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Research paper

Inverse majority vertex covering number of a graph

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Abstract

A set of vertices S, which covers atleast half of the edges is a Majority vertex cover of G. The majority vertex covering number $\alpha_M(G)$ of G is the minimum number in a Majority vertex cover. In this paper, new parameter has been introduced Inverse majority vertex covering number of a graph G with respect to Majority vertex covering set. Also majority vertex covering number obtained for classic graphs and Cartesian product graph.

Keywords: *Majority Vertex Covering Number; Inverse Majority Vertex Set; Inverse Majority Vertex Covering Number of A Graph.*

1. Introduction

Definition 1.1 [4]:

A subset S of V(G) is said to be a majority dominating set (MDS) if at least half of the vertices of V(G) are either in S or adjacent to elements of S. A majority dominating set S is minimal if no proper subset of S is a majority dominating set. The minimum cardinality of a minimal majority dominating set is called majority domination number, denoted by $\gamma_M(G)$.

Definition 1.2: *[5] A vertex set v is called majority dominating vertex if the degree of v, if $d(v) \geq \left\lceil \frac{p}{2} \right\rceil - 1$. For $S \subseteq V(G)$, a vertex $v \in S$ is called an enclave of S if $N[v] \subseteq S$ and $v \in S$ is an isolate of S if $N(v) \subseteq V - S$.*

Definition 1.3: *[10] Let D be minimum vertex cover of G. If $V - D$ contains a vertex cover D' of G, then D' is called an inverse vertex cover with respect to D. The inverse vertex covering number $\alpha_0^{-1}(G)$ of G is the minimum cardinality of an inverse vertex cover of G.*

Definition 1.4: *[6] A set of vertices S, which covers at least half of the edges, is a majority vertex cover of G. The majority vertex covering number $\alpha_M(G)$ of G is the minimum number of vertices in a majority vertex cover.*

2. Inverse majority covering number of a graph

Definition 2.1: *A set D_M be the majority vertex cover of G. If $V - D_M$ contains a majority vertex cover D_M^{-1} of G then D_M^{-1} is called a inverse majority vertex cover with respect to D_M. The inverse majority vertex covering number $\alpha_M^{-1}(G)$ of G is the minimum vertex covering number of G.*

Proposition 2.2

For the graph $G = W_p$ with $p \geq 3$, $\alpha_M^{-1}(G) = \left\lceil \frac{p-1}{3} \right\rceil$.

Proof
Let $G = W_p = K_1 + C_{p-1}$ with $p \geq 3$ and $q = 2(p-1)$. The vertex set $V(G) = \{u, v_1, v_2, \dots, v_{p-1}\}$. The majority vertex covering number $\alpha_M^{-1}(G) = 1$ since $d(u) = p - 1$. Let D_M be majority vertex coving set. The inverse majority vertex covering set

$$D_M^{-1}(G) = \left\{v_1, v_2, v_3, \dots v_{\left\lceil \frac{p-1}{3} \right\rceil}\right\} \Rightarrow \langle N[V - D_M] \rangle = C_p$$

Then $|D_M^{-1}| = \left\lceil \frac{p-1}{3} \right\rceil = \left\lceil \frac{2(q-1)}{3} \right\rceil \geq \left\lceil \frac{q}{2} \right\rceil$

Therefore $\alpha_M^{-1}(G) = |D_M^{-1}| = \left\lceil \frac{p-1}{3} \right\rceil$.

Proposition 2.3:

For the fan graph $G = F_p$ with $\alpha_M^{-1}(G) = \left\lceil \frac{p-1}{3} \right\rceil$.

Theorem 2.4: *If the graph $G = S_{2m+1}$ is the spider graph then $\alpha_M^{-1}(G) = \left\lceil \frac{m}{2} \right\rceil$.*

Theorem 2.5: *For the friendship graph $G = F_{1,2n}$ then $\alpha_M^{-1}(G) = \left\lceil \frac{p}{2} \right\rceil$.*

Proposition 2.6:
If the graph G is complete bipartite graph $G = K_{m,n}$ with $m, n \geq 2$ then

$$n_M^{-1}(G) = \begin{cases} \left\lceil \frac{m}{2} \right\rceil, \text{ if } m \leq n \text{ and } m = \text{even and } m - n = 1 \\ \left\lceil \frac{m}{2} \right\rceil + 1, \text{ if } m \leq n \text{ and } m = \text{odd} \end{cases}$$

with $m, n \geq 2$.

