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Research paper

Integer Interval Value of Milne's Predictor and Milne's Corrector Method for First Order ODE

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Abstract

In this paper A new approaches to solve the approximate solution of the initial value problem for the first order ordinary differential equations and the solution can be used to compute y numerically specified the value of $[\underline{x} \quad \overline{x}]$ near to $[\underline{x}_0 \quad \overline{x}_0]$ in the interval analysis method and also used Milne's predictor and corrector for interval. In interval method gives a more accurate the approximate solution of life situation and numerical illustration are given

Keywords: *Interval analysis, Milnes predictor and corrector method, first order differential equation, ect..*

1. Introduction

Various scientific and engineering troubles are described in the form of ordinary differential Equations. If such action are unable to be solved analytically, we use computers And numerical methods to approximate them, usually performing all computations in Floating-point arithmetic. Presently there are a number of interval methods for approximating the initial value problem which consists in an ordinary differential equation and an initial value of the function that should be found. An interval method for ordinary differential equations using interval arithmetic was First described by R. E. Moore in 1965 . There are also interval methods based With The analytical methods of solution are consistent only to a selected class of first order ordinary differential equations. In general, the first order differential equations associated with the whole of various physical systems do not posses solution in closed construct and hence it intends be constrained to numerical techniques for solving a well known differential equation.
Found In this paper, we make a new way for the integer interval value of milne's predictor and corrector method derived.

2. Preliminaries: Interval Arithmetic

Let, $\tilde{c} = [c_1, c_2]$, $\tilde{d} = [d_1, d_2]$

(I). Addition

$$\tilde{c} + \tilde{d} = [c_1 + d_1, c_2 + d_2]$$

(II). Subtraction

$$\tilde{c} - \tilde{d} = [c_1 - d_2, c_2 - d_1]$$

(III). Multiplication

$$\tilde{c}.\tilde{d} = [\min(c_1d_1, c_1d_2, c_2d_1, c_2d_2), \max(c_1d_1, c_1d_2, c_2d_1, c_2d_2)]$$

(IV). Division

$$\frac{[i, j]}{[k, l]} = [i, j].[\frac{1}{l}, \frac{1}{k}] \text{ If } 0 \notin [k, l]$$

(V). $\mu\tilde{c} = [\mu c_1, \mu c_2]\, for \mu \geq 0$

$$[\mu c_2, \mu c_1]\, for \mu < 0$$

(VI). Inverse

$$[c_1, c_2]^{-1} = [\frac{1}{c_2}, \frac{1}{c_1}],\ for 0 \notin [c_1, c_2]$$

(VII). $[c_1, c_2]^n = [c_1{}^n, c_2{}^n], if c_1 \geq 0$

$$= [c_2{}^n, c_1{}^n], if c_2 < 0$$

$$= [0, \max\{c_1{}^n, c_2{}^n\}], otherwise$$

3. Proposed Method

Consider the interval integer initial value problem

$$\frac{d\tilde{y}}{d\tilde{x}} = \tilde{f}(x, y) \quad .... (1)$$

$$y(\tilde{x}_0) = \tilde{y}_0$$

Assume that

$$\tilde{y}_0 = y(\tilde{x}_0), \tilde{y}_1 = y(\tilde{x}_1), \tilde{y}_2 = y(\tilde{x}_2), \tilde{y}_3 = y(\tilde{x}_3)$$

where $\tilde{x}_{i+1} = \tilde{x}_i + \tilde{h}, i = 0,1,2,3$ are known .

These are the starting values

$$\frac{d\tilde{y}}{dx} = \tilde{f}(x, y) \Rightarrow \tilde{y} = \int \tilde{y} dx$$

By newtons forward interpolation formula

$$y(\tilde{x}_0 + p\tilde{h}) = \tilde{y}_0 + p\Delta\tilde{y}_0 + \frac{p(p-1)}{2!}\Delta^2\tilde{y}_0 + ..$$

$$\tilde{y}' = \tilde{y}_0 + p\Delta\tilde{y}'_0 + \frac{p(p-1)}{2!}\Delta^2\tilde{y}_0 +$$
$$\frac{p(p-1)(p-2)}{3!}\Delta^3\tilde{y}'_0$$

