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Research paper

An improved method for the harmonic contributions assessment of utility and customer in distribution systems: part A analytical study

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Abstract

A new method is proposed to find suitable definitions of electrical power components in non-sinusoidal conditions, which based on the analysis of three-phase instantaneous power flows of both fundamental and all harmonics of signals in three-phase non-sinusoidal system. This paper also introduces an attempt to get the physical essence of the proposed three-phase power components for any non-sinusoidal unbalanced three-phase system. Therefore, we can use the formulas of these definitions with modern digital measurement technology in order to reach to revenue meter which enables us to identify the responsible for the harmonic distortion between customers and distribution utilities. The aim of this paper is presenting an analytical study that will rely on it in the next paper part B, in order to assess the contributions of harmonic distortion for utility and the customer at the Point of Common Coupling (PCC).

Keywords: *Harmonic Distortion Responsibility; Harmonic Pricing; Power Definitions; Power Quality; Revenue Meter.*

1. Introduction

1.1. Background

Industrial process is heavily automated, to ameliorate its manufacturing efficiency and performance; consequently the industrial customers introduce highly nonlinear loads to the distribution systems, due to the excessive use of power electronic equipment. This can introduce a large amount of harmonic distortion throughout the system.
We should admit that, we are much more effective in producing harmonics than in their mitigation [1]. This situation should not be propagated; also it has already become an alarming situation in an engineering institute, where power quality is an essential requirement. To ensure quality power, it is necessary to carry out an analytical study for defining power components to identify the details of the quantities they pollute in each instance. The following cases may necessitate performing an analytical study [2]:

- Harmonic pricing
- Determining the quality of electric service,
- Detection of the harmonic distortion sources,
- Harmonic mitigation techniques.

1.2. Previous work

The conventional definition for power components and calculation methods became debatable under non-sinusoidal conditions; consequently power component definitions were a field of meaningful research which created many theories.
Budeanu [3] presented the first attempt in 1927, to find a solution for power definition under non-sinusoidal conditions. This definition apparent power S, distortion power D, active power P, and reactive power Q. Fryze [4] presented the second attempt in 1932, where the current sources is spilt to active components, and a non-active components.
Shepherd and Zakihani [5]contended that the conventional definition of Budeanu [3] for reactive power was based on a fallacy, and presented their definitions. Sharon 's definition[6] contended that the conventional definition of Budeanu [3] for active power may result in discontinuity in the reactive power compensation by linear devices.
Emanuel [7] considered that only two components were needed in the apparent power from a physical viewpoint : they were active power, and complementary power.
Kuster [8] proposed an innovative definition for power component calculation. They divided the current of a power circuit into active current ip, inductive current iql and residual inductive current iqlr.
Czarnecki [9] developed the notion of perpendicular decomposition of the current source, to decompose the voltage sources into generated , scattered, active, and reactive components to the three phase a symmetrical circuit [10]. Also he inserts unbalanced current components $i_u$on these components.
Most of authors follow the ideas formulated by Fryze [4] , which decompose the load currents and power signal into different divisions like: active and reactive [11]; orthogonal active and residual [12]; instantaneous active and instantaneous reactive [13]; active and inactive [14]; distortion less and distortion [15]; real and imaginary [16]; fundamental and non-fundamental [17]; active, reactive, distortion, non-active and apparent power [2].
Reference [18] used Park vector as a method to define active and nonactive power component in non-sinusoidal three-phase system.
Reference [19], presented an attempt to achieve the decomposition of total instantaneous power for the single-phase system.

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In Reference [20], the formulation of instantaneous reactive power theory is developed. Also the authors proposed the non-fundamental effective apparent power as new definition, which rely on the flow of instantaneous power [21].
In Reference [22], The power components defined in the IEEE Standard 1459-2010 [23] are revisit. As a result, new definitions for study and assessment of various power quantities like as non-fundamental power, distortion power, and harmonic apparent power are introduced.
All these fertile efforts are criticized among each other, [17], and [24-28]. Generally, there are some conflicts because of one or more of these reasons:

- extremely decomposition,
- shortage of physical essence,
- inadequate summation,
- sophisticated measurement,

Finally, this Survey summarizes formerly published methods and deduces that: despite the often repeated claims that existing definitions are capable to resolve all problems in every application, debate about these definitions is likely to continue for years to come. The only real consensus that has emerged is that the present definitions are not sufficient for economic studies under non-sinusoidal poly-phase systems [29].
Therefore, none of the above methods can produce a definition of power components, which satisfy all desired power properties under non-sinusoidal conditions

