



Guiding Principles to Foster Higher Order Thinking Skills in Teaching and Learning of Mathematics

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Abstract

Stepping into the new millennium, many societies around the world are engaging in serious and promising educational reforms. A holistic framework in the form of a comprehensive process of fostering higher order thinking skills (HOTS) in mathematics teaching and learning is needed to guide teachers in implementation suitable activities in the classroom. Therefore, this research is intended to develop a Guiding Principles of Fostering Higher Order Thinking Skills (GP-HOTS) in teaching and learning of mathematics for secondary school students. The sample of the study was 266 secondary mathematics teachers that were chosen using multilevel cluster sampling technique. An instrument namely the Fostering HOTS Questionnaire related to principles of fostering HOTS in teaching and learning of mathematics was used in this study. Data analyses involved the computation of face, content, reliability indices and validation of themes and items using factor analysis. The results showed that there are six principles of how teachers can foster HOTS in teaching and learning of mathematics namely practice of implementing assessment for learning optimally, practice of determining HOTS learning outcomes, practice of HOTS questioning strategies, practice of integrating Information, Communication and Technology, practice of active learning, and practice of developing habits of mind. These principles have satisfied level of content validity indices as well as revealed an excellent acceptable level of reliability index. The implication is that the developed GP-HOTS in this study can be used in enhancing teachers' mathematical knowledge for teaching, hence creating futuristic minded students

Keywords: Guiding principle, higher order thinking skills, mathematics teaching and learning, secondary school level.

1. Introduction

21st century learning is widely discussed among mathematics educators in almost every education setting. The main objective of mathematics education in most countries is to prepare its citizens for the challenges of life. From this perspective, one of the aims of mathematics education is to enhance individuals with effective problem-solving as well as the Higher Order Thinking Skills (HOTS). Hence, it is vital for teachers to be knowledgeable and skillful in their subject matter so that they will be able to equip students with the skills they need for future success.

2. Background of the Study

Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) results showed that the achievement of Malaysian pupils is low in mathematics, especially in answering questions requiring Higher Order Thinking Skills (HOTS).¹ Following the situation, Malaysian Education Blueprint (MEB) 2013-2025 suggests a transition in mathematical education such that enhancing the teaching and learning by fostering HOTS rather than emphasizing cognitive algorithmic skills.¹

Teachers are the implementers and are very important in determining the success of education aspirations in MEB 2013-2025.¹ Past studies have shown that the quality of teaching has

had a major impact on student engagement and achievement in the classroom.²⁻⁴ In fact, teachers' understanding of mathematics, curriculum and HOTS also contributes to classroom practice, professional development and student achievement (Loukes-Horsley et al., 1998). However, HOTS among mathematics teachers in Malaysia is still very low and mathematics teaching and learning is still a procedural that emphasizes algorithms.^{1,5} This is further supported by the findings of the Malaysian Higher Education Leadership Academy study; 50% of Malaysian teachers observed failed to deliver their lessons effectively, particularly to inculcate higher order thinking.¹ Several studies also indicated that teachers are reluctant to implement the agenda, lack of knowledge and pedagogical skills as well as teachers' attitude toward teaching.^{5,6} In addition, HOTS can only be taught once all the mathematical concepts are understood where this indicates the assumption of HOTS should be taught separately. Furthermore, there are many studies showed that the module is an important requirement of the teacher in implementing HOTS elements.^{5,8-9} However, the module is only temporary; ideal for the topic selected in the module, and may be suitable for a group of students depending on the target of the developed module. Thus, this study aims to develop a guiding principle to foster HOTS (GP-HOTS) teaching and learning of mathematics. GP-HOTS in teaching and learning of mathematics in this study would give an idea or an important way how mathematics teachers can integrate HOTS in their teaching process.

3. Objective of the Study

The main purpose of the study is to develop a guiding principle to foster HOTS in teaching and learning of mathematics. In particular, the objectives of the study are:

- i. to generate items and themes for GP-HOTS based on the theory, policy and best teaching and learning of mathematics practice that can foster HOTS,
- ii. to verify the items and themes for GP-HOTS in teaching and learning of mathematics, and
- iii. to determine the actual structure of the items and themes for GP-HOTS in teaching and learning of mathematics.

