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***Research paper***



# Solution of Nonlinear Coupled Burger and Linear Burgers Equation

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**Abstract**

This paper applies an analytical technique, called Homotopy Perturbation Method to determine various problems of partial differential equations and coupled Burger equations in one and two dimension equations. The final conclusion confirms that the HPM is eventually useful as well as resourceful in predicting the explanation of these kind of problems and it further concludes that HPM could be a broad utilization in modern engineering complication.

***Keywords***: *Homotopy Perturbation system; Coupled Burger equation; Partial differential equation.*

## 1. Introduction

This Linear or non–liner partial differential equation follows a different range of science. The board application like the particular equations is the fundamental reason how they have been considered unequivocally from many mathematicians and scientists across the globe. Unfortunately, sometimes there may be actual stuff to resolve either numerically or theoretically as there are so many methods to get comparative explanation of these type of equation. The HPM is capable in observing the comparative or analytic explanation of the liner and non–liner partial differential equation. The current decision defends the power, easily as well as simplicity of the system to implement. In this view we will illuminate the HPM represented by [1,2] as well as alternative algorithm of the HPM presented by Momani.

## 2. Homotopy Perturbation System

All Thorough analysis of the present method provides successive activity:

B(u)–g(r) = 0, r

along the boundary favor about

C(u, u/n ) = 0, r ,

position B is a general operant g(r) is an establish systematic activity, C is a boundary operant, along with is an impressive boundary like the domain .

The operant B may be generally divided within two operant, S and

T. Position S is linear while T is a nonlinear operant. Equation (1)

as it may be, thus recorded as success

S(u) +T(u)– g(r) = 0

Applying the homotopy method, personally formulate a homotopy

v(r, p): [0,1]R, and that satisfies

$H(v; p) = (1 – p)[S(v) – S(u_0)] + p[B(v)–g(r)] = 0$

or

$H(v; p) = S(v) – S(u_0) + pS(u_0) + pS(u_0) + p[T(v) – g(r)] = 0$

position p is called homotopy parameter along with $u_0$ is a fundamental comparative since the clarification from equation 1, satisfies the affecting boundary favour. Clearly, from the above result we obtain

$H(v; p) = S(v) – S(u_0) = 0$

H(v; 1) = B(v)–g(r) = 0

Along with developing system of p from zero to unity is just that of H(v, p) from $S(v) – S(u_0)$ to B(v) – g(r).

$S(v) – S(u_0)$ and B(v) –g(r) is called malformation and homotopy respectively

We can assume that affecting clarification of above equation as it may convey in the act of a series in p as follows

$V = v_0 + pv_1 + p^2v_2 + p^3v_3 +$

Using p = 1 results in the comparative clarification

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U = lim(p ➔1)v = $v_0 + v_1 + v_2 + v_3 + \ldots\ldots$

## 3. Problems

We have given some problem based on the above method

***Example 1:*** Suppose the coupled Burgers equation

$$u_s + u_{rr} - 2ur + (uv)_r = 0 \quad (1)$$

$$v_s + v_{rr} - 2vv_r + (uv)_r = 0 \quad (2)$$

the basic condition

$$U(r,0) = \sin r \ , v(r,0) = \sin r \quad (3)$$

The homotopy for equation of the above equation we get,

$$\frac{\partial u}{\partial s} = p[\frac{\partial u}{\partial s} + u_{rr} + 2uu_r - (uv)_r - D_s u] \quad (4)$$

$$\frac{\partial v}{\partial s} = p[\frac{\partial v}{\partial s} + v_{rr} + 2vv_r - (uv)_r - D_s v] \quad (5)$$

The explanation about equation (1) and (2) we get.

$$U = u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots. \quad (6)$$

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \ldots. \quad (7)$$

therefore, putting (6), (7) and the basic conditions (3) into the (4) and (5) and comparing the indication by like range of p. we get affecting coming usual of linear partial differential equations.