1.3. Motivation

Economic benefits from using power electronic equipment are much more visible than losses and bad consequences caused by harmonics produced by this equipment. As a result, the sources of harmonic distortion become increasingly distributed over distribution system.
Defining of non-sinusoidal power components is still one of the most controversial subjects. In this paper, a new method which is rely on the analytical study of three-phase instantaneous power flows of both fundamental and harmonic signals is presented. It is attempting to get an obvious physical interpretation, and to subedit this for each power component in three-phase non-sinusoidal unbalanced system.
The study of harmonic has become a significant component in the analysis and design of the distribution system. Indeed, the analysis of harmonic has been vastly used for:

- Planning of system ,
- Development of operating criteria,
- Designing of equipment,
- Troubleshooting,
- Realization standard compliance,
- Responsibility of harmonic distortion, etc.

The authors of this paper were interested in the trend of harmonic distortion responsibility, and they provided many research papers to determine responsibility for the harmonic distortion in single phase and balanced systems [30-34]. In fact, the balanced three-phase systems are uncommon; the same authors induced the proposed method to produce an analytical study of power components that can determine the responsibility of harmonic distortion between the utility and the customer at the Point of Common Coupling (PCC) in unbalanced systems.
By using this analysis, the voltages and currents measured at the Point of Common Coupling (PCC) are sufficient for determining the magnitude and direction of power for each harmonic order. Therefore the developed method can be performed in any power quality Analyzer, which observes the harmonic voltages and currents (amplitudes and phase angles) together.

2. Analytical study of power components under sinusoidal condition

This Analytical study is based on an instantaneous power analysis for the calculation of power components in three-phase four-wire unbalanced system under sinusoidal condition.

2.1. Instantaneous representation

The line-to-neutral instantaneous voltages at time instant (t) are as follows:

$$\mathrm{v_a(t)} = \sqrt{2}\,\mathrm{V_a}\sin(\mathrm{wt} + \alpha_a) \quad (1)$$
$$\mathrm{v_b(t)} = \sqrt{2}\,\mathrm{V_b}\sin(\mathrm{wt} + \alpha_b - (2\pi/3)) \quad (2)$$
$$\mathrm{v_c(t)} = \sqrt{2}\,\mathrm{V_c}\sin(\mathrm{wt} + \alpha_c + (2\pi/3)) \quad (3)$$

The line instantaneous currents at time instant (t) are as follows:

$$i_a(t) = \sqrt{2}\,I_a\,sin(wt + \beta_a) \quad (4)$$
$$i_b(t) = \sqrt{2}\,I_b\,sin(wt + \beta_b - (2\pi/3)) \quad (5)$$
$$i_c(t) = \sqrt{2}\,I_c\,sin(wt + \beta_c + (2\pi/3)) \quad (6)$$

Where the angular frequency(w), voltage phase angles(α), current phase angles(β). Using (1), and (4), the instantaneous power per phase (a) is defined as:

$$u_a(t) = v_a(t)i_a(t) = 2V_aI_a\,sin(wt + \alpha_a)\,sin(wt + \beta_a)$$
$$= V_aI_a\big[cos(\alpha_a - \beta_a) - cos(2wt + \alpha_a + \beta_a)\big]$$
$$V_aI_a\big[cos(\alpha_a - \beta_a) - sin(2wt + \alpha_a + \beta_a - \pi/2)\big] \quad (7)$$

$$u_a(t) = \quad (8)$$

$$V_aI_a\,cos(\alpha_a - \beta_a) + V_aI_a\,cos(\alpha_a - \beta_a)\,sin(2wt + 2\beta_a - \pi/2) + V_aI_a\,sin(\alpha_a - \beta_a)\,sin(2wt + 2\beta_a)$$
(9)

$$u_a = P_a + p_a(t) + q_a(t) \quad (10)$$

$$u_a = P_a + s_a(t) \quad (11)$$

Where:

- $s_a(t)$: the instantaneous phasor power for phase (a)
- $p_a(t)$: the instantaneous active power for phase (a)
- $q_a(t)$: the instantaneous reactive power for phase (a)
- P_a : the average or active power for phase (a)
- $P_a = V_aI_a\,cos(\alpha_a - \beta_a)$, the amplitude of instantaneous active power for phase (a)
- $Q_a = V_aI_a\,sin(\alpha_a - \beta_a)$, the amplitude of instantaneous reactive power for phase (a)