For this article, only objective (iii) will be presented. Further details for the results of objectives (i) and (ii) can be referred to Nor'ain, Marzita and Mazlini.¹⁰

METHODOLOGY

This study employed a quantitative approach using the survey data-gathering method. For this study, the population was all national secondary school mathematics teachers in Peninsular Malaysia. In order to obtain the sample, a multi-stage cluster sampling was used.¹¹ The states of Peninsular Malaysia are divided into zone clusters namely the northern zone (Perlis, Kedah, Penang), the central zone (Perak, Selangor, Federal Territory), the southern zone (Negeri Sembilan, Melaka, Johor) and the eastern zone (Pahang, Terengganu, Kelantan). For each zone, a state was chosen by simple random sampling. For each selected state, the districts are randomly selected and the list of school names was obtained from the Department of State Education website. Furthermore, for each school in the district, the questionnaire was posted to the school to get a random sample of the study. The result of this study was 266 secondary school mathematics teachers. There is no consensus in the literature regarding the size of the sample appropriate to carry out the study involving factor analysis.¹² (MacCallum, Widaman, Zhang, & Hong (1999). MacCallum et al. (1999) suggested a sample size between 100 and 200 is

An instrument related to guiding principles to foster HOTS in teaching and learning of mathematics was used in this study, namely the Fostering HOTS Questionnaire (FHQ). The FHQ is used to determine the opinion of secondary school mathematics teachers on the important aspects on how to foster HOTS in mathematics teaching and learning. The instrument was divided into two sections. The first section enquires demographic information with fixed-choice items which allows for numerical comparison relatively easy. Items are about position, designation, name of school, teaching experience, gender, highest academic qualification, professional qualification, area of expertise and subject taught. The second section of the instrument consists of five Likert scales items that measure the themes of GP-HOTS.

All themes and items for the Guiding Principles of Fostering Higher Order Thinking Skills in Teaching and Learning of Mathematics were determined through extensive document analysis.¹⁰ It was designed in such a manner that the form of data received can be weighted and prioritized so that it can be linked to a practical decision guiding principles for fostering HOTS in teaching and learning of mathematics.

The results of document analysis showed that there were seven main themes of how teachers can foster HOTS in teaching and learning of mathematics namely determining the learning outcomes (total items 7; A1-A7), planning questioning strategies (total items 6; B1-B16), practicing active learning (total items 16; C1-C16), developing habits of mind (total items 7; D1-D7), practicing reflective thinking (total items 6; E1-E6), implementing optimally assessment for learning (total items 7; F1-F7) and integrating Information, Communication and Technology (total items 6; G1-G6). Using Content Validity Index (CVI) computational for five evaluators, the CVI for face validity was 0.91 and content validity was 0.96, indicated evidence of good face and content validity. The acceptable standard for index of average congruity recommended by Waltz¹³ is .90 for five

evaluators. In addition, the result of the internal consistency tests for the instrument was reported to be 0.97, hence is excellent acceptable level for this research purpose.¹¹

Data analyses in this study involved the validation of themes and items using the factorial analysis method of principal axis factoring for GP-HOTS. Principal axis factoring is a commonly used method of reducing items for a large variable to a small set of data, including important factors that summarize important information in a variable.¹⁴ Prior to the factor analysis, some assumptions and practical considerations were made so that analysis and findings were meaningful and had valid interpretations. Seven assumptions expressed by Coakes and Steed¹⁴ are considered: (1) Sufficient sample size; (2) Data is normally distributed; (3) Data is linear; (4) No outliers data; (5) Multicollinearity and singularity; (6) Factorable correlation matrices; and (7) Outliers between variables. This analysis was conducted to identify and arrange a large number of questionnaire items into constructs of one particular variable from the study sample.¹⁵

The following are the steps taken in this factor analysis procedure as suggested by Coakes and Steed¹⁴ and Hair et al.¹⁵