$$\frac{\partial u_0}{\partial s} = 0 \ , u_0 = \sin r$$

$$\frac{\partial v_0}{\partial s} = 0 \ , v_0 = \sin r$$

$$\frac{\partial u_1}{\partial s} = \frac{\partial u_0}{\partial s} + (u_0)_{rr} + 2u_0(u_0)_r - u_0(v_0)_r - v_0(u_0)_r - D_s u_0 \ , u_1(r,0) = 0$$

$$u_1 = -s\sin r$$

$$v_1 = -s\sin r$$

$$u_2 = \frac{s^2}{2!}\sin r$$

$$v_2 = \frac{s^2}{2!}\sin r$$

The homotopy perturbation solution for equation

$$u_0(r,s) = \sin r$$

$$v_0(r,s) = \sin r$$

$$u_1(r,s) = s\sin r$$

$$v_1(r,s) = s\sin r$$

$$u_2(r,s) = \frac{s^2}{2!}\sin r$$

$$v_2(r,s) = \frac{s^2}{2!}\sin r$$

$$u_3(r,s) = \frac{s^3}{3!}\sin r$$

$$v_3(r,s) = \frac{s^3}{3!}\sin r$$

:

In the continuous process of component, we get the n –terms approximate solution for equation (1), (2) we get.

$$U(r,s) = \sum_{i=0}^{n-1} u_i(r,s) = \sin r(1 - s + \frac{s^2}{2!} - \frac{s^3}{3!} + \ldots)$$

$$v(r,s) = \sum_{i=0}^{n-1} v_i(r,s) = \sin r(1 - s + \frac{s^2}{2!} - \frac{s^3}{3!} + \ldots)$$

Accurate solution for the one– dimensional coupled burger equations.

$$U(s, r) = \sin r\, e^{-s}$$

$$v(s, r) = \sin r\, e^{-s}$$

***Example 2:*** Consider the one-dimension linear burger equation

$$\phi_t + a\phi_x - \varepsilon\phi_{xx} = 0 \quad (1)$$

Basic case

$$\phi(x,0) = e^x \quad (2)$$

The homotopy for equation is

$$\frac{\partial \phi}{\partial t} = p[\frac{\partial \phi}{\partial t} - a\phi_x + \varepsilon\phi_{xx} - D_t\phi \quad (3)$$

The solution of equation can be written as a power series in p

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \cdots \quad (4)$$

Putting above and basic condition (2) into (3) and comparing the indication with exact range of p, we get the henceforth set of linear partial differential equations.

$$\frac{\partial \phi_0}{\partial t} = 0, \quad \phi(x,0) = e^x$$

$$\frac{\partial \phi_1}{\partial t} = \frac{\partial \phi_0}{\partial t} - a(\phi_0)_x + \in (\phi_0)_{xx} - D_t\phi_0, \quad \phi_1(x,0) = 0$$

$$\frac{\partial \phi_1}{\partial t} = -ae^x + \in e^x \ , \phi_1(x,0) = 0$$

$$\phi_1 = -tae^x + \in te^x \ ,$$

$$\frac{\partial \phi_2}{\partial t} = \frac{\partial \phi_1}{\partial t} - a(\phi_1)_x + \in (\phi_1)_{xx} - D_t\phi_1 \ , \quad \phi_2(x,0) = 0$$

$$\phi_2 = a^2\frac{t^2}{2}e^x - a \in e^x t^2 + \in^2 e^x\frac{t^2}{2}$$

:

$$\phi_0 = e^x$$

$$\phi_1 = -tae^x + \in te^x \ ,$$

$$\phi_2 = a^2\frac{t^2}{2}e^x - a \in e^x t^2 + \in^2 e^x\frac{t^2}{2}$$

$$\phi_3 = -a^3\frac{t^3}{3!}e^x - a^2 \in e^x\frac{t^3}{2} - a \in^2 e^x\frac{t^3}{2} + \in^3 e^x\frac{t^3}{3!}$$