As the same, the instantaneous power per phase (b) is defined as:

$$u_b(t) = v_b(t)i_b(t) \quad (12)$$

$$u_b(t) = V_bI_b\,cos(\alpha_b - \beta_b) + V_bI_b\,cos(\alpha_b - \beta_b)\,sin(2wt + 2\beta_b - (11\pi/6)) + V_bI_b\,sin(\alpha_b - \beta_b)\,sin(2wt + 2\beta_b - (4\pi/3)) \quad (13)$$

$$u_b = P_b + p_b(t) + q_b(t) = P_b + s_b(t) \quad (14)$$

Also, the instantaneous power per phase (c) is defined as;

$$u_c(t) = v_c(t)i_c(t) \quad (15)$$

$$u_c(t) = V_cI_c\,cos(\alpha_c - \beta_c) + V_cI_c\,cos(\alpha_c - \beta_c)\,sin(2wt + 2\beta_c + (5\pi/6)) + V_cI_c\,sin(\alpha_c - \beta_c)\,sin(2wt + 2\beta_c + (4\pi/3)) \quad (16)$$

$$u_c(t) = P_c + p_c(t) + q_c(t) = P_c + s_c(t) \quad (17)$$

The total instantaneous power $u(t)$ is defined as:

$$u(t) = u_a(t) + u_b(t) + u_c(t) \quad (18)$$

$$u(t) = V_a I_a\left[cos(\alpha_a - \beta_a) - sin(2wt + \alpha_a + \beta_a - \pi/2)\right] + V_b I_b\left[cos(\alpha_b - \beta_b) - sin(2wt + \alpha_b + \beta_b - (11\pi/6))\right] + V_c I_c[cos(\alpha_c - \beta_c) - sin(2wt + \alpha_c + \beta_c + (5\pi/6))] \quad (19)$$

$$u(t) = \begin{bmatrix} V_a I_a cos(\alpha_a - \beta_a) + \\ V_b I_b cos(\alpha_a - \beta_a) + \\ V_c I_c cos(\alpha_a - \beta_a) \end{bmatrix} + \begin{bmatrix} V_a I_a cos(\alpha_a - \beta_a) sin(2wt + 2\beta_a - \pi/2) + \\ V_b I_b cos(\alpha_b - \beta_b) sin(2wt + 2\beta_b - (11\pi/6)) + \\ V_c I_c cos(\alpha_c - \beta_c) sin(2wt + 2\beta_c + (5\pi/6)) \end{bmatrix} + \begin{bmatrix} V_a I_a sin(\alpha_a - \beta_a) sin(2wt + 2\beta_a) + \\ V_b I_b sin(\alpha_b - \beta_b) sin(2wt + 2\beta_b - (4\pi/3)) + \\ V_c I_c sin(\alpha_c - \beta_c) sin(2wt + 2\beta_c + (4\pi/3)) \end{bmatrix} \quad (20)$$

$$u(t) = P + p(t) + q(t) = P + s(t) \quad (21)$$

Where:

- $s(t)$: the three –phase instantaneous phasor power
- $p(t)$: the three–phase instantaneous active power
- $q(t)$: the three-phase instantaneous reactive power
- P : the three–phase average or active power (true power)

Fig.1. display the instantaneous power per phase $u_a(t)$, which follow a sinusoidal oscillation with a double frequency ($2f = 2w/2\pi$) shifted up by the average active power per phase $P_a(t)$.

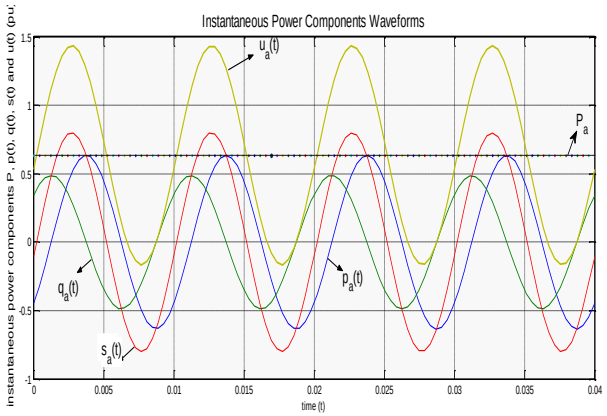


Fig. 1: Instantaneous Power Components Waveforms per Phase (A).

The sinusoidal oscillation is described by a function known as the instantaneous phasor power per phase $s_a(t)$, which can be divided into two orthogonal sinusoidal functions; instantaneous active power $p_a(t)$, and instantaneous reactive power $q_a(t)$.

