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Research paper



Application of wavelet analysis and its interpretation in rotating machines monitoring and fault diagnosis. A review

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Abstract

Vibration analysis is a key element of predictive maintenance of rotating machines. Several signal analysis methods are used to obtain useful information from vibration signatures. This signal highlights the changes in time domain (root mean square), in the frequency spectrum (Fourier Transform) and in the time-frequency (Short Time Fourier Transform and Wavelet Transform). Currently, the most of these methods use spectral analysis based on Fourier Transform (FT). However, these methods exhibit some limitations: it is the case of non-stationary signals.
The purposes of this review were: (1) to present the theoretical framework of wavelet analysis, (2) to outline the different experimental interpretations of the use of wavelet transform for the monitoring of gear and bearing defects and (3) to highlight the procedure of the choice of a wavelet analyzing and its order for a vibratory signal.

Keywords:*Fault Diagnosis; Fourier Transform; Predictive Maintenance; Vibration Analysis; Wavelet Transforms*.

1. Introduction

With the increasing demand for high performance, safety and reliability of industrial systems, the need for early diagnosis of defects has proved to be necessary and insistent. In fact, companies are actually looking for a relevant maintenance strategy that ensures the proper functioning of the production and achieves maximum economy with efficiency and quality [1]–[3]. Conditional preventive maintenance based on vibratory analysis is moving in this direction. Thanks to its various techniques, it makes it possible to establish in-depth diagnostics on the defects of rotating machines and to avoid, subsequently, unprogrammed production stoppages [4]–[6].
There are many analysis techniques, which have been fully developed and established over the years for processing vibration signals to obtain diagnostic information about progressing faults [7]. The conventional techniques for processing measured data contain the frequency domain technique, time domain technique and time-frequency domain technique. These methods have been widely employed to detect rotating machinery failures [8].
Undoubtedly, Fourier analysis has been the dominant signal processing tool for detecting the various defects frequently encountered in rotating machines. However, there are some crucial restrictions of the Fourier Transform (FT): the signal to be analyzed must be strictly periodic or stationary; otherwise, the resulting Fourier spectrum will make little physical sense [9].
Actually, the rotating machine vibration signals are often non-stationary and represent non-linear processes and their frequency components will change with time. Therefore, methods based on the FT are not suitable for non-stationary signals and they provide only limited performance for machinery diagnostics [10]. Hence, to overcome these problems, the time-frequency methods have received growing attention and gained reliable acceptance in the field of condition monitoring [11]. Among the time-frequency analysis methods, wavelets are the most widespread tools in vibration signal analysis [12].
The early work in wavelets was in the 1980's by Morlet, Grossmann, Meyer, Mallat and others, but it was the paper by Ingrid Daubechies in 1988 that caught the attention of the larger applied mathematicians in signal processing, statistics and numerical analysis [13]. In some areas, it is the first truly new tool we have had in many years. Indeed, use of Wavelet Transforms (WT) requires a new point of view and a new method of interpreting representations that we are still learning how to exploit [14].
The fundamental idea of WT is the decomposition of the signal in approximation and in detail signals by crossing through two complementary filters: high pass and low pass [15]. This process is done by choosing suitable basis functions called wavelet mother. Due to its strong capability in time and frequency domain, WT is applied recently by many researchers in the field of mechanical faults diagnosis [16]. Indeed, wavelet analysis has been used in gear diagnosis, rolling bearing diagnosis, compressor diagnosis and diesel engine diagnosis [17].

2. Wavelet transform (WT)

Though the technique itself is not new, its application to fault diagnosis studies is. Initially mathematicians have introduced and interested in WT to characterize the different functional spaces [18]. Physicists, meanwhile, consider the WT as a modern mathe-

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matical development for signal processing in both time and frequency domain [19]. WT may also be utilized to estimate power spectrum or power spectral density of signals that are non-stationary as well as a phase spectral for non-stationary signals [20].

The FT, and most techniques derived from it, are inappropriate to treat non-stationary signals due to the absence of any temporary information. A technique that is suitable for reating non-stationary signals is Short Time Fourier Transform [21]. However, the major disadvantage for this technique is the resolution of the frequency obtained, which remains constant for the whole signal as the same window is applied. On the other hand, WT allows a high frequency resolution at low frequencies, and high time resolution at high frequencies, as desired [22].

Morlet set up the first concept of wavelets in 1984 [23]. But, the major step in the development is the introduction of the notion of multi-resolution by Mallat. To obtain a scale function, it uses a low-pass filter and to get the details, the signal part that changes the pace, it uses a high-pass filter. This method allows, as he remarks, "an interpretation of the image which does not vary with the scale" [24].

Nowadays, the WT is considered as the main signal analysis which proves very effective in different fields especially in the field of fault detection. It has been widely applied to detect faults in various components of rotating machines such as gears, bearings and electric motors.

2.1. Continuous wavelet transform (CWT)

When referring to the FT approach, it can be observed that the signal is decomposed into a set of functions which are continuous and of infinite duration. Thus, the spectrum in question corresponds to the expansion coefficients. Along the same line of reasoning, the CWT consists of decomposing the signals in time-localised waveforms usually referred to as wavelets [25]. This is a linear transformation which obtained by continuously comparing the signal with a wavelet at different positions and scale.Mathematically, the CWT is written as shown in (1):

$$CWT\ (a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \Psi^* \left(\frac{t-b}{a}\right) dt \qquad (1)$$

Wherex(t)is the signal in the time domain;

$\Psi \in L^2(R)$is the mother wavelet or wavelet function;

a) $\in R^+ - \{0\}$is scale factor, theis used to ensure energy preservation;

b) $\in R$is time factor;

The time localization of the wavelet is described by the b translation parameter, while the a dilation determines the width or scale of the wavelet. It is worth noting that, by decreasing the a scale parameter, the oscillation frequency of the wavelet increases, but the duration of the oscillation also decreases, so it can be noted that exactly the same number of cycles is contained within each wavelet [26].

Fig. 1: Shows A Signal in the Time Domain, the Corresponding CWT and the Third View of the CWT.

