{x^2+y^2}}+\frac{\dot{K}(j\omega)}{\sqrt{(x-2d)^2+y^2}}\right) \tag{1.1}$$

$$\dot{U}_2(x,y,d)=\frac{\dot{I}_0\dot{\rho}_1}{2\pi}\left(\frac{1+\dot{K}(j\omega)}{\sqrt{x^2+y^2}}\right) \tag{1.2}$$

Where $\dot{K}(j\omega)=\frac{\dot{\rho}_2(j\omega)-\dot{\rho}_1(j\omega)}{\dot{\rho}_2(j\omega)+\dot{\rho}_1(j\omega)}$ is the complex frequency-dependent contrast coefficient determined by the resistivity ρ and permittivity ε of the first and second media: $\dot{\rho}_{1,2}(j\omega)=\frac{\rho_{1,2}}{1+j\omega\varepsilon_{1,2}\rho_{1,2}}$.

In (1.1) and (1.2) the first terms determine the normal component of the potential due to the influence of the real source A; the second terms determine the abnormal component of the potential, describing an impact of heterogeneity and caused by imaginary source: $\dot{A}'$.

In accordance with obtained expressions for the potential the ratio for complex vectors of the geoelectric field in the first medium take the form

$$\dot{E}_1^x(x,y,d)=-\frac{\partial\dot{U}_1(x,y,d)}{\partial x}=\frac{\dot{I}_0\dot{\rho}_1}{2\pi}\left(\frac{x}{\left(\sqrt{x^2+y^2}\right)^3}+\frac{\dot{K}(x-2d)}{\left(\sqrt{(x-2d)^2+y^2}\right)^3}\right) \tag{2.1}$$

$$\dot{E}_1^y(x,y,d)=-\frac{\partial\dot{U}_1(x,y,d)}{\partial y}=\frac{\dot{I}_0\dot{\rho}_1}{2\pi}\left(\frac{y}{\left(\sqrt{x^2+y^2}\right)^3}+\frac{\dot{K}y}{\left(\sqrt{(x-2d)^2+y^2}\right)^3}\right) \tag{2.2}$$

And for the second environment

$$\dot{E}_2^x(x,y,d)=-\frac{\partial\dot{U}_2(x,y,d)}{\partial x}=\frac{\dot{I}_0\dot{\rho}_1}{2\pi}\left(1+\dot{K}\right)\frac{x}{\left(\sqrt{x^2+y^2}\right)^3} \tag{3.1}$$

$$\dot{E}_2^y(x,y,d)=-\frac{\partial\dot{U}_2(x,y,d)}{\partial y}=\frac{\dot{I}_0\dot{\rho}_1}{2\pi}\left(1+\dot{K}\right)\frac{y}{\left(\sqrt{x^2+y^2}\right)^3} \tag{3.2}$$

Based on the principle of stationarity and linearity of the geoelectric section, change in tension under the influence of current geodynamic variations of the object of control under the condition of initial balancing of the measuring system can be determined by the expression

$$\Delta\dot{E}_{1,2}^{x,y}(x,y,d,\bar{\alpha})=\sum_{l=1}^{L}\frac{\partial\dot{E}_{1,2}^{x,y}(x,y,d,\bar{\alpha}^0)}{\partial\alpha_l}\Delta\alpha_l \tag{4}$$

Where $\bar{\alpha}=(\alpha_1^0+\Delta\alpha_1,...,\alpha_L^0+\Delta\alpha_L)$ and $\bar{\alpha}^0=(\alpha_1^0,...,\alpha_L^0)$ are the current and nominal value of the vector of geodynamic parameters with dimension L. In the simplest case, geodynamics of a medium with a sufficient degree of accuracy is described by changing of one parameter. In particular, for the considered plane boundary of the two media such parameter can be the boundary shift Δd. In other cases, the variable parameters can be the slope rotation of the plane boundary, or the approximating planes coefficients that are the fragments of the piecewise linear boundary [11-13].