2.2. Phasor representation

From the literature survey, it can be seen that the key point of the problem with the power component definition is the residual component of the total power excluding the average real power [35]. From this viewpoint, focusing on sinusoidal power components only excluding the average active power components is presented, since there is a consensus in their definitions. Fig.2. show that the sinusoidal power components can be presented as vectors on a phasor diagram.

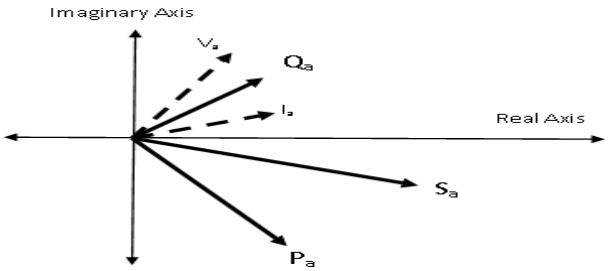


Fig. 2: Phasor Diagram of the Power Component Vectors per Phase (A).

Dash line … Va&Ia vectors (rotating speedω)
Solid line ----- power vectors (rotating speed2ω)

The above diagram is known and accepted for the common engineer. If we consider the line current of phase (a) as the reference for angles; i.e.$\beta_a = 0$, so the phasor current Ia will lie on the real axis. Fig.3. shows the power components in phasor diagram.

In this case, the phase difference between Va&Ia equal $(\alpha_a - \beta_a) = \alpha_a$; so the phase shift of reactive power Qa and active power Pa will always equal zero and $(-\pi/2)$ respectively. In addition, the phase shift of phasor power Sa will ways $(\alpha_a - \pi/2)$. This apparently stems from the conventional power triangle, and is beneficial for the study of power flow.

The power vectors in Fig.3. Will rotate by $(\pi/2)$ anticlockwise direction to find Fig.4. very similar to the traditional four-quadrant power flow directions, which cited by reference[36]. Consequently, phasor power per phase $s_a(t)$ has been represented in phasor diagram as a vector S_a which contains two perpendicular axes as reference direction for active and reactive power flow in Fig.4.

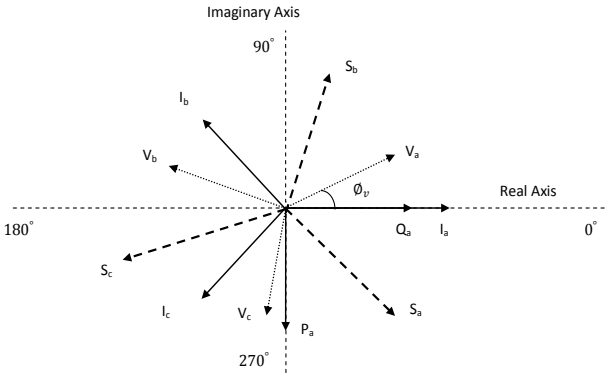


Fig. 3: Three- Phase Power Components Current Phase (A) as A Reference Axis.

3. Analytical study of power components under non sinusoidal condition

The analytical study for calculating power components for three-phase unbalanced condition extends and adapts from the procedures applied for identification of power components in single-phase two-wire systems. [31].

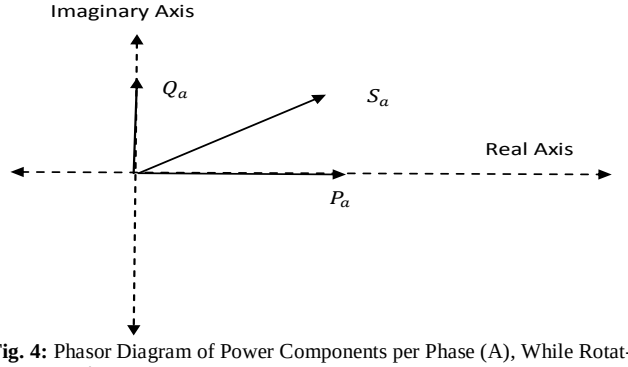


Fig. 4: Phasor Diagram of Power Components per Phase (A), While Rotating It by $\pi/2$ Anticlockwise.