The scalogram is defined as the square of the CWT module. It is used for the diagnosis of defects in rotating machines. However, because of the difficulty of interpreting the scalogram, the use of CWT is still relatively rare in engineering applications for fault detection of rotating machinery [21], [26].

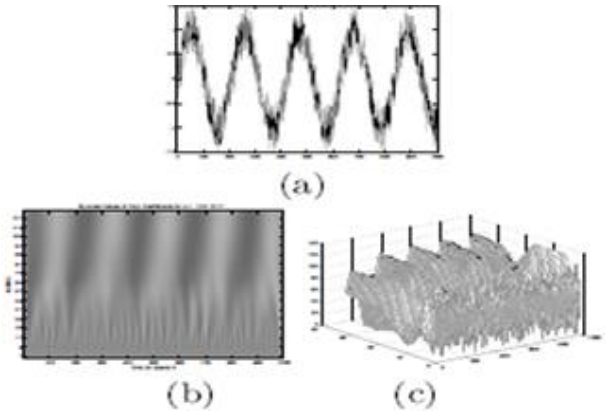


Fig. 1: A) Signal in Time Domain, B) Its CWT, C) 3d View of the CWT.

2.2. Discrete wavelet transform (DWT)

The DWT is derived from the discretization of CWT (a,b) by replacing and translation parameters by integer power of 2m/2. It is called dyadic grid [27].

Various derivations of the DWT appeared after Mallat's developments [28]. This tool consists of the processing of a discrete signal, x(i), at different frequencies and resolution levels or scales, decomposing the signals in approximation (A) and detail (D). The approximation information is obtained by means of a low pass filter, and the detail information using a high pass filter, as shown in Figure 2 [21].

Mathematically, DWT can be formulated as observed in (2):

$$DWT(j,k) = \frac{1}{\sqrt{2^j}} \int x(t)\ \Psi^* \left(\frac{t-k^{2j}}{2^j}\right) \qquad (2)$$

Here j is the scaling factor and k is the translation factor. It is obvious that the dilation factor is a power of 2.

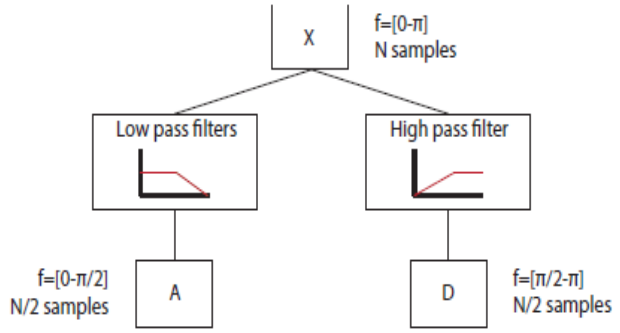


Fig. 2: DWT Decomposition of a Signal S) in Approximation Information A) and Detail Information D) Using Filter.

The different wavelet transforms in the discrete transform are Haar, Daubechies, Symlets, Coiflets, Biorthogonal, Reverse Biorthogonal and DMeyer.

2.3. Multiresolution analysis (MRA)

A very useful implementation of DWT, called multiresolution analysis, its concept was developed by Meyer and Mallat in 1986 [29]. MRA consists on the application of the DWT in a recursive way until the desired decomposition level is reached. The main disadvantage of MRA is that only approximation information could be decomposed in the different frequency bands, as shown in Figure 3.

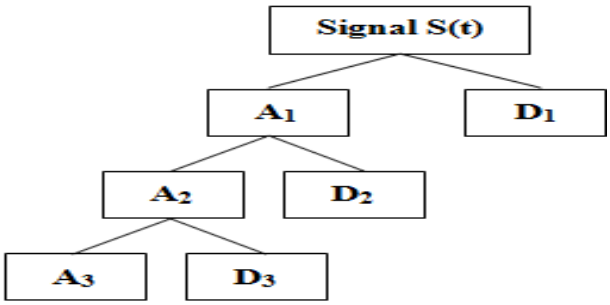


Fig. 3:MRA Decomposition Procedure until Decomposition Level 3 of Decomposition.

At any decomposition level, the signal can be expressed as the sum of approximation and detail coefficients as follows in Equation 3:

$$S=Aj+\sum DI\ (i\leq j) \quad (3)$$

Where
Aj=Approximation coefficients at jth level
Di=Detail coefficients

2.4. Multiresolution analysis (MRA)

In about 1992, Coifman, Meyer and Wickerhauser developed the Wavelet Packets Transform (WPT) to overcome the incapability of MRA to decompose high frequency bands[16]. Using WPT, all the approximation and detail information can be decomposed until the desired level [21].
This process can be observed in Figure 4.

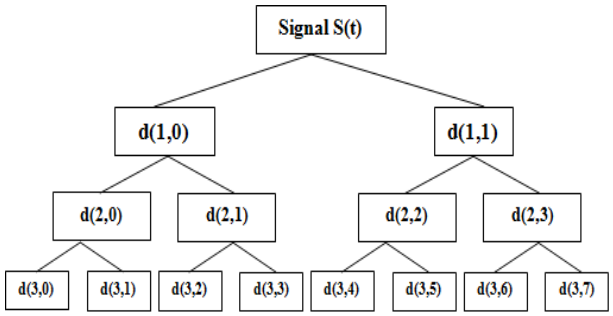


Fig. 4:WPT Decomposition Process Until Level 3 of Decomposition, Where Each Decomposition is Performed Based on DWT (See Figure 2)

The term d(k, j) represents the coefficients obtained for a packet, where k is the decomposition level (here, the number of packets obtained is 2k) and j is the position of the packet within the decomposition level. Then, each correlation vector d(k, j) has the structure of Equation 4.[30]

$$D(k,j)=\{d1(k,j),\ldots,dN(k,j)\}=di\{(k,j)\} \quad (4)$$

3. An application overview of wavelet in fault diagnosis

Recently, signal processing based on the WT has become increasingly introduced in various engineering fields: biomedical engineering, civil engineering, manufacturing engineering…
This part summarizes the application of WT in fault diagnosis of rotary machines, with emphasis on their key components such as gears and bearings.