Thus, change in the field strength at variation Δd (schematically shown in figure 2) takes the form:

$$\Delta\dot{E}_{1,2}^{x,y}(x,y,d,\Delta d)=\frac{\partial\dot{E}_{1,2}^{x,y}(x,y,d)}{\partial d}\Delta d \tag{5}$$

For the 1st medium

$$\frac{\partial\dot{E}_1^x(x,y,d)}{\partial d}=\frac{\dot{I}_0\dot{\rho}_1\dot{K}}{2\pi}\left(\frac{6(x-2d)^2}{\left(\sqrt{(x-2d)^2+y^2}\right)^5}-\frac{2}{\left(\sqrt{(x-2d)^2+y^2}\right)^3}\right) \tag{6.1}$$

$$\frac{\partial\dot{E}_1^y(x,y,d)}{\partial d}=\frac{\dot{I}_0\dot{\rho}_1\dot{K}}{2\pi}\left(\frac{6y(x-2d)}{\left(\sqrt{(x-2d)^2+y^2}\right)^5}\right) \tag{6.2}$$

For the 2nd medium, components of the intensity (3) do not contain a controlled geodynamic parameter, therefore

$$\frac{\partial\dot{E}_2^{x,y}(x,y,d)}{\partial d}=0 \tag{7}$$

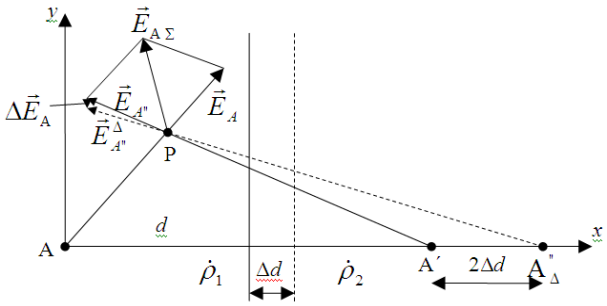


Fig. 2: Field Change On $\Delta\vec{E}_A$, Caused By Geodynamic Shift of the Boundary Δd for the Source A, Located in the Point P.

To implement an adjustment of the measuring system, it is necessary to control the currents of additional sources to create a field, reverse to the geodynamic change of field of the controlled medium. In general, the field control is carried out on many receiving points with coordinates (x_m, y_m), condition of compensation of geodynamic variations of the object for a point takes the form:

$$\Delta\dot{E}^{x,y}(x_m,y_m,d,\Delta d)=-\sum_{n=1}^{N}\frac{\dot{I}_n^{\hat{e}}}{\dot{I}_0}\dot{E}^{x,y}(x_m-x_n^{\hat{e}},y_m-\acute{o}_n^{\hat{e}},d+\Delta d-x_n^{\hat{e}}) \tag{8}$$

Where $\dot{I}_n^{\hat{e}}, x_n^{\hat{e}}, \acute{o}_n^{\hat{e}}$ are currents and coordinates of additional compensation sources? According to (7), compensation is not required for the second environment, so the lower indices are omitted.

One of the possible approaches for the practical use of this method in the systems of geodynamic control in geotechnical monitoring of objects is the use of approximation algorithms for determining the parameters of compensation signals.

Parametric approximation

When implementing the proposed method of compensation of geodynamic changes on the basis of parametric approximation, various options are possible. As the parameters of the objective minimization function, variations of different individual parameters of the intensity vector can be used: longitudinal or transverse component, modulus or phase vector or its individual components.
Approximation of the longitudinal component of the variation of the electric field intensity ΔEx or the transverse component ΔEy
On the basis of (8) condition of optimal compensation according to the criterion of minimum standard error (MSE) takes the form

$$\sigma_E^{x,y}(\mathbf{x}^{\hat{e}},\mathbf{\acute{o}}^{\hat{e}},\mathbf{I}^{\hat{e}})=\sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\left|\Delta\dot{E}_m^{x,y}+\tilde{\dot{E}}_{m\hat{e}}^{x,y}\right|^2},$$
$$\sigma_E^{x,y}=\sigma_E^{x}+\sigma_E^{y}\to\min, \tag{9}$$