3.1. Instantaneous representation

Although the instantaneous voltage and the instantaneous current are distorted, they are still periodic under unbalanced non-sinusoidal conditions and can be represented by Fourier series, which can decompose distorted signals into amplitude and phase angle of all the frequencies existing.
The line-to-neutral instantaneous voltages at time instant (t) are represented by the following set of equations:

$$\begin{aligned} v_a(t) &= \sum_{j=0}^{\infty}\sqrt{2}V_{aj}\,sin\big(j\omega t+\alpha_{aj}\big)\\ v_b(t) &= \sum_{j=0}^{\infty}\sqrt{2}V_{bj}\,sin\big(j\omega t+\alpha_{bj}-(2\pi/3)j\big)\\ v_c(t) &= \sum_{j=0}^{\infty}\sqrt{2}V_{cj}\,sin\big(j\omega t+\alpha_{cj}+(2\pi/3)\,j\big) \end{aligned} \quad (22)$$

Similarly, the line currents can be written;

$$\begin{aligned} i_a(t) &= \sum_{k=0}^{\infty}\sqrt{2}I_{ak}\,sin(k\omega t+\beta_{ak})\\ i_b(t) &= \sum_{k=0}^{\infty}\sqrt{2}I_{bk}\,sin(k\omega t+\beta_{bk}-(2\pi/3)\,k)\\ i_c(t) &= \sum_{k=0}^{\infty}\sqrt{2}I_{ck}\,sin(k\omega t+\beta_{ck}+(2\pi/3)\,k) \end{aligned} \quad (23)$$

Where:

- j & k : are integer numbers.
- V_j : jth harmonic voltage for each phase (RMS value),
- I_k : kth harmonic current for each phase (RMS value).
- α_j : The phase angle of the jth harmonic voltage V_j with respect to the chosen origin.
- β_j : The phase angle of kth harmonic current I_k with respect to the chosen origin.

We will take the fundamental current of phase ''a'' (I_{a1}) as reference axis i.e.($\beta_{a1}=0$). This choice is beneficial, because it allows all the harmonic powers to be

- plotted in the same two-dimensional plane (with rotation speed different)
- compared with the power created by the fundamental voltage V_1 and current I_1(magnitude and direction)

Using "(22)," "(23)", the instantaneous power per phase (a) is calculated by the multiplying of instantaneous voltage and current per phase.

$$u_a(t)=v_a(t)i_a(t) \quad (24)$$

$$u_a(t)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}2V_{aj}I_{ak}\big[sin\big(j\omega t+\alpha_{aj}\big)\,sin(k\omega t+\beta_{ak})\big] \quad (25)$$

$$u_a(t)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}V_{aj}I_{ak}\begin{bmatrix} sin\Big((j-k)\omega t+\alpha_{aj}-\beta_{ak}+\pi/2\Big)\\ +sin\Big((j+k)\omega t+\alpha_{aj}+\beta_{ak}-\pi/2\Big)\end{bmatrix} \quad (26)$$

Similar;

$$u_b(t)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}V_{bj}I_{bk}\begin{bmatrix} sin\Big((j-k)\omega t+\alpha_{bj}-\beta_{bk}-(2\pi/3)\,(j-k)+\pi/2\Big)\\ +si\,n\Big((j+k)\omega t+\alpha_{bj}+\beta_{bk}-(2\pi/3)\,(j+k)-\pi/2\Big)\end{bmatrix} \quad (27)$$

Similar;

$$u_c(t)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}V_{cj}I_{ck}\begin{bmatrix} sin\Big((j-k)\omega t+\alpha_{cj}-\beta_{ck}+(2\pi/3)\,(j-k)+\pi/2\Big)\\ +si\,n\Big((j+k)\omega t+\alpha_{cj}+\beta_{ck}+(2\pi/3)\,(j+k)-\pi/2\Big)\end{bmatrix} \quad (28)$$

For three–phase non-sinusoidal unbalanced system, the total instantaneous power $u(t)$ is defined as:

$$u(t)=u_a(t)+u_b(t)+u_c(t) \quad (29)$$

Using "(26)", "(27)", and "(28)", the total instantaneous power $u(t)$ is defined as:

$$u(t)=\sum_{j=0,k=0}^{\infty}\begin{bmatrix} V_{aj}I_{ak}\,sin\Big((j-k)\omega t+\alpha_{aj}-\beta_{ak}+\pi/2\Big)\\ +V_{bj}I_{bk}\,sin\Big((j-k)\omega t+\alpha_{bj}-\beta_{bk}-(2\pi/3)\,(j-k)+\pi/2\Big)\\ +V_{cj}I_{ck}\,sin\Big((j-k)\omega t+\alpha_{cj}-\beta_{ck}+(2\pi/3)\,(j-k)+\pi/2\Big)\end{bmatrix}+\sum_{j=0,k=0}^{\infty}\begin{bmatrix} V_{aj}I_{ak}\,sin\Big((j+k)\omega t+\alpha_{aj}+\beta_{ak}-\pi/2\Big)\\ +V_{bj}I_{bk}\,sin\Big((j+k)\omega t+\alpha_{bj}+\beta_{bk}-(2\pi/3)\,(j+k)-\pi/2\Big)\\ +V_{cj}I_{ck}\,sin\Big((j+k)\omega t+\alpha_{cj}+\beta_{ck}+(2\pi/3)\,(j+k)-\pi/2\Big)\end{bmatrix} \quad (30)$$