3. Inverse Majority vertex covering number for Grid graph

Definition 3.1: *[3] Let G and H be any two connected graphs with the vertex set $\{u_1, u_2, u_3, \ldots, u_n\}$ and $\{v_1, v_2, v_3, \ldots, v_n\}$ respectively. The Cartesian products graph $K = G \times H$ has $V(K) = V(G) \times V(H)$ and vertices (u_1, v_1) and (u_2, v_2) in $V(K)$ are adjacent if and only if either $u_1 = u_2$ and $v_1 v_2$ in $V(H)$ or $v_1 = v_2$ and $u_1 u_2$ in $E(G)$. The grid graph is $P_i \times P_j$ and the cylinder is $C_i \times P_j$ for $i, j \geq 3$.*

Theorem 3.2: *If the graph $G = P_2 \times P_j$ is grid graph with $j \geq 3$ then $\alpha_M^{-1}(G) = \left\lceil \frac{j+1}{2} \right\rceil$.*

Proof

Consider the graph $G = P_2 \times P_j$ is ladder graph with vertex set $\{v_{11}, v_{12}, v_{13}, \ldots, v_{1j}\}$ and the $\{v_{21}, v_{22}, v_{23}, \ldots, v_{2j}\}$, $\Delta(G) = 3$, $d(v_{i1}) = d(v_{ij}) = 2$ and $p = 2j$, $q = 2(j-1) + j$.

Case (i) j = even Let the majority vertex covering set is $D_M^{-1} = \{v_{13}, v_{15}, v_{17}, \ldots, v_{1j-1}, v_{22}\}$with $d(v_i, v_j) = 2$

$\Rightarrow |D_M^{-1}| = \left\lceil \frac{j-2}{2} \right\rceil + 1$and $|\langle D_M^{-1} \rangle| = 3\left(\left\lceil \frac{j-2}{2} \right\rceil + 1\right) > \left\lceil \frac{2(j-1)+j}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$.

Therefore, $|D_M^{-1}| \leq \alpha_M^{-1}(G)\alpha_M^{-1}(G)$.
Suppose $D_M^{-1} = \{v_{13}, v_{15}, v_{17}, \ldots, v_{1j-1}\} \Rightarrow |D_M^{-1}| = \left\lceil \frac{p-2}{2} \right\rceil$

and $|\langle D_M^{-1} \rangle| = 3\left(\left\lceil \frac{j-2}{2} \right\rceil\right) < \left\lceil \frac{2(j-1)+j}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$.
Therefore, $|D_M^{-1}| \geq \alpha_M^{-1}(G)$. Hence, $\alpha_M^{-1}(G) = \left\lceil \frac{j-2}{2} \right\rceil + 1 = \left\lceil \frac{j+1}{2} \right\rceil$.

Case (ii) j = odd

If $j = 3$ then $D_M^{-1} = \{v_{11}, v_{22}\} \Rightarrow |D_M^{-1}| = \left\lceil \frac{j-2}{2} \right\rceil + 1$ and $|\langle D_M^{-1} \rangle| = 3\left(\left\lceil \frac{j-2}{2} \right\rceil + 1\right) > \left\lceil \frac{q}{2} \right\rceil$.

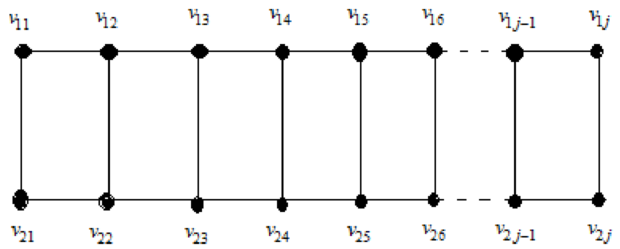
If $j > 3$ then $D_M^{-1} = \{v_{13}, v_{15}, v_{17}, \ldots, v_{1j-2}, v_{22}, v_{24}\} \Rightarrow |D_M^{-1}| = \left\lceil \frac{j-3}{2} \right\rceil + 2$ and
$|\langle D_M^{-1} \rangle| = 3\left(\left\lceil \frac{j-3}{2} \right\rceil + 2\right) > \left\lceil \frac{2(j-1)+j}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$. Therefore, $|D_M^{-1}| \leq \alpha_M^{-1}(G)$.

Suppose

$D_M^{-1} = \{v_{13}, v_{15}, v_{17}, \ldots, v_{1j-2}, v_{22}\} \Rightarrow |D_M^{-1}| = \left\lceil \frac{j-3}{2} \right\rceil + 1$

and $|\langle D_M^{-1} \rangle| = 3\left(\left\lceil \frac{j-3}{2} \right\rceil + 1\right) < \left\lceil \frac{2(j-1)+j}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$.

Therefore, $|D_M^{-1}| \geq \alpha_M^{-1}(G)$. Hence, $\alpha_M^{-1}(G) = \left\lceil \frac{j-3}{2} \right\rceil + 2 = \left\lceil \frac{j+1}{2} \right\rceil$.



