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***Research paper***

# Aircraft pitch control tracking with sliding mode control

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**Abstract**

Sliding mode control (SMC) is one of the robust and nonlinear control methods. An aircraft flying at high angles of attack is considered nonlinear due to flow separations, which cause aerodynamic characteristics in the region to be nonlinear. This paper presents the comparative assessment for the flight control based on linear SMC and integral SMC implemented on the nonlinear longitudinal model of a fighter aircraft. The controller objective is to track the pitch angle and the pitch rate throughout the high angles of attack envelope. Numerical treatments are carried out on selected conditions and the controller performances are studied based on their transient responses. Obtained results show that both SMCs are applicable for high angles of attack.

***Keywords***: *F18-HARV; pitch control; sliding mode control.*

## 1. Introduction

Aircraft dynamics can be highly nonlinear when it is flying at high angles of attack. The necessity for an aircraft to fly at high angles of attack may be low but it may be encountered in some situations such as avoiding collision with terrains, flying during bad weather conditions, executing evasive manoeuvres, reducing landing distance or during other emergency situations. However, controlling a nonlinear flight can be challenging. The major hurdles in designing flight control system for a nonlinear model are the modelling of uncertainties and parameter variations that describe the characteristics of the aircraft itself. Traditionally, flight control systems are designed based on linear mathematical models at various flight conditions. A controller parameter is then varied with flight conditions in the gain scheduled approach [1]. However, a great amount of assessment is required to provide enough reference at the off-design point. On the other hand, nonlinear control technique such as dynamic inversion [2-3] has proved the robustness for a nonlinear aircraft model. In this paper, the implementation of the sliding mode control (SMC) on a nonlinear aircraft model is presented.

SMC is a nonlinear control that is insensitive to disturbance and model parameter variations. This characteristic makes the SMC a robust controller, which means that stability is guaranteed if there are changes in the dynamics model to some level of performance [4]. SMC entails two terms, which are full state feedback control (or equivalent control) and switching control. The equivalent control is said to be continuous that slide the dynamics on a selected surface by setting the error asymptotically approaching zero while selection of the latter control action will attract the switching surface to the system state by forcing the systems trajectories back on the surface. This control approach has been initiated by Emilyanov in the early 1950. Since then, it has been applied to various control problems including spacecraft attitude control [5], air-to-air missile control [6] and aircraft flight control. Though there are already works dealt with the SMC on aircraft controls with different set of objectives, the SMCs are either designed based on linearization of the aircraft model [7-8], implemented for lateral-directional motion of the aircraft [9] or designed as a reconfigurable control for a damage aircraft [10].

This paper attempts on applying a linear SMC to a nonlinear longitudinal model aircraft in tracking the pitch control. Further application of integral SMC on the same model is compared to the former controller to evaluate the performances of both controllers. This core work on SMC for a nonlinear aircraft model could be a promising solution towards flying at high angles of attack.

## 2. Control design

SMC is a discontinuous controller with high frequency in the control input to cope with the nonlinearity associated with the system while stabilizing it at the same time [11]. Design of the controller involves two phases: the first is to select a sliding surface and the second is to force the trajectory of the system towards the sliding surface in the reaching phase and maintain it for a subsequent of time. In order to implement SMC into the nonlinear aircraft model, the system is defined in the following equation:

$$\dot{x} = f(x) + Bu + d \tag{1}$$

where $x$ and $u$ denote the state vector and control vector while $f$ denotes the nonlinear function describing the state vector, $B$ is the control matrix and $d$ represents the uncertainties and external disturbances. A suitable selection for the sliding surface is required depending on the control objective.