Equation (30) will be divided into two parts depending on the values of j and k as follow:

i) If $j=k=n$, the part of $u(t)$ that is multiplying the voltages and currents of the same harmonic order will be called distortionless power.
ii) If $j\neq k$, the part of $u(t)$ that is multiplying voltages and currents of different harmonic order will be called distortion power. The voltages and currents have the same harmonic order "n" When $j=k=n$, the distortionless power$u_d(t)$can be expressed as:

$$\begin{aligned} u_d(t) &= \sum_{\substack{n=0\\ j=k=n}}^{\infty}P_{2n}+\\ &\sum_{\substack{n=0\\ j=k=n}}^{\infty}\begin{bmatrix} V_{an}I_{an}\,cos(\alpha_{an}-\beta_{an})\,sin(\,2n\omega t+2\beta_{an}-\pi/2)\\ +V_{bn}I_{bn}\,cos(\alpha_{bn}-\beta_{bn})\,sin(\,2n\omega t+2\beta_{bn}-(2\pi/3)\,(2n)-\pi/2)\\ +V_{cn}I_{cn}\,cos(\alpha_{cn}-\beta_{cn})\,sin(\,2n\omega t+2\beta_{cn}+(2\pi/3)\,(2n)-\pi/2)\end{bmatrix}\\ &+\sum_{\substack{n=0\\ j=k=n}}^{\infty}\begin{bmatrix} V_{an}I_{an}\,sin(\alpha_{an}-\beta_{an})\,sin(\,2n\omega t+2\beta_{an})\\ +V_{bn}I_{bn}\,sin(\alpha_{bn}-\beta_{bn})\,sin\big(\,2n\omega t+2\beta_{bn}-(2\pi/3)\,(2n)\big)\\ +V_{cn}I_{cn}\,sin(\alpha_{cn}-\beta_{cn})\,sin\big(\,2n\omega t+2\beta_{cn}+(2\pi/3)\,(2n)\big)\end{bmatrix} \end{aligned} \quad (31)$$

$$u_d(t)=\sum_{\substack{n=0\\ j=k=n}}^{\infty}[P_{2n}+p_{2n}(t)+q_{2n}(t)]=\sum_{\substack{n=0\\ j=k=n}}^{\infty}[P_{2n}+s_{2n}(t)] \quad (32)$$

Where:

- $s_{2n}(t)$: the three-phase instantaneous phasor power
- $p_{2n}(t)$: the three-phase instantaneous active power
- P_{2n} : the three-phase average or active power (true power)
- $q_{2n}(t)$: the three-phase instantaneous reactive power

i) The voltages and currents have different harmonic order When$j\neq k$, the distortion power $d(t)$ can be expressed as:

$$d(t)=\sum_{n=1}^{\infty}d_n(t) \quad (33)$$

Where, $d_n(t)$ is the nth harmonic distortion power,

$$\begin{aligned} d_n(t) &= \sum_{\substack{j=0,k=0,j\neq k\\ j-k=n,j>k}}^{\infty}\begin{bmatrix} V_{aj}I_{ak}\,sin\big(n\omega t+\alpha_{aj}-\beta_{ak}+\pi/2\big)\\ +V_{bj}I_{bk}\,sin\big(n\omega t+\alpha_{bj}-\beta_{bk}-(2\pi/3)\,(n)+\pi/2\big)\\ +V_{cj}I_{ck}\,sin\big(n\omega t+\alpha_{cj}-\beta_{ck}+(2\pi/3)\,(n)+\pi/2\big)\end{bmatrix}\\ &+\sum_{\substack{j=0,k=0,j\neq k\\ k-j=n,j<k}}^{\infty}\begin{bmatrix} V_{ak}I_{aj}\,sin\big(n\omega t-\alpha_{ak}+\beta_{aj}+\pi/2\big)\\ +V_{bk}I_{bj}\,sin\big(n\omega t-\alpha_{bk}+\beta_{bj}-(2\pi/3)\,(n)+\pi/2\big)\\ +V_{ck}I_{cj}\,sin\big(n\omega t-\alpha_{ck}+\beta_{cj}+(2\pi/3)\,(n)+\pi/2\big)\end{bmatrix}\\ &+\sum_{\substack{j=0,k=0,j\neq k\\ j+k=n,}}^{\infty}\begin{bmatrix} V_{aj}I_{ak}\,sin\big(n\omega t+\alpha_{aj}+\beta_{ak}-\pi/2\big)\\ +V_{bj}I_{bk}\,sin\big(n\omega t+\alpha_{bj}+\beta_{bk}-(2\pi/3)\,(n)-\pi/2\big)\\ +V_{cj}I_{ck}\,sin\big(n\omega t+\alpha_{cj}+\beta_{ck}+(2\pi/3)\,(n)-\pi/2\big)\end{bmatrix} \end{aligned} \quad (34)$$