Integrating this between $\tilde{x}_0$ and $\tilde{x}_4$

$$\tilde{y}_4 = \tilde{y}_0 + \int_{\tilde{x}_0}^{\tilde{x}_4}(\tilde{y}'_0 + \frac{p(p-1)}{2!}\Delta^2\tilde{y}'_0)dx$$

Where $p = \frac{\tilde{x} - \tilde{x}_0}{\tilde{h}}$, $p\tilde{h} = \tilde{x} - \tilde{x}_0, dx = \tilde{h}dp$

$$\tilde{y}_4 = \tilde{y}_0 +$$
$$\tilde{h}\int_{[0 \quad 0]}^{[4 \quad 4]} [\tilde{y}'_0 + p\Delta\tilde{y}'_0 + \frac{p^2 - p}{2}\Delta^2\tilde{y}'_0 + \frac{p^3 - 3p^2 + 2p}{6}\Delta^3\tilde{y}'_0]dp$$

$$\tilde{y}_4 = \tilde{y}_0 + \tilde{h}[[4 \quad 4]\tilde{y}_0 + [8 \quad 8]\Delta\tilde{y}'_0$$
$$+ \frac{[20 \quad 20]}{[3 \quad 3]}\Delta^2\tilde{y}'_0 + \frac{[8 \quad 8]}{[3 \quad 3]}\Delta^3\tilde{y}'_0 + ..]$$

Neglecting fourth and higher order differences and expressing

$$\Delta\tilde{y}'_0 = \tilde{y}'_1 - \tilde{y}'_0$$
$$\Delta^2\tilde{y}_0 = \tilde{y}'_2 - 2\tilde{y}'_1 + \tilde{y}'_0$$
$$\Delta^3\tilde{y}_0 = \tilde{y}'_3 - 3\tilde{y}'_2 + 3\tilde{y}'_1 - \tilde{y}'_0$$

We have

$$\tilde{y}_4 = \tilde{y}_0 + \frac{[4 \quad 4]}{[3 \quad 3]}\tilde{h}[[2 \quad 2]\tilde{y}'_1 - \tilde{y}'_2 + 2\tilde{y}'_3]$$
$$+ \frac{[14 \quad 14]}{[45 \quad 45]}\tilde{h}\tilde{y}^V(\xi)$$

Where $\tilde{x}_0 < \xi < \tilde{x}_4$

In general

$$\tilde{y}_{n+1} = \tilde{y}_{n-3} + \frac{[4 \quad 4]}{[3 \quad 3]}$$
$$\tilde{h}[[2 \quad 2]\tilde{y}'_{n-2} - \tilde{y}'_{n-1} + 2\tilde{y}'_n]...(1)$$

And the error $= \frac{[14 \quad 14]}{[45 \quad 45]}\tilde{h}\tilde{y}^V(\xi)$

Where $\tilde{x}_{n-3} < \xi < \tilde{x}_{n+1}$

Expression (1) is called Milnes predictor formula.To obtain correct formula,consider

$$\int_{\tilde{x}_0}^{\tilde{x}_2}\tilde{y}'dx = \int_{\tilde{x}_0}^{\tilde{x}_2}(\tilde{y}'_0 + p\Delta\tilde{y}'_0 + \frac{p(p-1)}{2!}\Delta^2\tilde{y}'_0)dx$$

$$\tilde{y}_2 = \tilde{y}_0 + \tilde{h}\int_0^{[2]}[\tilde{y}'_0 + p\Delta\tilde{y}'_0 + \frac{p^2 - p}{2}\Delta^2\tilde{y}'_0 + \frac{p^3 - 3p^2 + 2p}{6}\Delta^3\tilde{y}'_0]dx$$

$$= \tilde{y}_0 + \frac{\tilde{h}}{[3 \quad 3]}[[\tilde{y}'_0 + [4 \quad 4]\tilde{y}'_1 + \tilde{y}'_2]$$
$$- \frac{\tilde{h}^V}{[90 \quad 90]}\Delta^4\tilde{y}'_0$$