### 2.1. Linear sliding surface

For linear SMC, the sliding surface is a function of tracking error in which, it is chosen such that:

$$s = \dot{e} + ke \tag{2}$$

where $e$ is defined as the difference between output measurement and set point and $k$ determine the slope of the sliding surface. The tracking error is given by:

$$e = x - x_d \quad (3)$$

The subscript d represents the demand value or set point. The time derivatives of the sliding surface in Eqn. 2 can be written as the following:

$$\dot{s} = \ddot{e} + k\dot{e} \quad (4)$$

Substituting $e$ into the Eqn. 4 gives that:

$$\dot{s} = (\ddot{x} - \ddot{x}_d) + k(\dot{x} - \dot{x}_d) \quad (5)$$

where $k$ is a positive gain determining the slope of the sliding surface. To satisfy the reachability condition, a candidate function based on the Lyapunov function given by:

$$V = \frac{1}{2}s^2 \quad (6)$$

is sought from its derivative which are given by:

$$\dot{V} = s\dot{s} \leq -\eta|s| \quad (7)$$

where constant $\eta \geq 0$. Then solving for the $s$ gives the following term:

$$\dot{s} \leq -\eta\, sgn\,(s) \quad (8)$$

This term is always associated with the switching control in SMC. From Eqn. 5 and Eqn. 8, the control, $u$, can be redefined by substituting the expression for $x$ from Eqn. 1, which is:

$$u = \frac{1}{kB}[-(\ddot{x} - \ddot{x}_d) + k(-f + \dot{x}_d)] - \eta\, sgn(s) \quad (9)$$

with the first part of the equation represent the equivalent control and the second part represent the switching control.

### 2.2. Integral sliding surface

Integral SMC is applied to eliminate the effect of the system initial error. It enforces a sliding mode to converge from the beginning of the system response. For integral SMC, an integral action is introduced by augmenting the system with an integrator as in the following equation:

$$s = \dot{e} + k_1 e + k_2 \int e\ dt \quad (10)$$

where $k_1$ and $k_2$ are positive gains. Applying the similar steps as in the linear SMC, the control input equation is given as:

$$u = \frac{-1}{k_1 B}[(\ddot{x} - \ddot{x}_d) + k_1(f - \dot{x}_d) + k_2(x - x_d)] - \eta\, sgn(s) \quad (11)$$

However, there is a major drawback in using discontinuous controller as the switching control. It caused chattering on the control input which is not recommended as it will tear the mechanical system of the control input due to the rise in oscillation. One method to reduce the chattering effect is by substituting the signum function with a continuous function such as saturation function. Thus, in this paper, the signum function in Eqn. 9 and Eqn. 11 then is replaced with the saturation function.

## 3. Application of SMC on aircraft dynamics

The aircraft model used in this work is based on the F-18 fighter aircraft [12]. The dynamic equations of the aircraft longitudinal motion adopted here are given by:

$$\dot{V} = \frac{1}{m}(-\bar{q}SC_D + 2T\cos\alpha\cos 1.98^o - mg\sin\theta\cos\alpha) \quad (12)$$

$$\dot{\alpha} = \frac{1}{mV}(-\bar{q}SC_L + 2T\sin\alpha\cos 1.98^o - mg\sin\theta\sin\alpha - mg\cos\theta\cos\alpha) \quad (13)$$

$$\dot{q} = \frac{1}{I_y}[\bar{q}S\bar{c}(C_M) + l_z(\bar{q}SC_L\sin\alpha - \bar{q}SC_D\cos\alpha) + l_x(\bar{q}SC_L\cos\alpha + \bar{q}SC_D\sin\alpha) + l_{ze}(2T\cos 1.98^0)] \quad (14)$$

$$\dot{\theta} = q \quad (15)$$

where $m$, $g$, $\bar{q}$, $S$, and $I_y$ are the mass, gravitational acceleration, dynamic pressure, wing area and inertia, respectively. These longitudinal dynamic motions can be written under nonlinear affine state representation as in previous Eqn. 1. The state vector is defined by $x = [V, \alpha, q, \theta]^T$ which represents the velocity, angle of attack, pitch rate and pitch angle. The control vector is determined by one input, which is $u = \delta_h$ or the stabilator deflection angle, in which the thrust is kept constant throughout the study. $C_D$, $C_L$, and $C_M$ denote the aerodynamic coefficients for drag, lift and moment. The aerodynamics coefficients are defined as follows [12]:

$$C_D = C_{D_0} + C_{D_{\delta_h}}\delta_h + \frac{\bar{c}}{2V}C_{D_q}q \quad (16)$$

$$C_L = C_{L_0} + C_{L_{\delta_h}}\delta_h + \frac{\bar{c}}{2V}C_{L_q}q \quad (17)$$

$$C_M = C_{M_0} + C_{M_{\delta_h}}\delta_h + \frac{\bar{c}}{2V}C_{M_q}q \quad (18)$$

Therefore, input control $\delta_h$ for the aircraft can be found by solving Eqn. 9 and Eqn. 11. The state variable $x$ is replaced by θ and the control scheme is illustrated as in Figure 1.