Continuing this process, we get,

$$\Phi(x, t) = e^x + -tae^x + \in te^x + a^2\frac{t^2}{2}e^x - a \in e^x t^2$$
$$+ \in^2 e^x\frac{t^2}{2} - a^3\frac{t^3}{3!}e^x - a^2 \in e^x\frac{t^3}{2} - a \in^2 e^x\frac{t^3}{2} + \in^3 e^x\frac{t^3}{3!} + \ldots..$$

Hence

$\phi$(x,t) = $e^x e^{-t(a-\in)}$

This is Accurate solution for the one dimensional linear burger equations.

***Example 3:*** Suppose the two-dimension burger equation.

$\frac{\partial u}{\partial t}$ +u$\frac{\partial u}{\partial x}$ +v$\frac{\partial u}{\partial y}$ = [$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$] (1)

$\frac{\partial v}{\partial t}$ +u$\frac{\partial u}{\partial x}$ +v$\frac{\partial u}{\partial y}$ = [$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}$] (2)

Basic case

U(x, y, 0) = x + y; v(x, y, 0) = x – y (3)

The homotopy equation (1), (2) are.

$\frac{\partial u}{\partial t}$ = p[$\frac{\partial u}{\partial t}$–u$u_x$ –v$u_y$+$u_{xx}$+$u_{yy}$ − $D_t u$] (4)

$\frac{\partial v}{\partial t}$ = p[$\frac{\partial v}{\partial t}$–u$u_x$ –v$u_y$+$v_{xx}$+$v_{yy}$ − $D_t v$] (5)

The explanation of equation (1), (2) as power series in p

U = $u_0$ +p$u_1$ +$p^2u_2$ +$p^3u_3$ +…. (6)

v = $v_0$ +p$v_1$ +$p^2v_2$ +$p^3v_3$ +…. (7)

therefore, putting (6), (7) and the basic conditions (3) into the (4) and (5) and comparing the indication by exact range of p, we get the henceforth set of linear PDE

$\frac{\partial u_0}{\partial t}$ = 0, $u_0$ = x + y

$\frac{\partial v_0}{\partial t}$ = 0, $v_0$ = x – y

$\frac{\partial u_1}{\partial t}$ = [$\frac{\partial u_0}{\partial t}$ – $u_0(u_0)_x$ – $v_0(u_0)_y$ + $(u_0)_{xx}$ + $(u_0)_{yy}$ –$D_t u_0$];
$u_1$(x,y, 0) =0

$\frac{\partial v_1}{\partial t}$ = [$\frac{\partial v_0}{\partial t}$ $u_0(u_0)_x$ –$v_0(u_0)_y$ + $(v_0)_{xx}$ + $(v_0)_{yy}$ –$D_t v_0$];
$v_1$(x,y, 0) =0

$\frac{\partial u_1}{\partial t}$= – (x + y) (1) – (x – y) (1) + 0 + 0

$u_1$= –2xt

$v_1$= –2yt

$u_2$ = 2x$t^2$ +2y$t^2$

$v_2$ = 2x$t^2$ – 2y$t^2$

$u_0$= x + y

$u_1$= –2xt

$u_2$ = 2x$t^2$ + 2y$t^2$

$u_3$ = 4y$t^4$ – 4y$t^3$

:

$v_0$= x – y

$v_1$= –2yt

$v_2$ = 2x$t^2$ – 2y$t^2$

$v_3$ = 4x$t^4$– 4y$t^3$

:

Therefore, explication of the burger standard obtained by homotopy perturbation method

U(x, y, t) = $\sum_{i=0}^{n-1} u_i$(x,y,t)

= $u_0 + u_1$+$u_2$+…..