From "(32)", and "(33)", the total instantaneous power $u(t)$ is:

$$u(t)=u_d(t)+d(t) \quad (35)$$

$$u(t)=\sum_{n=0}^{\infty}[P_{2n}+s_{2n}(t)]+\sum_{n=1}^{\infty}d_n(t) \quad (36)$$

Because the harmonic order of voltages and currents is always an integer number, the nth instantaneous power $u_n(t)$ as,

$$u_n(t)=P_n+s_n(t)+d_n(t) \quad (37)$$

Where, adding phasor power $s_n(t)$ to distortion power $d_n(t)$ as both of them has the same harmonic order n, will produce the so-called fictitious power $f_n(t)$

$$f_n(t) = s_n(t) + d_n(t) \quad (38)$$
$$u_n(t) = P_n + f_n(t) \quad (39)$$

Finally, the total instantaneous power $u(t)$ as:

$$u(t) = \sum_{n=0}^{\infty} u_n(t) \quad (40)$$

The waveforms of the sixth order harmonic power components per phase are shown in Fig.5.

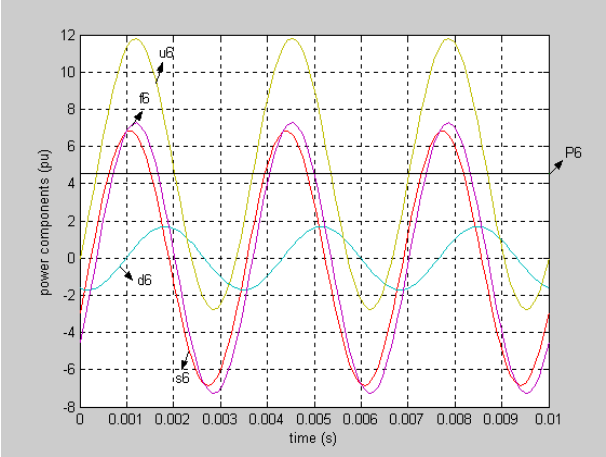


Fig. 5: The Waveforms of Sixth Order Harmonic Power Components per Phase.

3.2. Phasor representation

The phasor diagram in Fig.6 represents the nth harmonic non-sinusoidal power components per phase (a). By rotating the vectors of power components by $(\pi/2)$ anticlockwise, we get the traditional power flow direction [31]. These power components will be second order harmonic power (they will be subscripted by number 2). By this way, power components for each harmonic order can be compared in magnitude and direction with power generated by V_1&I_1 (useful power).
Because all the harmonic power components have the same frequency(nf), they have been represented in the phasor diagram. As a result, all the harmonic power components can be represented in a complex form in four quadrant reference direction. Note that all the phase angles of three-phase power components of the previous equations have increased by $(\pi/2)$ to get the traditional power flow direction.

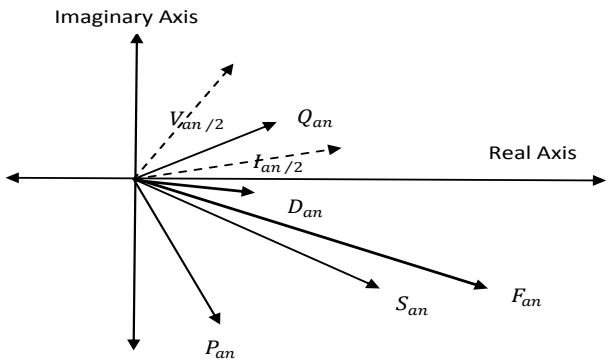


Fig. 6: Phasor Diagram of the Nth Harmonic Power Component Vectors per Phase (A).