3.1. Gear fault diagnosis

The vibration generated by the gears is complicated in its structure but gives a lot of information. Many signal processing techniques can be used for the detection and diagnosis of gear vibration defects by vibration analysis.Among these most advanced techniques we found the time-frequency approach based on the wavelet transform. In 1993, Wang et McFadden[31] published the first research about the application of the wavelet transform to the analysis of gear vibration signals. Afterwards, several studies were used in order to detect early signs of incipient mechanical failure. In fact, Wang[32] is returned, in 1996, to develop the application of the wavelet transform by describing the time-frequency and time-scale distributions to show links between these transforms in different dimensional spaces. Sung et al.[33]employed WT to detect the location of tooth defects in a gear system with high accuracy. They properly verified the effectiveness of WT in detecting and locating the fault in the gearbox in which gears rotate in very close angular speeds.
Baydar and Ball [7] used the time–frequency representation of both the vibration and acoustic signals to detect two commonly encountered local faults, tooth breakage and tooth crack. Three different analysing wavelets, Morlet, Mexican hat and the Gabor-based wavelet were initially examined for their performance in detecting fault conditions in the gear. It can appreciate that the Gabor-based wavelet generated the best performance although a good results were obtained by Morlet mother wavelet.
Zheng et al. [34] proposed a new approach of gear fault diagnosis based on continuous wavelettransform. They experimentally investigated the feature of gear fault advancement using the Time-Averaged Wavelet Spectrum (TAWS) based on CWT using Morlet wavelet function. The results of this study showed that TAWS can effectively extract gear fault information as shown in Figure 5.

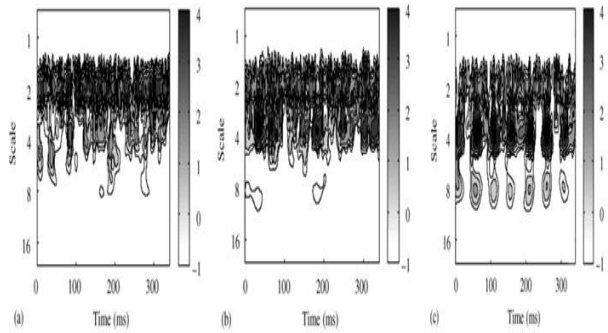


Fig. 5: Contour Plots of Wavelet Power Spectrum: (A) 0% Gear Fault Advancement; (B) 50% Gear Fault Advancement; (C) 100% Gear Fault Advancement.

Amarnath et al. [35] presented an experimental studies conducted on the gearbox include the both healthy and faulty gear with gradual removal of tooth. They used the CWT coefficients to show the severity of fault in different angular positions 120°, 180° and 240°. They conclude that the CWT technique is a promising tool for the detection of developing faults in gears. Li Li et al. [11]choosed the Haar wavelet to diagnosis normal, crack and breakage signals as shown in figure 6 . The analysis results demonstrate that Haar wavelet continuous wavelet transform (HCWT) can diagnose both the faults and the fault types of gears.

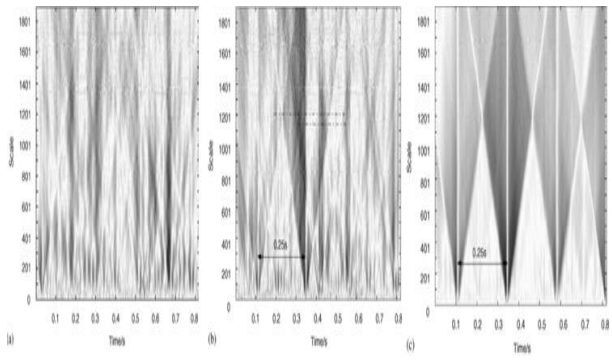


Fig. 6: HCWT of Gear Signals A) Normal, B) Tooth Crack, and C) Tooth Breakage.

Razafindrazato et al. [36] showed through a comparison of classical methods, the interest of decomposition by wavelets. It appears that the wavelet continuous transform constitutes one of the most advantageous for the detection of defects on geared motors, as shown Figure 7.

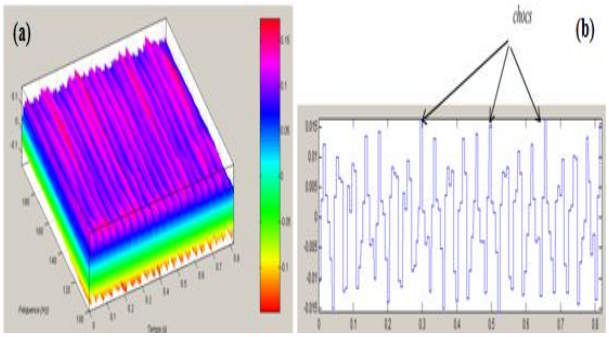


Fig. 7: A) Continuous Wavelet Decomposition, B) Discrete Wavelet Decomposition for the Measured Signal.

Boudiaf et al. [37] proposed a comparative study between wavelet transform (WT) and envelope detection (ED) for gear fault diagnosis. They compared the peaks of gear fault at the rotation frequencies of the shaft for different methods ED and WT (Morletanalyzing WT). The experimental results revealed that the WT is at least as good as the ED for the detection of gear faults. Kiran et al. [38] analysed the vibration signal processing techniques using FFT and CWT technique. This experimental study indicates that CWT (Morlet mother WT) method is effective in gear fault diagnosis, as shown in Figure 8.

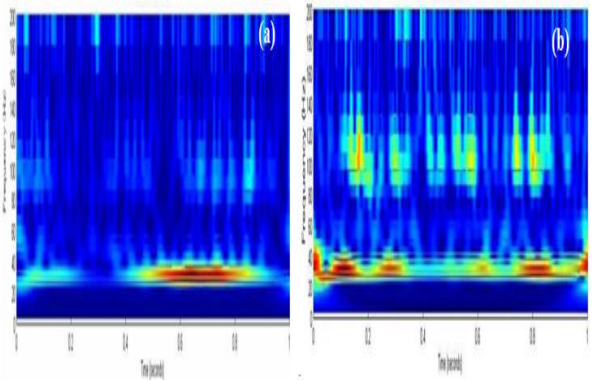


Fig. 8: Time-Frequency Plot of the CWT of (A) Healthy Gear and (B) Faulty Gear.