Where the arguments of $\Delta\dot{E}_m^{x,y}=\Delta\dot{E}^{x,y}(x_m,y_m,d,\Delta d)$ are omitted and marked $\tilde{\dot{E}}_{m\hat{e}}^{x,y}=\sum_{n=1}^{N}\frac{\dot{I}_n^{\hat{e}}}{\dot{I}_0}\dot{E}^{x,y}(x_m-x_n^{\hat{e}},y_m-\acute{o}_n^{\hat{e}},d+\Delta d-x_n^{\hat{e}})$.
Approximation of the amplitude components of electrical field variations

$$\sigma_{|E|}^{x,y}(\mathbf{x}^{\hat{e}},\mathbf{\acute{o}}^{\hat{e}},\mathbf{I}^{\hat{e}})=\sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\left(\left|\Delta\dot{E}_m^{x,y}\right|+\left|\tilde{\dot{E}}_{m\hat{e}}^{x,y}\right|\right)^2}\to\min. \tag{10}$$

Approximation of the phase components of electric field intensity variations

$$\sigma_{\varphi}^{x,y}(\mathbf{x}^{\hat{e}},\mathbf{\acute{o}}^{\hat{e}},\mathbf{I}^{\hat{e}})=\sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\left[\arg\left(\Delta\dot{E}_m^{x,y}\right)+\arg\left(\tilde{\dot{E}}_{m\hat{e}}^{x,y}\right)\right]^2}\to\min \tag{11}$$

To increase the sensitivity of the measuring device, it seems promising to use information about the phase component of the field changes in the first place, neglecting the amplitude component. This idea is the basis of the phase method of registration of isotropic and anisotropic near-surface layers [14]. The method has better noise immunity in comparison with the amplitude method most widely used in electromagnetic sounding and profiling.
Approximation of components of the electric field of heterogeneity by the field of extended additional sources
Based on the expression for a field potential of the linear electrode [15], approximate value of the potential approximated by the system of vertical linear electrodes with length L_n has the form

$$\tilde{\dot{U}}_m=\frac{\dot{\rho}}{2\pi}\sum_{n=1}^{N}\frac{\dot{I}_n}{2L_n}\ln\frac{\sqrt{(L_n-y)^2+r_{mn}^{\ 2}}+L_n-z}{\sqrt{(L_n+y)^2+r_{mn}^{\ 2}}-L_n-z} \tag{12}$$

The expression (12) can be used as a normal component of the potential (1) for the problem of minimizing an approximation of the electric field potential. The potential of a curvilinear source (for example, a cable segment) can be obtained by integrating the potential of a point source along the corresponding curve.
In practice, during the geodynamic research there is a problem associated with inability to provide a uniform spatial distribution of the points of registration of the electromagnetic field. This is due to the fact that most of the geodynamic objects are located in areas of complex development, typical for most industrial enterprises or in conditions of limited access. When solving the problems of geodynamic control, the collection system used is initially stationary and uneven. Due to the fact that the distances from the object and to the source of the probing signal are initially different, errors in the registration of the electromagnetic field at different points of observation have different dispersions. Therefore, a distinctive feature of the application of processing methods in geodynamic measurement systems, which allows to significantly reduce the error of geodynamic estimates, is the use of weight coefficients [16].

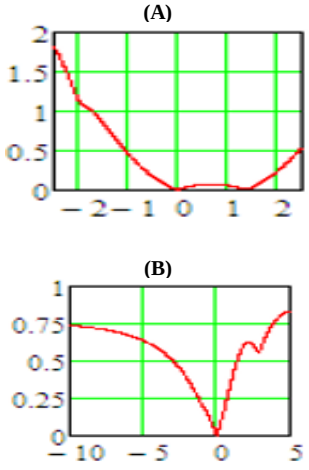
Weighting coefficients δ_{mn} are determined based on the dependence of the variance of approximation errors of the field parameter Λ from the argument, and, accordingly, are determined by the localization of the field registration point r_m relative to the source of the probing field r_n and object of study:

$$\delta_{mn}=1/L_n(\mathbf{r}_m),\ \sum_{n=1}^{N}\sum_{m=1}^{M}\delta_{m,n}=1 \tag{13}$$