Dash line … V&I vectors (rotating speed$n\omega/2$)
Solid line ----- power vectors (rotating speed $n\omega$)

4. Physical essence of power components

It is an attempt to get physical interpretation of the proposed three-phase power components for periodic current and voltage waveforms for any non-sinusoidal unbalanced three-phase four wire system.
It is needful to remember, that all power components connect with their associated energies. Now, any power components can be measured if the formula for its calculation has been found. This section summarizes the Analysis and conclusion of the physical essence of the following components, for each n order power of harmonic.

4.1. Real (average) active power- (P_n)

It is representing the power dissipated in a component of system resistance (constant value).
The physical essence: it is the rate of the net transferred active energy (dissipative one) supplied by/ or to the system produced by the voltage and current having the same harmonic order $n/2$.

4.2. Instantaneous active power-$p_n(t)$

A sinusoidal function, which represent the active power oscillation with amplitude equals P_n. It is the associated power component with the system resistance.
The physical essence: it is the rate of the active energy oscillations between utility (supply) and customer (load) with nth harmonic frequency.

4.3. Instantaneous reactive power-$q_n(t)$

A sinusoidal function, which represent the reactive power oscillation with amplitude equals Q_n. It is the associated power component with the potential or kinetic energy.
The physical essence: it is the rate of the reactive energy (quad-ergy) oscillations between utility (supply) and customer (load) with nth harmonic frequency.

4.4. Instantaneous phasor power-$s_n(t)$

A sinusoidal function, which represent the phasor power oscillation of with amplitude equals S_n. It is the summation of the two above active and reactive sinusoidal power.
The physical essence: it is the rate of the phasor energy oscillations (summation of the active and reactive energy) between utility (supply) and customer (load) with nth harmonic frequency.

4.5. Distortion power-$d_n(t)$

A sinusoidal function, which represents the summation of all distortion power. It is generated by the cross products of harmonic voltages and harmonic currents of different frequencies, which produce nf frequency functions.
The physical essence: it is the rate of the distortion energy oscillations between utility (supply) and customer (load) with nth harmonic frequency.
The authors [31] have validated experimentally different definition ns of distortion power. They deduce that existent definitions are not correct, neither from a physical nor numerical viewpoint due to the involvement of like voltages and currents in distortion power (excepting the Budeanu's one). The proposed formulas of distortion power have overcome this feebleness of those incorrect formulas aforesaid.

4.6. Fictitious power- $f_n(t)$

A sinusoidal function, which represents the summation of distortion power and phasor power, these distortion power and phasor power have the same frequency nf.

We have employed the same terminologies and symbols used in the IEEE dictionary [37], but with various meaning. In the IEEE dictionary, the fictitious power F is the vector summation of both distortion and reactive power F=jQ+kD (nonactive power). But here, the factitious power is the total sinusoidal (nonactive) part of the nth harmonic instantaneous power $u_n(t)$.
We can deem it as a quantity corresponding to the IEEE power vectors, but in a two-dimensional plane, for each individual harmonic order n.
The physical essence: it is the rate of the fictitious energy oscillations (summation of distortion, reactive and active energies) between utility (supply) and customer (load) with nth harmonic frequency.

4.7. nth harmonic instantaneous power-$\mathbf{u_n(t)}$

The summation of the fictitious power $f_n(t)$ and the active (average) power P_n for each harmonic order n.
The physical essence: it is the rate of the fictitious energy oscillations plus active energy dissipation between utility (supply) and customer (load) with nth harmonic frequency.

4.8. Total instantaneous power-$\mathbf{u(t)}$

The summation of all instantaneous power for each harmonic order n.
The physical essence: it is the rate of all the fictitious energies oscillations plus the active energies dissipation between utility (supply) and customer (load).

5. Conclusion

Although the final objective of this analytical study is determining the responsibility of harmonic distortion in part B of this paper, but we have reached a new method for defining of the power component in the three-phase unbalanced system, which has interesting advantages such as:

- Using the same terminology and symbols of power components employed in the IEEE dictionary.
- The components of instantaneous sinusoidal power have been presented in the phasor forms as vectors.
- These vectors can be represented in a phasor diagram.
- Through a quick look to this phasor diagram, one can uprightly determine the direction of power components flow.
- All the power components can be derived from direct measurment of periodic signal of voltages and currents at a single point.
- Using the common units of power (W, VAR and VA).
- There is no the need for complex mathematical models, the power component definitions are clearly presented.

The next paper part B will present a new approach for determining the contribution of the harmonic distortion for each harmonic order power and consequently the responsibility of harmonic distortion at the PCC between the utility and the customer can be determined.

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