In Ref. [8]Saravanan and Sreenath applied discrete family wavelets such as Haar, Dmey, Daubachies, Coiflet, Biorthogonal, Reverse Biorthogonal and Symlet to the analysis of the vibration signals produced by a bevel gear box in various conditions and faults. They illustrated the powerfulness and flexibility of the discrete wavelet transform (DWT) to decompose linear and nonlinear processing of vibration signal and to classify efficiencies of each analyzing wavelet used.

3.2. Bearing fault diagnosis

For a long time, rolling element bearings were sources of concern for Maintainers. These highly reliable organs are often a cause of a major failure leading to discontinuation of production, even very serious operating incidents [39]. Several methods exist to exploit the vibration signal by highlighting the changes in the time domain, the frequency domain and the time-frequency domain. Yiakopoulos and Antoniadis [40] proposed a new effective demodulation method based on the Discrete Wavelet Transform (DWT) to diagnosis three different types of bearing faults. They experimentally found that the choice of both the specific wavelet family (Morlet) and number of the wavelet levels (typical value being three levels of approximation) give an effective demodulation method. Nikolaou and Antoniadis [41] evaluated the use of Wavelet Packet Transform (WPT) to diagnosis a localized bearing fault . They concluded that the WPT has the advantage of flexibility and efficient computational implementation compared to using filters or CWT as shown in Figures 9 and 10.

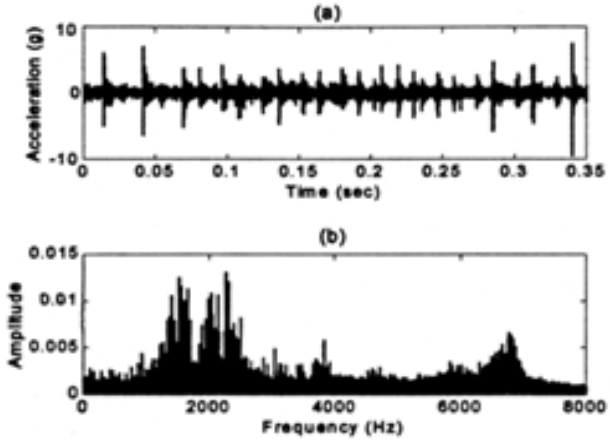


Fig. 9: A) Time Waveform and B) Spectrum of Vibration Signal Measured on A Bearing with an Inner Race Fault.

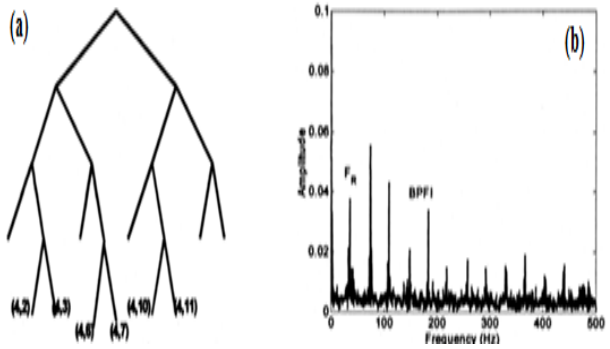


Fig. 10: A) Wavelet Packet Decomposition Tree of the Inner Race Fault Signal, B) Spectrum of the Squared Best Coefficient Vector, Obtained By Implementation of the WP Decomposition Tree of Fig 9(A).

Sun and Tang [42] made an attempt to identify the location (in time) of defect-induced bursts in the vibration signals using the singularity analysis across all scales of the continuous wavelet transform. They showed through numerical experiments, that singularity analysis is indeed quite useful for bearing defect diagnosis. The proposed method has the ability to discriminate noise from the signal significantly and is robust to bearing.Milisen and Lenaerts[43] compered different methods of detecting defects in bearings.Experimentally, they found that with the wavelet transform a very slight line can be observed around the default frequency BPFI but nothing really significant, as shown Figure 11. Therefore they concluded that the WT is poorly suited for the detection of rolling defects comparing to the envelope analysis.

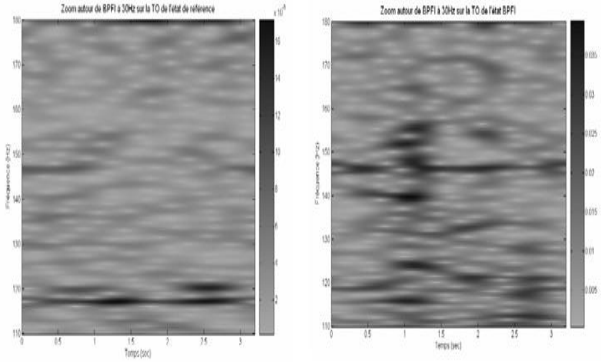
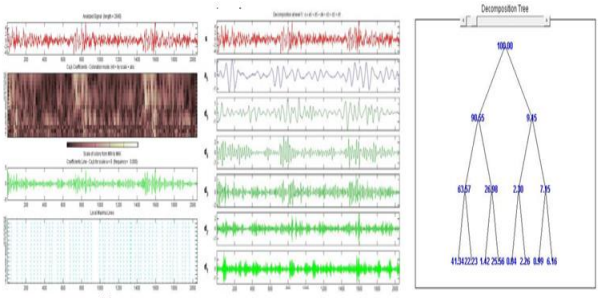


Fig. 11:WT in the Range 110Hz-180Hz, A) Healthy Bearing B) Faulty Bearing.