Where $L_n(\mathbf{r}_m)$ is the function that determines the spatial location of the registration point.
Approximation MSE of the field parameter Λ with introduction of weight coefficients

$$\sigma_{\varphi}^{x,y}(I,\varphi)=\sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\delta_{mn}\left(\Lambda+\tilde{\Lambda}\right)^2}\to\min \tag{14}$$

Where $\delta_{mn}=1$ at uniform grid registration.
Mathematical modeling of geodynamic changes compensation by the field of additional sources
Based on the solution of minimization problem (9), an approximation of the components of variation of electric field intensity ΔE_x and ΔE_y by the field of additional compensation sources is performed. The calculation is performed for two variants of spatial placement of the receiving electrodes: 1) single receiving electrode (M=1) at the point $x_m=d/2,\ y_m=d/2$; 2) uniform grid of receiving electrodes 6x5 (M=30). In both cases, accepted: $I_0=1A$; $d=5$ м; $\Delta d=0.5$ m; $\rho_1=1$ Ohm·m; $\rho_2=10\rho_1$. For the solution of a minimization problem of the chosen nonlinear method based on the conjugate gradient (nonlinear conjugate gradient method) [17, 18] and software MathCAD parameter TOL (tolerance, Convergence Tolerance) is assumed to be TOL=10-6.
In the 1st case compensation MSE of the section boundary shift on Δd by one additional source (N=1) at optimal (in accordance with (9)) source parameters ($I^{\tilde{n}}$=18 mA, $x^{\tilde{n}}$=4.95 m, $y^{\tilde{n}}$=1.06 m) was 1.238·10-6 V/m, which is significantly less than initial approximation error (7.065·10-4 at $I_{im}^{\tilde{n}}$=0.1 A, $x_{im}^{\tilde{n}}=y_{im}^{\tilde{n}}=-5$ m). The section boundary offset compensation MSE when the parameters of the additional source are changed (offset along the abscissa axis Δx, offset along the ordinate axis on Δy, and change of currents on ΔI) is shown in figure 3. Figure 3 illustrates the good landscape of optimization problem and the apparent absence of local minima (with the exception of $\Delta x\approx$1.4 m in Fig. 3a).
MSE, mV/m



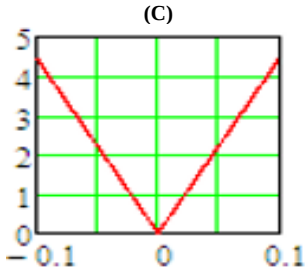
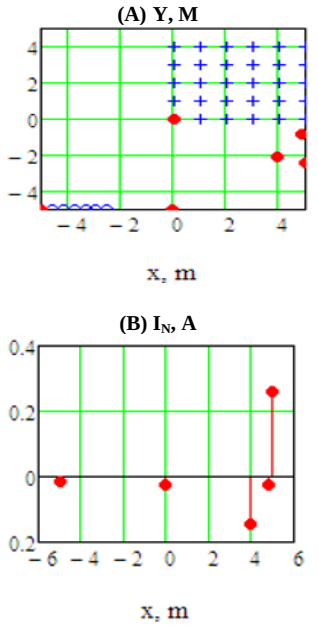


Fig. 3: Compensation MSE for the Boundary Displacement when Changing the Parameters of Additional Source (N=1, Ic=18ma, Xc=4.95 M, Yc=1.06 M) for the Field Measurements at One Point (M=1, Xm=D/2, Ym=D/2): A) Displacement Along the Abscissa Axis on Δx; B) Displacement Along the Ordinates Axis on Δy; C) Change In Currents on ΔI.