Theorem 3.3: *If the graph $G = P_3 \times P_j$ with $j \geq 4$ then$\alpha_M^{-1}(G) = \left\lceil \frac{3(j-2)}{4} \right\rceil$.*

Proof

Let $G = P_3 \times P_j$ be the grid graph with $V(G) = \{v_{11}, v_{12}, v_{13}, \ldots, v_{1j}, v_{21}, v_{22}, \ldots, v_{2j}, v_{31}, v_{32}, \ldots v_{3j}\}$ then $p = 3j$ and $= 3(2j-1)$, $\Delta(G) = 4$. $V_1(G) = \{v_{11}, v_{12}, v_{13}, \ldots, v_{1j}\}$, $V_2(G) = \{v_{21}, v_{22}, v_{23}, \ldots, v_{2j}\}$, $V_3(G) = \{v_{31}, v_{32}, v_{33}, \ldots, v_{3j}\}$ I, II and III row vertices respectively. $d(v_{2j}) = 4$ and $d(v_{1j}) = d(v_{3j}) = 3$..

Case (i) $j = even$

Let $D_M^{-1}(G) = \{v_{23}, v_{25}, v_{27}, \ldots v_{2j-1}, v_{12}, v_{14}, \ldots, v_{1j-2}\}$. Then $|D_M^{-1}| = \left\lceil \frac{j-2}{2} \right\rceil + \left\lceil \frac{j}{4} \right\rceil = \left\lceil \frac{3(j-2)}{4} \right\rceil \Rightarrow |\langle D_M^{-1} \rangle| = 4\left\lceil \frac{j-2}{2} \right\rceil + 3\left\lceil \frac{j}{4} \right\rceil > \left\lceil \frac{3(2j-1)}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$. Since $d(v_{2j}) = 4$ and $d(v_{1j}) = d(v_{3j}) = 3$. Therefore $|D_M^{-1}| \leq \alpha_M^{-1}(G)$. If $|D_M^{-1}| = \left\lceil \frac{j-2}{2} \right\rceil + \left\lceil \frac{j}{4} \right\rceil - 1$ then $|\langle D_M^{-1} \rangle| = 4\left\lceil \frac{j-2}{2} \right\rceil + 2\left\lceil \frac{j}{4} \right\rceil < \left\lceil \frac{3(2j-1)}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$. Therefore $|D_M^{-1}| > \alpha_M^{-1}(G)$. Hence $|D_M^{-1}| = \alpha_M^{-1}(G) = \left\lceil \frac{3(j-2)}{4} \right\rceil$.

Case (ii) $j = odd$

Choose set

$D_M^{-1}(G) = \{v_{23}, v_{25}, v_{27}, \ldots, v_{2j-2}\} \Rightarrow |D_M^{-1}| = \left\lceil \frac{j-3}{2} \right\rceil + \left\lceil \frac{j}{4} \right\rceil = \left\lceil \frac{j-2}{2} \right\rceil + \left\lceil \frac{j}{4} \right\rceil = \left\lceil \frac{3(j-2)}{4} \right\rceil \Rightarrow |\langle D_M^{-1} \rangle| = 4\left\lceil \frac{j-3}{2} \right\rceil + 3\left\lceil \frac{j}{4} \right\rceil > \left\lceil \frac{3(2j-1)}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$.

Therefore $|D_M^{-1}| \leq \alpha_M^{-1}(G)$. Hence $\alpha_M^{-1} = \left\lceil \frac{3(j-2)}{4} \right\rceil$.

Corollary 3.4:

For the Grid graph $G = P_4 \times P_j$ with $j \geq 4$. Then $\alpha_M^{-1} = \left\lceil \frac{p}{8} \right\rceil$.

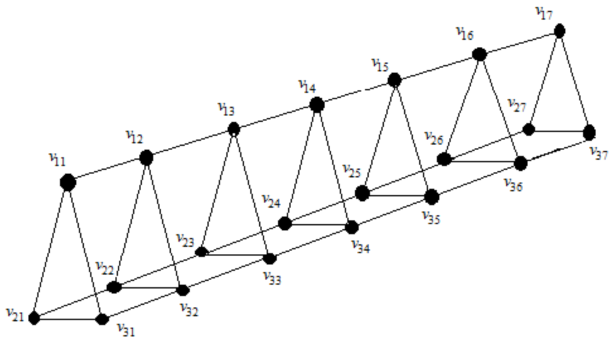
3.5. Majority vertex covering number for Cylinder graph

Theorem 3.5.1: *For a Cylinder graph $= C_3 \times P_j, \alpha_M^{-1}(C_3 \times P_j) = \left\lceil \frac{3(j-1)}{8} \right\rceil$.*