= x + y–2xt + 2x$t^2$ +2y$t^2$ + 4y$t^4$– 4x$t^3$ +…

= x(1 +2$t^2$ +4$t^4$ +8$t^6$ +…) –2xt(1+2$t^2$ +4$t^4$ +8$t^6$ …) +y(1 +2$t^2$+4$t^4$ +8$t^6$+…)

= (x + y – 2xt) (1 +2$t^2$+ 4$t^4$ +8$t^6$+…)

becomes a geometric progression

=$\frac{x+y-2xt}{1-2t^2}$

V(x,y, t) = $\sum_{i=1}^{n-1} v_i$(x,y,t)

V = $v_0 + v_1$+$v_2$+…..

= x–y–2yt+2x$t^2$– 2y$t^2$+4x$t^4$–4y$t^3$+…

= $\frac{x-y-2yt}{1-2t^2}$

This is the exact solution of two dimensional Burger's equation.

***Example 4:*** Suppose the one-dimension differential equation.

$u_t$ = $\lambda u_{xx}$

initial condition

U(x, 0) = $a^x$+bsin $h(c_1$x) +$c_2$ cos $h(c_3$x)

The homotopy for the above equation is

$\frac{\partial u}{\partial t}$ = p[$\frac{\partial u}{\partial t}$ +$\lambda u_{xx}$ − $D_t$u]

We will have the solution of given problem as power series in p

U = $u_0$ +p$u_1$ +$p^2u_2$ +$p^3u_3$ +….

Therefore, putting

$\frac{\partial u_0}{\partial t}$ = 0

$u_0$ = $a^x$+bsin $h(c_1$x) +$c_2$ cos $h(c_3$x)

$\frac{\partial u_1}{\partial t}$= [$\frac{\partial u_1}{\partial t}$ +$\lambda(u_0)_{xx}$ –$D_t u_1$)] ; $u_1$(x,0) = 0

= $\lambda(a^x(\log a)^2$+b$c_1{}^2$ sin $h(c_1$x) +$c_2 c^2{}_3$ cos $h(c_3$ x))

$u_1$= $\lambda t(a^x(\log a)^2$+b$c_1{}^2$ sin $h(c_1$x) +$c_2 c^2{}_3$ cos $h(c_3$ x))

$u_2$= $\lambda^2 \frac{t^2}{2!}(a^x(\log a)^4$+b$c_1{}^4$ sin $h(c_1$x) +$c_2 c^4{}_3$ cos $h(c_3$ x))

$u_3$= $\lambda^3 \frac{t^3}{3!}(a^x(\log a)^6$+b$c_1{}^6$ sin $h(c_1$x) +$c_2 c^6{}_3$ cos $h(c_3$ x))

:

Therefore, solution of above problem can be derived by Homotopy Perturbation Method

U(x, t) = $\sum_{i=1}^{n-1} u_i$(x,t) = $u_0 + u_1 + u_2 + u_3 + \ldots$

= $a^x$+bsin $h(c_1 x) + c_2 \cos h(c_3 x) + \lambda t(a^x (\log a)^2 + b c_1{}^2 \sin h(c_1 x) + c_2 c^2{}_3 \cos h(c_3 x)) + \lambda^2 \frac{t^2}{2!}(a^x (\log a)^4 + b c_1{}^4 \sin h(c_1 x) + c_2 c^4{}_3 \cos h(c_3 x)) + \ldots$

$a^x e^{\lambda t(\log a)^2}$ +bsin $h(c_1 x) e^{\lambda t c_1{}^2} + c_2 \cos h(c_3 x) e^{\lambda t c_3{}^2}$.

This is the accurate solution of one-dimension Heat equation.