- a) Items with anti-image correlation matrix $<.500$ will be aborted.
- b) Items that do not belong to any of the factors are omitted and items that fall into more than one factor will not be accepted and will be dropped.
- c) Items that have values similar to other items will also be dropped.
- d) Only items with factor loading values exceeding or equal to 0.50 will be maintained within each component.
- e) The items will also be dropped if there is an unusual load with the constructed construct.
- f) Sphericity Bartlett test should be significant ($p <.05$) to measure the correlation between item or variable.
- g) Kaiser-Meyer-Olkin sample sufficiency test should be at least 0.60 and above. This method is used to measure the adequacy of the study sample.
- h) The number of factors is based on the suitability of the item, at least three items per factor.
- i) Factor construction is based on previous theories and studies

4. Findings and Discussion

Respondent Profiles

Respondents in this study consisted of 266 secondary mathematic teachers. Out of the 266 mathematics teachers involved, 78.9% were female teachers. The majority of mathematics teachers in this study have six to 20 years of experience. Only seven teachers have more than 30 years teaching experience. Almost 90% of these math teachers have a Bachelor's Degree, 8.9% Master's Degree and about .4% has Doctor of Philosophy's degree. In terms of professional qualifications, almost all respondents have a degree in Education.

Results for Kaiser-Meyer-Olkin and Bartlett Test

Table 1: Kaiser-Meyer-Olkin and Bartlett Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.	.948	
Bartlett's Test of Sphericity	Approx. Chi-Square	15867.105
	df	2080
	Sig.	.000

Table 1 shows Kaiser-Meyer-Olkin and Bartlett Test. The Bartlett test shows $p <.05$ and the Kaiser-Meyer-Olkin sampling sufficiency test shows a greater value than .6.¹⁵ Measurement of sampling adequacy can be referred to the anti-correlation image which indicating all measurements of acceptable sampling adequacy that have values greater than .5 except item C13 having a value of .494.

Communality

The communality values for items with values ranging from .480 to .852. Only items C7, C9, C12, C13 and E3 have a relatively low value with the respective values are .275, .491, .480, .545 and .379. Another item shows the contribution of the variance proportion for each item to a considerable factor that exceeds .6. This indicates that each item can explain the factors that were involved in this study.

Total Variance Explained

Table 2 also shows three levels of total variance explained. At the first stage, the factor is shown with its Eigen value, percentage of

contributed variance and its cumulative percentage. If referenced to the Eigen value, it is expected that there are 11 factors that can be extracted because it has an Eigen value greater than 1. If 11 factors are extracted, then the contribution of the variance is 71.786%. The next stage, after 11 factors were extracted, it is found that only 7 remaining factors have Eigen values greater than 1 and the percentage contribution of the variance decreased to 61.189%. The third column shows Eigen's contribution with percentage contribution similar to the second stage.

Table 2: Total Variance Explained

Factor	Total Variance Explained								
	Initial Eigen Value			Extraction Sums of Squared Loadings			Rotation Sum of Squared Loadings		
	Total	% variance	Cumulative %	Total	% Variance	Cumulative %	Total	% Variance	Cumulative %
1	29.308	45.090	45.090	29.003	44.620	44.620	9.063	13.943	13.943
2	3.078	4.735	49.825	2.823	4.343	48.962	6.784	10.437	24.380
3	2.948	4.535	54.360	2.672	4.110	53.073	5.781	8.894	33.273
4	1.972	3.034	57.394	1.680	2.585	55.658	5.391	8.294	41.567
5	1.665	2.561	59.955	1.320	2.031	57.688	5.144	7.913	49.480
6	1.501	2.310	62.265	1.194	1.837	59.525	3.259	5.013	54.494
7	1.457	2.242	64.506	1.081	1.664	61.189	2.536	3.901	58.395
8	1.300	2.000	66.507	.933	1.435	62.624	1.559	2.398	60.792
9	1.168	1.797	68.304	.790	1.216	63.840	1.247	1.918	62.711
10	1.155	1.777	70.081	.668	1.028	64.869	1.235	1.899	64.610
11	1.108	1.705	71.786	.625	.961	65.830	.793	1.219	65.830
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Scree Plot

Figure 1 shows the Scree Plot that graphically graphs Eigen values for each factor. The Scree Plot diagram shows that there are six or seven dominant factors in this study.