Gao and Yan [44] evaluated the effectiveness of WPT in detecting transient features from a time-varying signal. They applied this technique to realistic vibration signals measured from a bearing test system. The result showed that WPT has been able to identify all the transient elements embedded within both the analytically formulated test signal and the realistic bearing vibration signal. In the Ref. [45]Djebala et al. proposed discrete wavelet analysis (DWA) as an efficient tool allowing a clear detection in time and in frequency. They applied this approach to simulated signals and the results found were validated by the experiment. The results show DWA's ability to detect defects of various sizes and types and in several configurations, including in the industrial environment. Tsiafis et al. [46] employed Mexican hat CWT to detect specific faults on a roller bearing such as faults in cage, rolling balls and inner and outer ring. They found that wavelet transform offers the safe discrimination between the healthy and the faulty roller bearings using a low cost test procedure cost application. Sun et al. [47] proposed the combination of wavelet transform and envelope spectrum correlation analysis to diagnose rolling element bearing faults. Experimentally, they concluded that the proposed method is suitable for bearing systems with low Signal to noise ratio (SNR) whose characteristic fault frequency may not be easy to extract. Shakya et al. [48] presented a very exhaustive comparison of the majority of time, frequency and time-frequency domain vibration-based damage identification parameters. It was proved that the CWT time slice data are best suited for early damage identification for all the cases considered. Kulkarni and Sahasrabudhe[29] applied a methodology for fault diagnosis of rolling element bearings based DWT and WPT. The results of this study show that wavelet packet node energy coefficients are sensitive to the faults in the bearing. Himanshu and Rahul [49] explained the procedure for detecting bearing faults using FFT and by using WT more specifically HAAR wavelet up to two levels of approximations and detail components. Then, from decomposition tree they found the conclusion that as the condition of tool deteriorating the decomposition peak gives clear representation. Pricop et al. [50] presented the results of an experimental study applied on a diagnostic of bearings through three Wavelet methods: DWT, CWT and WPT with different mother wavelet (see Figure 11) . They concluded that hybrid approach is effective for detection of faults with different features found in rotating equipment.



4. Choice of the wavelet analyzing and its order in the vibration signal analysis

The choice of the wavelet (type and order) adapted to the analysis of a vibratory signal are an essential challenge. Indeed, in several researches on the application of WT, the choice of the appropriate mother wavelet is made based on the literature. There are many types of mother wavelets such as:Haar, Daubechies, Gaussian, Meyer, Mexican Hat, Morlet, Coiflet, Symlet, Biorthogonal and others. As an example, Bouzouane et al. [51] tested two types of wavelets on real cases: Daubechies and Morlet. The results obtained showed that the multi-resolution method (Daubechies) was better adapted to unbalance identification and monitoring and that the continuous transform (Morlet) was more suited to the analysis of non-stationary signals such as those generated by gears.

Moreover, the selection between the three versions of the WT: CWT, DWT and PWT depends on the nature of the signal and the defect studied [52]. Therefore, the CWT has been widely used for fault diagnostics of rotary machines, which can be seen through the several researches mentioned above. However, by utilizing these wavelet coefficients as input, the phenomena of coefficient overlapping and energy leakage are generated. So, the results generated by CWTs are difficult to be interpreted by inexperienced operators [53]. Hence, to overcome the deficiencies making the CWT still limited to be used in vibration based machine fault diagnosis, various hybrid approaches have been developed. For example: the combination of CWT and Envelope Analysis [54], the integration of CWT and the correlation filtering [55], the CWT together with autocorrelation enhancement [56] and so on.

Compared with the CWT, the DWT, thanks to its multi-resolution analysis ability, proved more suitable for detecting fault from non-stationary signals sampled on rotary machines. In addition, the DWT method has been widely adopted in many studies to remove strong background noise and enhance fault-related information contained in measured signals [57]–[59]. Furthermore, DWT coefficients allow extracting of a good set of fault-related features which helps to identify machine defects in a much effective way [57], [60], [61].Also, the DWT can be used as a hybrid approach by integrating it with other techniques. As an example, Wu and Hsu [62] proposed system consisted of a combination of signal feature extraction using DWT technique and fault identification using fuzzy–logic inference.

The performance of WPT in high frequency region makes this technique an attractive tool for detecting and differentiating transient components with high frequency characteristics. So, many research works, which based on the comparison of WPT technique and other time-frequency techniques, has concluded that the WPT was able to identify defect induced transient components embedded within the vibration signal [63]. Identical to the DWT, wavelet coefficients of the WPT method are widely used to extract useful information to characterize machine defects [64], [65]. The WPT technique can therefore represent a signal in many different which allowing an optimal decomposition for an improved diagnosis of machine defects[66].

Finally, similar to the CWT and the DWT, features extracted from the combination of the WPT and other methods give the excellent results [67], [68].

On the other hand, the original signal using WT can be decomposed into approximations and details versions with different frequency bands by using a successive low-pass and high-pass filtering. The decomposed levels will not change their information in the time domain [69]. However, useful information can be contained in some sub-bands. So, the fault can be detected from a given level of resolution [70]. Thus, the appropriate number of levels of decomposition (nLS) depends on the sampling frequency (fs) of the analyzed signal. For each of the diagnostic approaches based on wavelet decomposition, the level number must be chosen judiciously in order to allow the high level signals (approximation and detail) to cover the entire range of frequencies along which the component due to the defects changes during all operating regimes.

Therefore, the minimum number of decomposition levels required can be calculated by satisfying the following condition as (5)[71].

$$2^{-(nLS+1)}\, fs<f \tag{5}$$

Indeed, the level of decomposition of the approximation signal which includes the harmonics around the fundamental is the integer (nLs) expressed by the (6):

$$NLS = INT\left(\frac{\log(\frac{fs}{f})}{\log(2)}\right) \tag{6}$$

With "int" for integers, fs: sampling frequency, f: main frequency.

Clearly, the use of these equations gives a theoretical value of the level of decomposition of a signal. But practically, it is necessary

to seek the optimal level of decomposition allowing the extract of useful information with avoiding especially the relatively long and complicated calculation processes [72].

5. Conclusion

In conclusion, this paper reviews the use of WT for rotating machines monitoring and fault diagnosis. The revision of the literature has briefly covered the theoretical framework of wavelet analysis, its multiple ways of application in both gears and bearings defect, the choice of the wavelet analyzing and its order, then the presentation of important results. Through this study, it can be conclude that the WT is a powerful tool used in fault diagnostics. But some difficulties occurred which made the applications of the wavelets have still not achieved a wide status comparing to the FT. Indeed, the main obstacle for the popularization of the WT is the absence of a standard or general method to select and interpret the wavelet function for different cases.