Proof : Consider the graph $G = C_3 \times P_j$ with $j \geq 4$ and $V(G)$ be the vertex set with cardinality $3j$, $E(G)$ be edge set with $|E(G)| = 3j + 3(j-1)$. Let $V_1(G)$, $V_2(G)$ and $V_3(G) \subseteq V(G)$ be the I, II and III row vertices respectively. $d(v_{ij}) = 4$, $v_{ij} \in V_1(G), V_3(G)$, $ij \neq 12, 1j, 21, 2j, 31, 3j$ and $d(v_{ij}) = 3$.
If j is even then the inverse majority vertex covering$D_M^{-1}(G) = \{v_{13}, v_{15}, v_{17}, \ldots, v_{32}, v_{34}, \ldots v_{ij}\}$, $v_{ij} \in V_1(G), V_3(G) \Rightarrow |D_M^{-1}(G)| = \left\lceil \frac{3(2j-1)}{8} \right\rceil, |\langle N[D_M^{-1}] \rangle| = 4\left\lceil \frac{j-2}{2} \right\rceil + 4\left\lceil \frac{j-2}{2} \right\rceil = 8\left\lceil \frac{j-2}{2} \right\rceil = 4\left\lceil \frac{3(2j-1)}{8} \right\rceil$, $4\left\lceil \frac{3(2j-1)}{8} \right\rceil \geq \left\lceil \frac{3(2j-1)}{2} \right\rceil \geq \left\lceil \frac{q}{2} \right\rceil$. Therefore $|D_M^{-1}(G)| = \left\lceil \frac{3(2j-1)}{8} \right\rceil \leq \alpha_M^{-1}(C_3 \times P_j)$. Suppose $|D_M^{-1}(G)| = \left\lceil \frac{3(2j-1)}{8} \right\rceil - 1 \Rightarrow 4\left\lceil \frac{3(2j-1)}{8} \right\rceil - 1 < \left\lceil \frac{3(2j-1)}{2} \right\rceil - 1 < \left\lceil \frac{q}{2} \right\rceil$. Therefore $|D_M^{-1}(G)| = \left\lceil \frac{3(2j-1)}{8} \right\rceil > \alpha_M^{-1}(C_3 \times P_j)$. Hence $|D_M^{-1}(G)| = \alpha_M^{-1}(C_3 \times P_j) = \left\lceil \frac{3(2j-1)}{8} \right\rceil$. If $j = odd$ then $|D_M^{-1}(G)| = \left\lceil \frac{3(2j-1)}{8} \right\rceil \Rightarrow |\langle N[D_M^{-1}] \rangle| = 4\left\lceil \frac{j-3}{2} \right\rceil + 4\left\lceil \frac{j-2}{2} \right\rceil = 4\left\lceil \frac{3(2j-1)}{8} \right\rceil \geq \left\lceil \frac{3(2j-1)}{2} \right\rceil \geq \left\lceil \frac{q}{2} \right\rceil$.
Therefore $\alpha_M^{-1}(C_3 \times P_j) = \left\lceil \frac{3(2j-1)}{8} \right\rceil$.

3.5.2. Example

Consider the graph $G = C_3 \times P_7$ with $p = 3(7) = 21, q = 3(7) + 3(6) = 39$ then $\left\lceil \frac{q}{2} \right\rceil = \left\lceil \frac{39}{2} \right\rceil = 20$.



$D_M^{-1}(G) = \{v_{13}, v_{15}, v_{22}, v_{24}, v_{26}\}$ is a majority vertex covering set of G

$|D_M^{-1}| = \alpha_M^{-1} = \left\lceil \frac{3(2(7)-1)}{2} \right\rceil$=20.

4. Bounds on Inverse majority vertex covering number

Proposition 4.1:
For any graph G without isolated vertices then

$\alpha_M(G) \leq \alpha_M^{-1}(G)$

This bound is sharp for $K_{m,m}$ with$m \geq 2$.
Proposition 4.2:
For any connected graph $\boldsymbol{G}$ then

$\gamma_M(G) \leq \alpha_M^{-1}(G)$

This bound holds for$G = C_{2n}$.
Proposition 4.3
For any connected graph G,

$\alpha_M(G) + \alpha_M^{-1}(G) = p - 2.$

Proposition 4.4:

For any graph $\boldsymbol{G}$ without isolated vertices ,

$\gamma_M(G) \leq \alpha_M(G) \leq \alpha_M^{-1}(G).$

Proposition 4.5:
For any connected graph $\boldsymbol{G}$ then,

$\alpha_M^{-1}(G) \leq p - \gamma_M(G)$

Observation 4.6:
i) If the graph $G = P_i \times P_j$ then $\alpha_M(G) = \alpha_M^{-1}(G)$
ii) If the graph $G = C_3 \times P_j$ then $\alpha_M(G) = \alpha_M^{-1}(G)$

5. Conclusion

The researcher has introduced the new parameter inverse majority vertex covering of a graph $\boldsymbol{G}$. This number defined $\boldsymbol{\alpha_M^{-1}(G)}$ and determined for some classes of graph and also obtained the bounds. This generalized for Cartesian product graphs

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