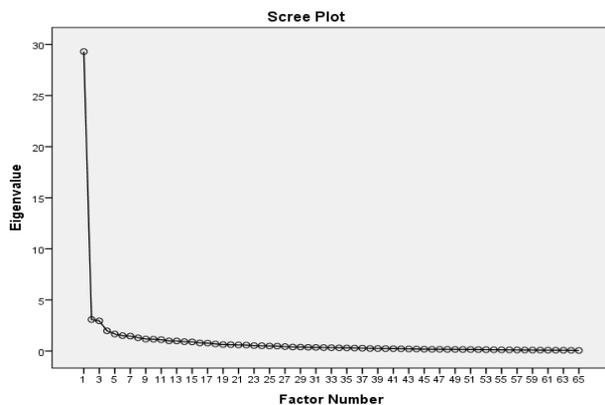


Figure 1: Scree Plot

Rotated Factor Matrix

The factor matrix shows how much a variable correlates with the factor to be formed. The complex variables are proposed using the Rotated Factor Matrix.¹⁴ as the factor extraction method has not produced a major component of the key factor. The purpose of this factor rotation is to obtain a simpler structural factor for easy interpretation. There were 11 factors with the factor loading factor for each item. However, it is found that factors 7, 8 9, 10 and 11 have only less than three items, so all these factors are dropped.^{14, 15} This finding reinforced the findings of the Scree Plot diagram showing that there are six or seven factors that can be taken into account in this study. However, only six factors are considered in this study with the loading factor for each item exceeding .5.¹⁵ The results showed that there are six main themes of how teachers can foster HOTS in teaching and learning of mathematics namely practice of implementing assessment for learning optimally, practice of determining the HOTS learning outcomes, practice of HOTS questioning strategies, practice of integrating ICT, practice

of active learning, and practice of developing habits of mind. Theme 1 has 16 items that composed of items come from the implementing optimally assessment for learning theme. The name of the theme is modified by adding the word practice as the word means the actual application or use of an idea, belief, or method as well as the customary, habitual, or expected procedure or way of doing of something. After being reviewed, items such as E6 and D7 are related to assessment strategies. As an example item E6, "Providing various methods for collecting information related to student learning outcomes in order to improve teaching techniques can be implemented" indicates that by providing a variety of methods for collecting information regarding learning outcomes is aimed for assessment purposes, which in line with the definition of assessment by William.¹⁶ Theme 2 also has seven items similar to the original item and no item has been added or dropped. The name of this theme is amended to "practice of determining HOTS learning outcomes", so that the theme is focusing on fostering HOTS. Theme 3 has six items and is given the name as "practice of HOTS questioning strategy". The name of the theme is changed so that the new theme is geared towards teacher's practice of using questioning skills to foster HOTS. The new items for this theme include the items from the original theme but 10 items were dropped due to low loading factor, which is less than .5. When reviewed, these six items are in line with recommendations by other researchers.¹⁷⁻¹⁹ They stressed the need to use a variety of questioning strategies or techniques. For example, Wetzel¹⁹ suggested that teachers are encouraged to ask further questions to foster HOTS, namely (1) What additional information is needed to solve the problem?, (2) How does the evidence support the conclusions made?, and (3) How the data obtained related to the findings? Theme 4 has six items related to ICT usage with amended new name as "practice of integrating ICT. No items were dropped but the order of the items changed according to the value of the loading factor respectively. Several researcher have demonstrated that the integrating of ICT in teaching and learning such as using blogs, wikis and web 2.0 supported inquiry thinking, creativity, critical reflection and discourse, which indicates the characteristic of HOTS²⁰⁻²¹

Theme 5 which named as "practice of active learning" has five items. A total of 11 items were dropped due to having a loading factor of less than .5. This item is in line with the Protheroe's recommendation that suggests some of the things that need to be done in mathematics to achieve an effective environment for HOTS, such as active involvement in learning mathematics.²² Education Alliance (EA)²³ encourages teaching activities characterized by student-centered learning and focus on teaching inquiry and problem solving such as conducting cooperative learning and scaffolding to create concepts, procedures and understanding. Miri et al.²⁴, as well as Staples and Truxaw²⁵ also emphasize that activity selection must be able to provide students with the opportunity to practice mathematical skills learned while they are thinking critically.