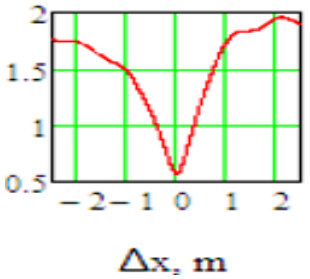
In the 2nd case (a uniform grid of receiving electrodes 6x5, M=30), the compensation MSE by one additional source (N=1) with the same initial approximation as in the case 1 is significantly higher: 1.003·10-3. It is important to increase the number of sources. Figure 4 shows the offset compensation of the section boundary by seven additional sources (N=7). The resulting compensation MSE was 5.727·10-4 V/m with an initial approximation error of $2.365 \cdot 10^{-3}$ V/m, there are also no visible local minima on the graphs of MSE dependence from the source parameters (figure 4c, 4d, 4e).

Dependence of the compensation MSE from the number of additional sources confirms that for field measurements at a single point (M=1, x_m=d/2, y_m=d/2), one source N=1 (figure 5a) is enough, for field measurements in a distributed array of points 6x5 (M=30, figure 5b), it is important to increase the number of sources.

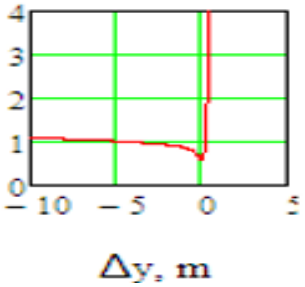


MSE, MV/m

(C)



(D)



(E)

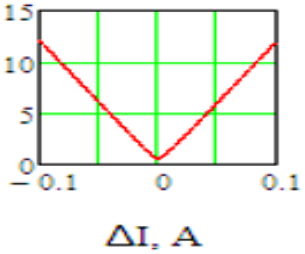


Fig. 4: Compensation of the Boundary Offset by Seven Additional Sources (N=7) for the Field Measuring in the Array of Points 6x5 (M=30): A) Location of the Receiving Electrodes (Crosses) and Emitting Electrodes (Hollow Circles Show the Initial Value, Filled Circles – End Approach); B) Final Distribution of the Currents and Abscissas Of Additional Sources (Approximation Result); C) Compensation MSE Of The Sources Displacement On Δx; D) Compensation MSE of the Sources Displacement on Δy; D) Compensation MSE when Changing the Sources Currents on ΔI.

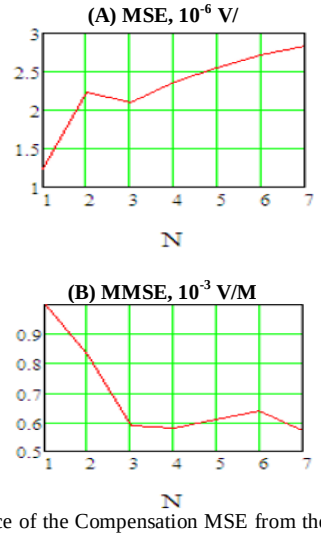


Fig. 5: Dependence of the Compensation MSE from the Number of Additional Sources N: A) for the Measurements of Field at One Point (M=1, X_m=D/2, Y_m=D/2), Only One Source N=1 Is Enough; B) for Measurements of Field in the Array of Points 6x5 (M=30).

Thus, mathematical modeling has confirmed a high accuracy of compensation of real geoelectric fields by the field of additional sources in the conditions of both narrow and wide area of the receiving electrodes location of the probing installation, which determines an efficiency of method in the conditions of complex construction.

3. Conclusion

The paper substantiates the use of compensation methods of geodynamic variations to increase the sensitivity of multipolar geoelectric control systems. Developed method involves an implementation of compensation of changes in the components of electric field in the control points caused by the natural geodynamic mismatch of the probing system from the object under research, using additional sources. Selection of source parameters is based on the solution of approximation problem of variations of the transverse and longitudinal component of the field caused by geodynamics of the controlled medium. Mathematical modeling has confirmed a high accuracy of compensation of real geoelectric fields by the field of additional sources in the conditions of both narrow and wide area of location of the receiving electrodes of the probing installation, which determines an efficiency of the method in the conditions of complex development, characteristic of modern conditions of territories formation. Further development of the method involves its mathematical modeling and experimental testing for independent or simultaneous geodynamic changes in various geoelectric parameters of the controlled medium: a rotation or slope of the plane boundary, or coefficients of the approximating planes, which are fragments of a piecewise linear boundary. In addition, it should be noted that this method can be successfully used to create an effect of presence of geodynamic variations due to additional electrodes, which simplifies the task of installing the control system in real conditions.