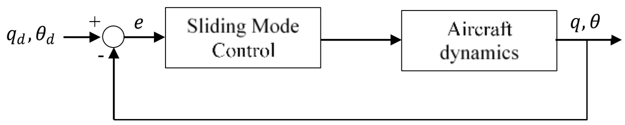


**Fig. 1:** SMC control scheme

## 4. Simulation study

The performance of the control law is evaluated by numerical treatments on the selected fighter aircraft model. The considered aircraft has the following general parameters as shown in Table 1.

**Table 1:** Aircraft parameters [12]

| Parameter | Value |
|---|---|
| Mass, $\boldsymbol{m}$ (kg) | 16224.63 |
| Wing span, $\boldsymbol{b}$ (m) | 11.41 |
| Wing area, $\boldsymbol{S}$(m$^2$) | 37.16 |
| Mean aerodynamic chord, $\bar{\boldsymbol{c}}$ (m) | 3.51 |
| Pitch inertia, $\boldsymbol{I_y}$ (kg.m$^2$) | 236246.3 |
| Total thrust, $\boldsymbol{T}$ (kN) | 14131 |

The simulation is performed from a trimmed condition of velocity, $V$ = 100 m/s, at an altitude of $h$ = 4572 m and angle of attack, $\alpha$ = 11.5°. Firstly, a ramp input is injected into the demand of pitch angle θ with an increment up to 20˚ and returned to its initial position again with another ramp input to simulate a climbing phase of the aircraft.

Figure 2 and Figure 3 show the effect of gain, $\eta$ to the pitch angle and pitch rate tracking for both the linear SMC and integral SMC respectively. Both linear SMC and integral SMC show the best signal response at $\eta = 0.1$. For $\eta > 0.1$, the response oscillates throughout the simulation time but still maintains to follow the demand signal. While at lower $\eta$, the response fails to follow the demand signal though the response is smoother compared to the former condition. However, though the signal is at its best for $\eta = 0.1$, the input signal from stabilator deflection $\delta_h$ has shown chattering effect from signum function, which is not acceptable as it causes mechanical tear on the actuator. Therefore, a further simulation study is done on the saturation function as shown in Figure

4 and Figure 5, in which the responses are smoother with less oscillations.