Theme 6 is "practice of developing habits of minds" which has only three items. Four items were dropped as the loading factor value was less than .5. The formation of mathematical habits is equally important in today's society in parallel with the encouragement to think deeply.²⁶ According to Gordon²⁶, when mind habits are raised in mathematical teaching, students can think more deeply and in a simpler way.

Overall, the six themes acquired have been renamed where all themes are starting with the word "Practice". This word is appropriate to illustrate that teachers are always performing activities that can nurture HOTS in the teaching and learning of mathematics continuously. Each theme is intertwined with each other and is always used throughout the teaching and learning process. GP-HOTS can be illustrated in Figure 2 below.

5. Conclusion

The result of the factor analysis has shown that there are six main themes for GP-HOTS; Theme 1- practice of implementing assessment for learning optimally, Theme 2 - practice of determining HOTS learning outcomes, Theme-3 practice of HOTS questioning strategies, Theme 4 - practice of integrating Information, Communication and Technology (ICT), Theme 5 - practice of active learning, and Theme 6 - practice of developing habits of mind. The total number of items for this guiding principle is 27 with each item composed of total items 16, 7, 6, 6, 5 and 3, respectively.

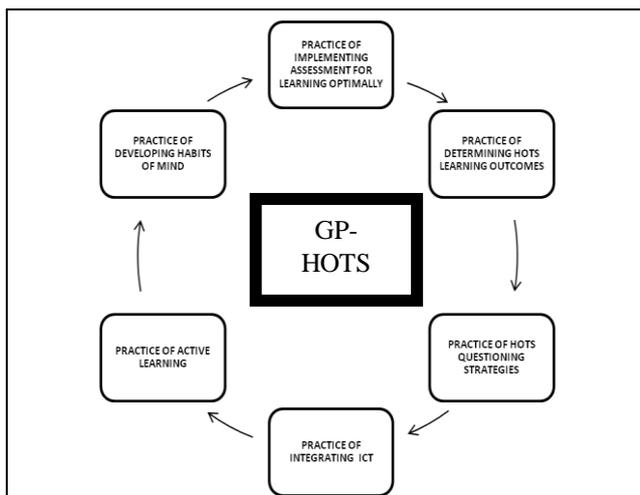


Figure 2: Guiding Principles of Fostering Higher Order Thinking Skills (GP-HOTS)

6. Recommendation

The developed GP-HOTS are very important in guiding all secondary school mathematics teachers in fostering students' HOTS during the mathematical teaching and learning processes. GP-HOTS emphasizes on how teachers can plan activities that can

foster the development of students' HOTS. Teachers are not required to use certain modules but can use this principle and diversify their teaching methods in developing students' HOTS according to the learning outcomes that will be achieved in the lesson. This study is in line with the aspirations of MEB 2013-2025 which emphasizes HOTS in teaching and learning of Mathematics, hence it is important in helping to improve the 21st century teacher pedagogy and produce futuristic-minded students. The developed guiding principle also adds pedagogical knowledge in line with the 21st century learning that emphasizes on thinking skills and problem solving (MOE, 2013). This knowledge contributes an idea to all educators, especially mathematic educators to constantly update the latest information on mathematical knowledge for teaching.²⁷ The Sultan Idris Education University (UPSI) which is the premier university teacher training centre that trains pre-service and in-service teachers can use this GP-HOTS as a fundamental aspect that should be employed by the teachers, especially in mathematics education. Therefore, UPSI can serve as a reference centre and contribute ideas to lecturers, teachers and other educators in providing guidance related to GP-HOTS to assist the Ministry of Education in achieving the transformation of education. In addition, this principle can be commercialized at all levels of primary and lower secondary education up to the level of higher education in the field of mathematics. As a result, schools are able to produce students with the mind and skills in line with the requirements of the job market. The GP-HOTS can also impact the development of national education in particular to achieve the aspiration of occupying one-third of TIMSS and PISA assessment tests over a period of 15 years.

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