Acknowledgement

This work was supported by the Ministry of Education and Science of the Russian Federation under Grant № 5.3606.2017/ PCH.

References

[1] Zaborovsky A. I. Electrical Exploration. - Moscow.: Publishing House of mining, fuel and Oil literature. 1943. - P. 101-103.
[2] Kuzichkin O.R., Kamshilin A.N., Kalinkina N. E. Organization of geoelectric monitoring of karst on the basis of equipotential electric prospecting methods / / Devices and systems. Management, control, diagnostics 2007, №12. - P. 48-53.
[3] Kuzichkin O.R. Data processing algorithms in a multipolar electrolocation systems // Radiotekhnika 2007, №6. - P. 60-63.
[4] Sharapov R., Kuzichkin O. Monitoring of Karst-Suffusion Formation in Area of Nuclear Power Plant // Proceedings of the 7th 2013 IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems (IDAACS), 12-14 September 2013, Berlin, Germany. Vol. 2, 2013. P. 810-813.
[5] E. N. Volkova, A. N. Kamshilin. Geoelectric monitoring. Methods, ways and means.//Materials of the International conference "Effective and Safe Coal Mining on the basis of Modern Achievements of Geomechanics". – M.: Science, 1996. P.p. 35-39.
[6] Bykov A.A., Kuzichkin O.R. Regression prediction algoritm of suffusion processes development during geoelectric monitoring. // Advances in Environmental Biology. 2014. № 8. P. 1404-1410.
[7] Kauffman A. A. Introduction to the theory of geophysical methods. Gravitational, electric and magnetic fields. / A. A. Kauffman – M.: Subsoil, 1997. – 520 p.
[8] Dmitriev V.I. Electromagnetic fields in inhomogeneous media. - Moscow: Moscow state University Publ., 1969.
[9] Chernyak, G. Ya. Electromagnetic methods in hydrogeology and engineering geology. - M.: Nedra. 1987. - 213 p.
[10] A. Bykov, O. Kuzichkin. Approximation of equivalent transfer function of the geoelectric section in geodynamic inspection // International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management (2014) P. 615 – 622.
[11] Kurilov I.A., Kuzichkin O.R. Application of the compensation control method in geodynamic monitoring. - Radio engineering and telecommunication systems. - №3, 2013. - P. 50-58.
[12] Bogolyubov A. N., Bogolyubov N. P. Method of mapping of rocks // the patent for invention RUS 2030768
[13] Kurilov I.A., Romanov D. N. Piecewise-linear continuous approximation of characteristics // Data, information and processing: Collection of scientific articles / under the editorship of S. S. Sadykov, D. E. Andrianov. - M.: Hotline-Telecom. - 2002. – P. 175 – 180.
[14] Oleg Kuzichkin, Anastasia Grecheneva, Ekaterina Mikhaleva, Nikolay Dorofeev and Baknin Maxim. Application of phase-metric measuring system for geodynamic control of karst processes// Journal of Engineering and Applied Science. 12 (Spesial Issue 4). 2017. 6858-6863 pp.
[15] Bursian V.R. Theory of electromagnetic fields used in electrical exploration. – 2nd edition. - Leningrad.: Nedra, 1972. - 368 p.
[16] Grecheneva A., Kuzichkin O., Bykov A., Dorofeev N., Romanov R. Information and analytical support for the processing of heterogeneous data of geotechnical monitoring // International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM. Volume 17, Issue 12, 2017, Pages 453-460.
[17] Maksimov Yu. A., Filipovska E. A. Algorithms for solving the problems of nonlinear programming. - Moscow: MEPhI, 1982.
[18] Y.-H. Dai and Y. Yuan, "Nonlinear conjugate gradient method with a strong global convergence property", SIAM J. Optim.10 (1999), no. 1, 177-182.