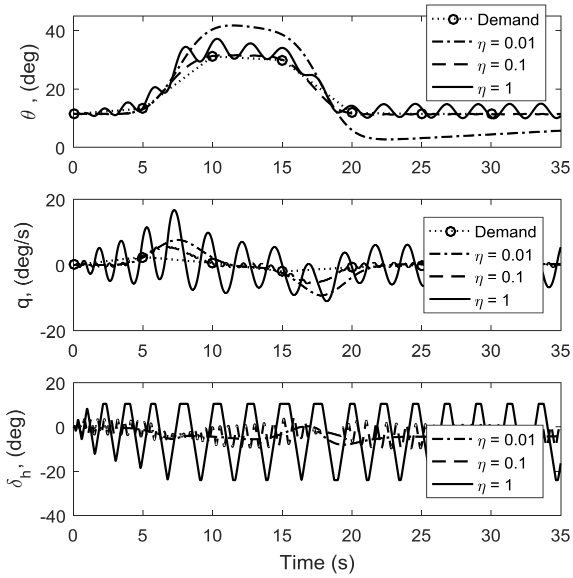


**Fig. 2:** Linear SMC at various $\eta$ using signum function.

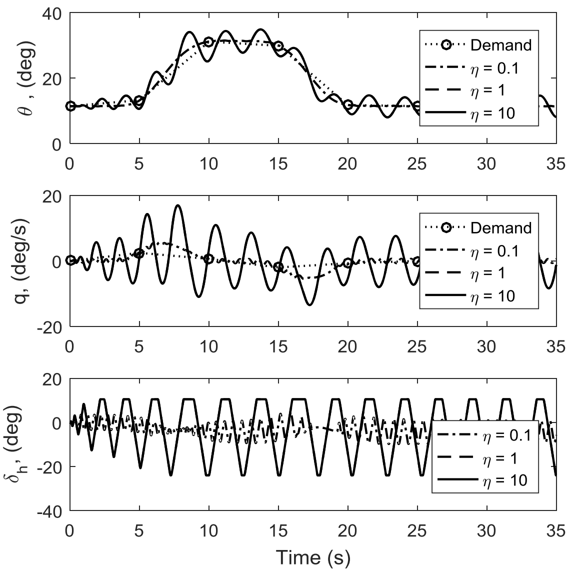


**Fig. 3:** Integral SMC at various $\eta$ using signum function.

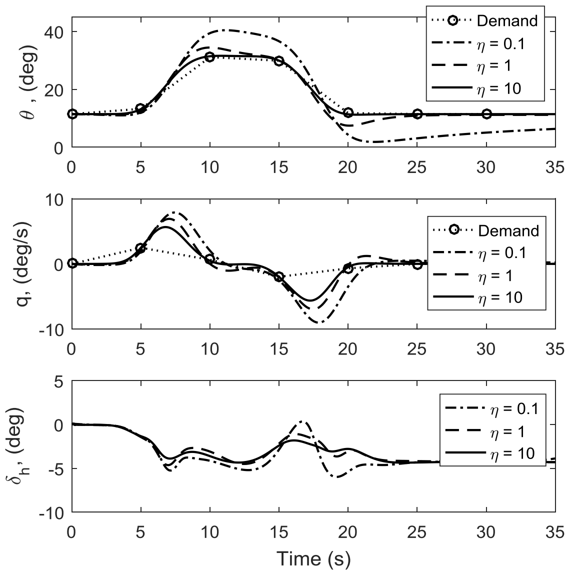


**Fig. 4:** Linear SMC at various $\eta$ using saturation function.

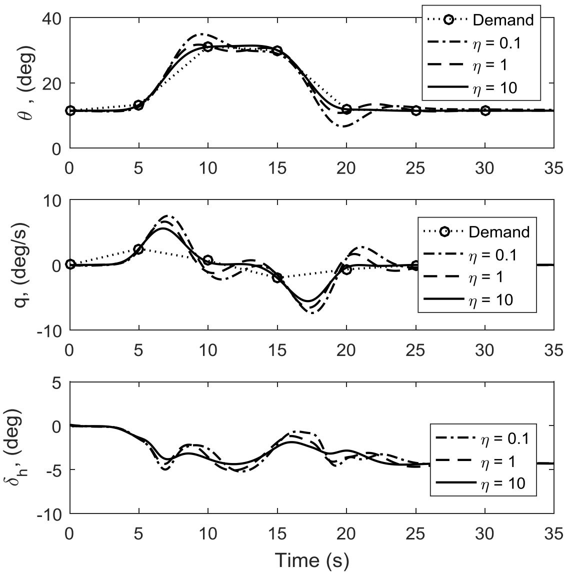


**Fig. 5:** Integral SMC at various $\eta$ using saturation function.

The objective of SMC is to force both the errors and its derivatives to zero so that the sliding surface is also tended to be zero in a finite time. The responses in Figure 4 and Figure 5 somehow show that at lower $\eta$, linear SMC displays some steady state errors that is not present with integral SMC. But the steady state errors with linear SMC are reduced as the $\eta$ is increased. Therefore, the tracking performance for the integral SMC shows that the controller manages to follow the demand value exactly regardless of the $\eta$ as it eliminates the steady state errors completely. This is supported in Figure 6 and Figure 7 where the tracking errors of each controllers based on the best $\eta$ is presented.