It can be proved to be an error

$$= -\frac{\tilde{h}^V}{[90 \quad 90]}\tilde{y}^V(\xi)$$ Where $\tilde{x}_0 < \xi < \tilde{x}_2$

$$\tilde{y}_2 = \tilde{y}_0 + \frac{\tilde{h}}{[3 \quad 3]}[[\tilde{y}'_0 + [4 \quad 4]\tilde{y}'_1 + \tilde{y}'_2]$$
$$- \frac{\tilde{h}^V}{[90 \quad 90]}\tilde{y}^V(\xi)$$

In general

$$\tilde{y}_{n+1} = \tilde{y}_{n-1} + \frac{\tilde{h}}{[3 \quad 3]}[\tilde{y}'_{n-1} + [4 \quad 4]\tilde{y}'_n$$
$$+ \tilde{y}'_{n+1}]...(2)$$

And the error $= -\frac{\tilde{h}^V}{[90 \quad 90]}\tilde{y}^V(\xi)$

Where $\tilde{x}_{n-3} < \xi < \tilde{x}_{n+1}$

Expression (2) is called Milnes corrector formula

In particular, to compute $\tilde{y}_4$ corresponding to

$\tilde{x}_4 = \tilde{x}_0 + [4 \quad 4]\tilde{h}$ where $\tilde{h}$ is the step size

Milnes predictor formula

$$\tilde{y}_{4,p} = \tilde{y}_0 + \frac{[4 \quad 4]\tilde{h}}{[3 \quad 3]}[[2 \quad 2]\tilde{y}'_1 - \tilde{y}'_2$$
$$+ [2 \quad 2]\tilde{y}'_3]$$

Milnes corrector formula

$$\tilde{y}'_{4,c} = \tilde{y}_2 + \frac{\tilde{h}}{[3 \quad 3]}[\tilde{y}'_2 + [4 \quad 4]\tilde{y}'_3 + \tilde{y}'_4]$$

Where $\tilde{y}'_4 = \tilde{f}(\tilde{x}_4, \tilde{y}_{4,p})$

4. Numerical Illustration

Prproblems 1. Using the milne's method compute $y[0.6 \quad 1]$ given

$$\frac{dy}{dx} = [1 \quad 1] + [1 \quad 1]y^2, y[0 \quad 0] = [0 \quad 0],$$

that $y[0.1 \quad 0.3] = [0.1014 \quad 0.3041]$,

$y[0.3 \quad 0.5] = [0.2114 \quad 0.6342]$,

$y[0.5 \quad 0.7] = [0.3421 \quad 1.0262]$

Solution:

We have the following table of values

$\tilde{x}$	$\tilde{y}$	$\tilde{y}'=[1\quad 1]+\tilde{y}^2$
$[0\quad 0]$	$[0\quad 0]$	$[1\quad 1]$
$[0.1\quad 0.3]$	$[0.1014\quad 0.3041]$	$[1.0103\quad 1.0925]$
$[0.3\quad 0.5]$	$[0.2114\quad 0.6342]$	$[1.0447\quad 1.4022]$
$[0.5\quad 0.7]$	$[0.3421\quad 1.0262]$	$[1.1170\quad 2.0530]$

To find y(0.8)

$\tilde{x}_4=[0.7\quad 0.9]$.Here $\tilde{h}=[0.2\quad 0.2]$

By Milnes predictor formula

$$\tilde{y}_{4,p}=\tilde{y}_0+\frac{[4\quad 4]\tilde{h}}{[3\quad 3]}[[2\quad 2]\tilde{y}'_1-\tilde{y}'_2+[2\quad 2]\tilde{y}'_3]$$

$$=[0\quad 0]+\frac{[4\quad 4]}{[3\quad 3]}$$

$$[0.2\quad 0.2][[2\quad 2][1.0103\quad 1.0925]$$

$$-[1.0447\quad 1.408]+$$

$$[2\quad 2][1.1170\quad 2.0530]]$$

$$=\frac{[0.8\quad 0.8]}{[3\quad 3]}[2.8466\quad 5.2463]$$

$$=[0.7591\quad 1.3990]$$

By Milnes corrector formula

$$\tilde{y}'_4=[1\quad 1]+\tilde{y}_4^2$$

$$=[1\quad 1]+[0.7591\quad 1.3990]^2$$

$$=[1.5762\quad 2.9572]$$

$$\tilde{y}'_{4,c}=\tilde{y}_2+\frac{\tilde{h}}{[3\quad 3]}[\tilde{y}'_2+[4\quad 4]\tilde{y}'_3+\tilde{y}'_4]$$