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References

[1] K. Vernekar, "Fault Detection of Gear Using Spectrum and Cepstrum Analysis," Proc. Indian Natl. Sci. Acad., vol. 81, no. 4, pp. 1177–1182, 2015.https://doi.org/10.16943/ptinsa/2015/v81i4/48270.

[2] F. Khathyri, B. Elkihel, A. al motalibBerrehili, F. Delaunois, and I. Manssouri, "Detection and localization of defects in composite material using the non-destructive testing methods: Ultrasonic and Infrared Thermography," Int. J. Emerg. Trends Eng. Dev. Issue, vol. 7, no. 2, pp. 36–45, 2017.

[3] R. K. Mobley and R. K. Mobley, "2 – Financial Implications and Cost Justification," in an Introduction to Predictive Maintenance, 2002, pp. 23–42.

[4] D. Dyer, D. Burch, R. Cheeser, C. Bethea, L. Bowman, and R. Moshage, "Vibration Monitoring for Predictive Maintenance in Central Energy Plants," no. September, pp. 1–70, 1993.

[5] R. K. Mobley and R. K. Mobley, "7 – Vibration Monitoring and Analysis," in an Introduction to Predictive Maintenance, 2002, pp. 114–171.

[6] R. K. Mobley and R. K. Mobley, "3 – Role of Maintenance Organization," in an Introduction to Predictive Maintenance, 2002, pp. 43–59.

[7] N. BAYDAR and A. BALL, "Detection of Gear Failures Via Vibration and Acoustic Signals Using Wavelet Transform," Mech. Syst. Signal Process., vol. 17, no. 4, pp. 787–804, 2003.https://doi.org/10.1006/mssp.2001.1435.

[8] S. Natarajan, "WAVELET ANALYSIS in FAULT DIAGNOSIS of SPUR," vol. 2, no. 7, pp. 8–19, 2015.

[9] S. Mallat and W. L. Hwang, "Singularity detection and processing with wavelets," IEEE Trans. Inf. Theory, vol. 38, no. 2, pp. 617–643, 1992.https://doi.org/10.1109/18.119727.

[10] H. Bendjama, S. Bouhouche, and M. S. Boucherit, "Application of Wavelet Transform for Fault Diagnosis in Rotating Machinery," Int. J. Mach. Learn. Comput, vol. 2, no. 1, pp. 82–87, 2012.https://doi.org/10.7763/IJMLC.2012.V2.93.

[11] L. Li, L. Qu, and X. Liao, "Haar wavelet for machine fault diagnosis," Mech. Syst. Signal Process, vol. 21, no. 4, pp. 1773–1786, 2007.https://doi.org/10.1016/j.ymssp.2006.07.006.

[12] V. DhanushAbhijit, V. Sugumaran, and K. I. Ramachandran, "Fault Diagnosis of Bearings using Vibration Signals and Wavelets," Indian J. Sci. Technol., vol. 9, no. 33, 2016.https://doi.org/10.17485/ijst/2016/v9i33/101325.

[13] I. Daubechies, "mallt," Commun. Pure Appl. Math., vol. 41, no. 7, pp. 909–996, Oct. 1988.https://doi.org/10.1002/cpa.3160410705.

[14] Z. K. Peng and F. L. Chu, "Application of the wavelet transform in machine condition monitoring and fault diagnostics: a review with bibliography," Mech. Syst. Signal Process., vol. 18, no. 2, pp. 199–221, Mar. 2004.https://doi.org/10.1016/S0888-3270(03)00075-X.

[15] C.-L. Liu, "A Tutorial of the Wavelet Transform Chapter 1 Overview." 2010.

[16] Y. Qin, Y. Mao, and B. Tang, "Vibration signal component separation by iteratively using basis pursuit and its application in mechanical fault detection," J. Sound Vib, vol. 332, no. 20, pp. 5217–5235, 2013.https://doi.org/10.1016/j.jsv.2013.04.021.

[17] H. ZHENG, Z. LI, and X. CHEN, "GEAR FAULT DIAGNOSIS BASED ON CONTINUOUS WAVELET TRANSFORM," Mech. Syst. Signal Process., vol. 16, no. 2–3, pp. 447–457, Mar. 2002.

[18] R. Moshage et al., "Vibration Monitoring for Predictive Maintenance in Central Energy Plants." 1993.

[19] A. Doncescu, "Ondelettes: Théorie ET Applications."

[20] X. Jiang and S. Mahadevan, "Wavelet spectrum analysis approach to model validation of dynamic systems," Mech. Syst. Signal Process, vol. 25, no. 2, pp. 575–590, Feb. 2011.https://doi.org/10.1016/j.ymssp.2010.05.012.

[21] M. Gómez, C. Castejón, and J. García-Prada, "Review of Recent Advances in the Application of the Wavelet Transform to Diagnose Cracked Rotors," Algorithms, vol. 9, no. 1, p. 19, Mar. 2016.https://doi.org/10.3390/a9010019.

[22] S. Delvecchio Advances in Wavelet Theory and Their Applications in Engineering, Physics and Technology. 2012.

[23] Henrique S.MALVAR, "Lapped Transforms for Efficient Transform/Subband Coding," IEEE Trans. Acoust. Signal Process. vol. 38, pp. 969–978, 1990.https://doi.org/10.1109/29.56057.

[24] S. Mallat, a Wavelet Tour of Signal Processing. 2008.

[25] J. LIN and M. J. ZUO, "GEARBOX FAULT DIAGNOSIS USING ADAPTIVE WAVELET FILTER," Mech. Syst. Signal Process., vol. 17, no. 6, pp. 1259–1269, Nov. 2003.https://doi.org/10.1006/mssp.2002.1507.

[26] L. M. Hee, M. S. Leong, and K. H. Hui, "Analysis of Residual Wavelet Scalogram for Machinery Fault Diagnosis," Adv. Mater. Res., vol. 845, pp. 113–117, Dec. 2013.