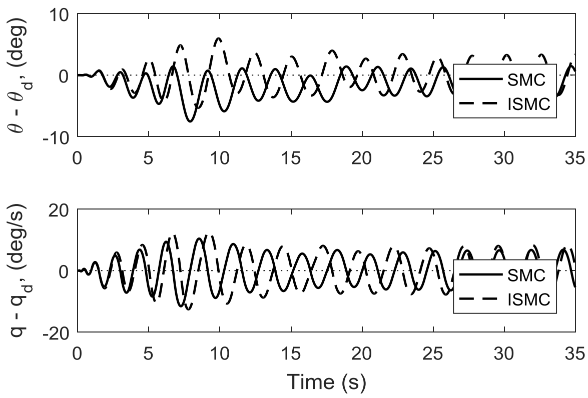


**Fig. 6:** Tracking error taken at $\eta = 0.1$ for both linear SMC and integral SMC based on signum function.

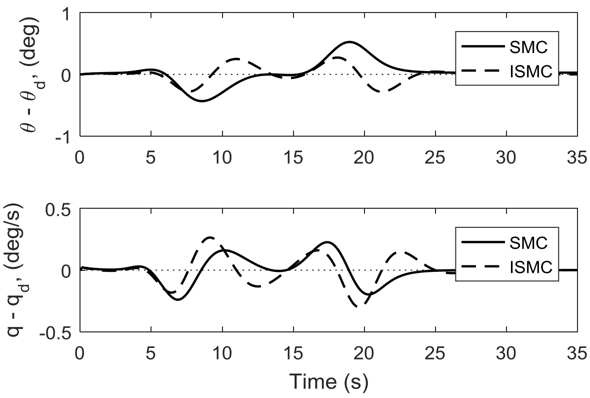


**Fig. 7:** Tracking error at $\eta = 0.1$ for both linear SMC and integral SMC based on saturation function.

To show that the aircraft is stabilized, a transient performance for the longitudinal state variables are generated as shown in Figure 8. Both linear SMC and integral SMC return the states to their equilibrium conditions though the controller input shows different performances. It can be seen as well that the aircraft model is an example of minimum phase system. When the stabilator is deflected to gain more climb angle, the aircraft suddenly lose the altitude for a short time before steadily gain the altitude to a higher level.

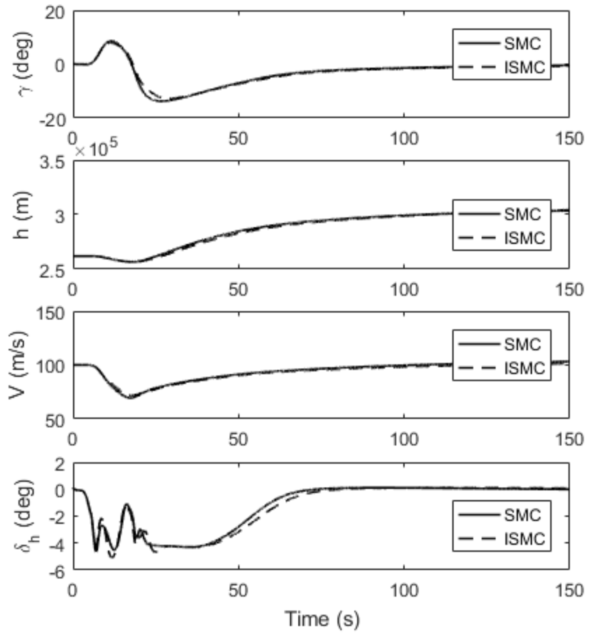


**Fig. 8:** Longitudinal states for both linear SMC and integral SMC with saturation function taken at $\eta = 1$.

## 5. Conclusion

In this paper, the implementation of linear SMC and integral SMC on a nonlinear longitudinal model of an aircraft is demonstrated. The performances of both controllers are compared and it has been shown that the integral SMC with a saturation function improves the performance with a smoother tracking of the reference value. It is also interesting to note that the performance for the linear SMC with a saturation function at higher $\eta$ is seen comparable to the performance of the integral SMC with a saturation function as well. Future works are expected to employ the SMCs on a full nonlinear state model into a cascade structure to demonstrate a global stability, i.e. multiple time scale, as SMC is not a recursive controller.

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