$$\tilde{y}'_{4,c}=[0.2114\quad 0.6342]$$

$$+\frac{[0.2\quad 0.2]}{[3\quad 3]}[[.0447\quad 1.4022]$$

$$+[4\quad 4][1.1170\quad 2.0530]+$$

$$[1.5762\quad 2.9572]$$

$$=[0.2114\quad 0.6342]+[0.4728\quad 0.8385]$$

$$=[0.6842\quad 1.4727]=1.0785$$

Problems 2. Using the Milne's method compute $y[0.3\quad 0.5]$ given that

$$\frac{dy}{dx}=x^2-y, y[0\quad 0]=[1\quad 1],$$

$$y[0\quad 0.2]=[0.4526\quad 1.3578],$$

$$y[0.1\quad 0.3]=[0.4107\quad 1.2320],$$

$$y[0.2\quad 0.4]=[0.3746\quad 1.1238]$$

Solution:

$\tilde{x}$	$\tilde{y}$	$\tilde{y}'=\tilde{x}^2-\tilde{y}$
$[0\quad 0]$	$[1\quad 1]$	$-[1\quad 1]$
$[0\quad 0.2]$	$[0.4526\quad 1.3578]$	$-[0.4526\quad 1.3178]$
$[0.1\quad 0.3]$	$[0.4107\quad 01.2320]$	$-[0.4007\quad 1.1420]$
$[0.2\quad 0.4]$	$[0.3746\quad 1.1238]$	$-[0.3346\quad 0.9638]$

To find, $y[0.3\quad 0.5]$, $\tilde{h}=[0.1\quad 0.1]$

By Milnes predictor formula

$$\tilde{y}_{4,p}=\tilde{y}_0+\frac{[4\quad 4]\tilde{h}}{[3\quad 3]}[[2\quad 2]\tilde{y}'_1-\tilde{y}'_2+[2\quad 2]\tilde{y}'_3]$$

$$=[1\quad 1]+\frac{[4\quad 4]}{[3\quad 3]}$$

$$[0.1\quad 0.1][-[2\quad 2][0.4526\quad 1.3178]$$

$$+[0.4007\quad 1.1420]-[2\quad 2][0.3346\quad 0.9638]]$$

$$=[1\quad 1]+\frac{[0.4\quad 0.4]}{[3\quad 3]}$$

$$[-[1.5744\quad 4.5632]+[0.4007\quad 1.1420]]$$

$$=[1\quad 1]-\frac{[1.6650\quad 0.1730]}{[3\quad 3]}$$

$$=[0.9423\quad 0.4450]=0.6937$$

$$\tilde{y}'_4=\tilde{x}_4^2-\tilde{y}_4$$

$$=[0.3\quad 0.5]^2-[0.9423\quad 0.4450]$$

$$=-[0.3550\quad 0.6923]$$

$$\tilde{y}'_{4,c}=\tilde{y}_2+\frac{\tilde{h}}{[3\quad 3]}[\tilde{y}'_2+[4\quad 4]\tilde{y}'_3+\tilde{y}'_4]$$

$$=[0.4107\quad 1.2320]+\frac{[0.1\quad 0.1]}{[3\quad 3]}$$

$$[-[0.4007\quad 1.1420]-[4\quad 4][0.3346\quad 0.9638]$$

$$-[0.3550\quad 0.6923]]$$

$$=[0.4107\quad 1.2320]+$$

$$[0.0333\quad 0.0333][-[2.0934\quad 5.6895]]$$

$$=[0.2212\quad 1.1623]=0.6918$$

5. Conclusion

The proposed interval valued Milnes predictor and corrector method is simple to learn and understand and works effectively to obtain the solutions of real life situations. The examples presented in this paper evidently show that the interval predictor corrector Methods can be applied successfully. Comparing the results with other methods of the same or similar order, we see the high compatibility of the interval solutions obtained. In this method is either to slow in case of $\tilde{h}$ being small or too inaccurate, in case of $\tilde{h}$ is not small for real life situations.

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