[27] J. Olkkonen, DISCRETE WAVELET TRANSFORMS - THEORY AND APPLICATIONS. 2011.

[28] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PATTERN Anal. Mach. Intell. VOL. II, vol. 11, no. 7, pp. 674–693, 1989.

[29] P. G. Kulkarni and A. D. Sahasrabudhe, "Application Of Wavelet Transform For Fault Diagnosisof Rolling Element Bearings," Int. J. Sci. Technol. reseach, vol. 2, no. 4, pp. 138–148, 2013.

[30] A. Raad, S. Kass, and J. Antoni, "CFA / VISHNO 2016 Etude de la décompositionen mode empiriquecombinéedans le cas des signaux de vibrations de roulements et d' engrenagesdansdiverses conditions," pp. 1839–1844, 2016.

[31] W. Wang and P. McFadden, "Application of the wavelet transform to gearbox vibration analysis," in Structural dynamics and vibrations - 16th Annual energy-sources technology conference, Structural dynamics and vibrations -1993-, 1993, pp. 15–20.

[32] W. J. Wang, "Wavelet transform in vibration analysis for mechanical fault diagnosis," 1996.

[33] C. K. Sung, H. M. Tai, and C. W. Chen, "Locating defects of a gear system by the technique of wavelet transform," Mech. Mach. Theory, vol. 35, no. 8, pp. 1169–1182, 2000.https://doi.org/10.1016/S0094-114X(99)00045-2.

[34] H. Zheng, Z. Li, and X. Chen, "GEAR FAULT DIAGNOSIS BASED ON CONTINUOUS WAVELET TRANSFORM," 2002.

[35] S. S. C. S. M.Amarnath, "Application of Wavelet transform to gearbox fault diagnosis.pdf." 2005.

[36] G. Marie, R. Razafindrazato, O. Riou, and J. F. Durastanti, "Détection de défaut sur motoréducteur à engrenageenutilisant la transforméeenondelettes," Rev. Sci. Maint., 2012.

[37] Z. Mentouri, S. Ziani, and S. Taleb, "Wavelet Transform and Envelope Detection for Gear Fault Diagnosis. A Comparative Study," Electr. Eng., no. 2, 2013.

[38] K. Vernekar, H. Kumar, and K. V. Gangadharan, "Gear Fault Detection Using Vibration Analysis and Continuous Wavelet Transform," Procedia Mater. Sci., vol. 5, pp. 1846–1852, 2014.https://doi.org/10.1016/j.mspro.2014.07.492.

[39] B.Abouelanouar, "A comparative experimental study of different methods in detection and monitoring bearing defects," Int. J. Adv. Sci. Tech. Res., vol. 1, no. 7, pp. 409–423, 2017.

[40] C. T. Yiakopoulos and I. A. Antoniadis, "Wavelet based demodulation of vibration signals generated by defects in rolling element bearings," Shock Vib., vol. 9, no. 6, pp. 293–306, 2001.https://doi.org/10.1155/2002/592436.

[41] N. G. Nikolaou and I. A. Antoniadis, "Rolling element bearing fault diagnosis using wavelet packets," NDT E Int., vol. 35, no. 3, pp. 197–205, 2001.https://doi.org/10.1016/S0963-8695(01)00044-5.
[42] Y. Sun Q.; Tan, "Singularityanalysis Using Continuous Wavelet Transform for Bearing Fault Diagnosis," Mech. Syst. Signal Process., vol. 16, pp. 1025–1041, 2002.https://doi.org/10.1006/mssp.2002.1474.
[43] X. Lou and K. A. Loparo, "Bearing fault diagnosis based on wavelet transform and fuzzy inference," Mech. Syst. Signal Process. vol. 18, no. 5, pp. 1077–1095, 2004.https://doi.org/10.1016/S0888-3270(03)00077-3.
[44] R. X. Gao and R. Yan, "Non-stationary signal processing for bearing health monitoring," Int. J. Manuf. Res., vol. 1, no. 1, p. 18, 2006.https://doi.org/10.1504/IJMR.2006.010701.
[45] A. Djebala, N. Ouelaa, and N. H. S. Guenfoud, "Application de la TransforméeenOndelettesDiscrètedans la Détection des Défauts de Roulements," 2009.
[46] I. Tsiafis, K.-D. Bouzakis, A. Kaplanis, A. Karamanidis, and T. Xenos, "Fault Diagnosis of Roller Bearings Using the Wavelet Transform," Rom. Rev. Precis. Mech. Opt. Mechatronics, no. 39, pp. 21–24, 2011.
[47] Q. Liu, F. Chen, Z. Zhou, and Q. Wei, "Fault diagnosis of rolling bearing based on wavelet package transform and ensemble empirical mode decomposition," Adv. Mech. Eng., vol. 2013, 2012.
[48] P. Shakya,a K. Darpe, and M. S. Kulkarni, "Vibration-based fault diagnosis in rolling element bearings : ranking of various time , frequency and time-frequency domain data-based damage identification parameters," Int. J. Cond. Monit., vol. 3, no. 2, pp. 1–10, 2013.
[49] R. Himanshu, "Importance of Vibration Parameters in Fault Diagnosis and Condition Monitoring of Bearing and Analysis of Wavelets," Int. J. Adv. Eng. Reseach Sci., vol. 3, no. 6, pp. 65–72, 2016.
[50] M. PRICOP, "Vibration Analysis for Detection and Localization the Faults of Rotating Machinery Using Wavelet Techiniques," Sci. Bull. Nav. Acad., vol. 19, no. 1, pp. 264–272, 2016.
[51] B.Bouzouane, A.Miloudi, N. Hamzaoui, and A. Benchaala, "Détection de défauts de machines tournantes per la méthode des ondelettes," in CongrèsFrançais de Mécanique, At Nice, France, 2003.
[52] L. H. Cherif, S. M. Debbal, and F. Bereksi-Reguig, "Choice of the wavelet analyzing in the phonocardiogram signal analysis using the discrete and the packet wavelet transform," Expert Syst. Appl., vol. 37, no. 2, pp. 913–918, Mar. 2010.https://doi.org/10.1016/j.eswa.2009.09.036.
[53] P. W. Tse and W. X. Yang, "The Practical Use of Wavelet Transforms and Their Limitations in Machine Fault Diagnosis ," Proc. Int. Symp. Mach. Cond. Monit. Diagnosis JSME Annu. Meet vol. 2002, pp. 9–16, and 2002.
[54] R. Yan and R. X. GAO, "Multi-scale enveloping spectrogram for vibration analysis in bearing defect diagnosis," Tribol. Int., vol. 42, no. 2, pp. 293–302, Feb. 2009.https://doi.org/10.1016/j.triboint.2008.06.013.
[55] S. Wang, Z. Zhu, Y. He, and W. Huang, "Adaptive Parameter Identification Based on Morlet Wavelet and Application in Gearbox Fault Feature Detection," EURASIP J. Adv. Signal Process., vol. 2010, no. 1, p. 842879, 2010.https://doi.org/10.1155/2010/842879.
[56] G. Y. Luo, D. Osypiw, and M. Irle, "On-Line Vibration Analysis with Fast Continuous Wavelet Algorithm for Condition Monitoring of Bearing," Modal Anal., vol. 9, no. 8, pp. 931–947, Aug. 2003.https://doi.org/10.1177/10775463030098002.
[57] R. Yan, R. X. GAO, and X. Chen, "Wavelets for fault diagnosis of rotary machines: A review with applications," Signal Processing, vol. 96, pp. 1–15, Mar. 2014.https://doi.org/10.1016/j.sigpro.2013.04.015.
[58] D. Wenliao, Y. Jin, L. Yanming, and L. Chengliang, "Adaptive wavelet filtering for bearing monitoring based on block bootstrapping and white noise test," Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci., vol. 226, no. 9, pp. 2345–2360, Sep. 2012.https://doi.org/10.1177/0954406211430780.
[59] Z. Li, X. Yan, C. Yuan, Z. Peng, and L. Li, "Virtual prototype and experimental research on gear multi-fault diagnosis using wavelet-autoregressive model and principal component analysis method," Mech. Syst. Signal Process., vol. 25, no. 7, pp. 2589–2607, Oct. 2011.https://doi.org/10.1016/j.ymssp.2011.02.017.
[60] P. Li, F. Kong, Q. He, and Y. Liu, "Multiscale slope feature extraction for rotating machinery fault diagnosis using wavelet analysis," Measurement, vol. 46, no. 1, pp. 497–505, Jan. 2013.https://doi.org/10.1016/j.measurement.2012.08.007.
[61] D. Wang, Q. Miao, and R. Kang, "Robust health evaluation of gearbox subject to tooth failure with wavelet decomposition," J. Sound Vib, vol. 324, no. 3–5, pp. 1141–1157, Jul. 2009.https://doi.org/10.1016/j.jsv.2009.02.013.
[62] J.-D. Wu and C.-C. Hsu, "Fault gear identification using vibration signal with discrete wavelet transform technique and fuzzy–logic inference," Expert Syst. Appl., vol. 36, no. 2, pp. 3785–3794, Mar. 2009.https://doi.org/10.1016/j.eswa.2008.02.026.
[63] R. X. Gao and R. Yan, "Non-stationary signal processing for bearing health monitoring," Int. J. Manuf. Res., vol. 1, no. 1, p. 18, 2006.https://doi.org/10.1504/IJMR.2006.010701.
[64] P. Boškoski and Đ. Juričić, "Fault detection of mechanical drives under variable operating conditions based on wavelet packet Rényi entropy signatures," Mech. Syst. Signal Process., vol. 31, pp. 369–381, Aug. 2012.https://doi.org/10.1016/j.ymssp.2012.04.016.
[65] Y. Feng and F. S. Schlindwein, "Normalized wavelet packets quantifiers for condition monitoring," Mech. Syst. Signal Process., vol. 23, no. 3, pp. 712–723, Apr. 2009.https://doi.org/10.1016/j.ymssp.2008.07.002.
[66] B. Liu, "Selection of wavelet packet basis for rotating machinery fault diagnosis," J. Sound Vib., vol. 284, no. 3–5, pp. 567–582, Jun. 2005.https://doi.org/10.1016/j.jsv.2004.06.047.
[67] Y. Yang, Y. He, J. Cheng, and D. Yu, "A gear fault diagnosis using Hilbert spectrum based on MODWPT and a comparison with EMD approach," Measurement, vol. 42, no. 4, pp. 542–551, May 2009.https://doi.org/10.1016/j.measurement.2008.09.011.
[68] G.-M. Xian and B.-Q. Zeng, "An intelligent fault diagnosis method based on wavelet packer analysis and hybrid support vector machines," Expert Syst. Appl., vol. 36, no. 10, pp. 12131–12136, Dec. 2009.https://doi.org/10.1016/j.eswa.2009.03.063.
[69] Z.-L. Gaing, "Wavelet-Based Neural Network for Power Disturbance Recognition and Classification," IEEE Trans. Power Deliv, vol. 19, no. 4, pp. 1560–1568, Oct. 2004.https://doi.org/10.1109/TPWRD.2004.835281.
[70] H. Bendjama, D. Idiou, K. Gherfi, and Y. Laib, "Selection of Wavelet Decomposition Levels for Vibration Monitoring of Rotating Machinery," ADVCOMP 2015 Ninth Int. Conf. Adv. Eng. Comput. Appl. Sci., no. c, pp. 96–100, 2015.
[71] T. Ishak, "Extraction d'indicateursrobustes pour le diagnostic des défautsmécaniques : Comparaison de L'EMD ET des ondelettes (WT)," UNIVERSITE FERHAT ABBAS - SETIF UFAS (ALGERIE), 2012.
[72] A. M. Gaouda, M. M. A. Salama, M. R. Sultan, and A. Y. Chikhani, "Power quality detection and classification using wavelet-multiresolution signal decomposition," IEEE Trans. Power Deliv., vol. 14, no. 4, pp. 1469–1476, 1999.https://doi.org/10.1109/61.796242.