***Example 5:*** Consider the one-dimension linear burger equation

$\phi_t$+a$\phi\phi_x - \varepsilon\phi_{xx}$ = 0 (1)

Basic case

$\phi$(x,0) =x (2)

The homotopy for equation (1) are

$\frac{\partial\phi}{\partial t}$= p[$\frac{\partial\phi}{\partial t}$ –a$\phi\phi_x + \varepsilon\phi_{xx} - D_t\phi$] (3)

The solution of equation (1) can be written as a power series in p

$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \cdots$ (4)

Putting (4) and basic condition (2) into (3) and comparing the indication with exact range of p, we get the henceforth set of linear partial differential equations.

$\frac{\partial\phi_0}{\partial t}$= 0, $\phi(x,0)$= x

$\frac{\partial\phi_1}{\partial t}$= $\frac{\partial\phi_0}{\partial t} - a\phi_0(\phi_0)_x + \in(\phi_0)_{xx} - D_t\phi_0$, $\phi_1(x,0) = 0$

$\frac{\partial\phi_1}{\partial t} = -a$x , $\phi_1(x,0) = 0$

$\phi_1 = -tax$

$\frac{\partial\phi_2}{\partial t}$ = $\frac{\partial\phi_1}{\partial t} - a\phi_0(\phi_1)_x - a\phi_1(\phi_0)_x + \in(\phi_1)_{xx} - D_t\phi_1$ , $\phi_2(x,0) = 0$

–a$(-atx - axt)$

$\phi_2$ =$2a^2\frac{t^2}{2}x$

$\phi_2$ =$a^2t^2x$

:

:

$\phi_0$ =x

$\phi_1 = -tax$

$\phi_2 = a^2t^2x$

$\phi_3$ =$-a^3t^3x$

Continuous in this process we get.

Φ(x, t) =x –$tax$+ $a^2t^2x$ $-a^3t^3x$ +……

Φ(x, t) = x(1–at +$a^2t^2$ $-a^3t^3$ +…)

Φ(x, t) = x$(1+at)^{-1}$

Φ(x, t) $\frac{x}{1+at}$

So accurate solution

H.P

***Example 6:*** Suppose the two-dimension burger equation.

$\frac{\partial\phi}{\partial t}$ +a $\phi\frac{\partial\phi}{\partial x}$ +a $\phi\frac{\partial\phi}{\partial y}$ = b [$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$] (1)

Basic case

$\phi$ (x, y, 0) = x + y (2)

The homotopy equation (1) is.

$\frac{\partial\phi}{\partial t}$ = p[$\frac{\partial\phi}{\partial t}$ -a$\phi\phi_x$ –a$\phi\phi_y$+b$\phi_{xx}$+b$\phi_{yy} - D_t\phi$] (3)

The explanation of equation (1), (2) as power series in p

$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \cdots$ (4)

Therefore, putting (4), and the basic conditions (2) into the (3) and comparing the indication by exact range of p. we get the henceforth set of linear PDE.

$\frac{\partial\phi_0}{\partial t}$ = 0 ,$\phi_0$ = x+y (5)

$\frac{\partial\phi_1}{\partial t}$ = [$\frac{\partial\phi_0}{\partial t} - a\phi_0(\phi_0)_x - a\phi_0(\phi_0)_y + b(\phi_0)_{xx} + b(\phi_0)_{yy} - D_t\phi_0$]; $\phi_1$(x,y,0) =0 (6)

$\frac{\partial\phi_1}{\partial t}$= –a(x+y)(1) – a(x+y)(1) +0+0 (7)

$\phi_1$= –2a(x+y)t

$\frac{\partial\phi_2}{\partial t}$ = [$\frac{\partial\phi_1}{\partial t} - a\phi_0(\phi_1)_x - a\phi_1(\phi_0)_x - a\phi_1(\phi_0)_y - a\phi_0(\phi_1)_y + b(\phi_0)_{xx} + b(\phi_0)_{yy} - D_t\phi_1$], $\phi_2$(x,y,0)=0 (8)

$\frac{\partial\phi_2}{\partial t}$ = –a(x+y) (–2at) –(–2at(x+y))1 –a(x+y)(–2at –a(–2at(x+y) (9)

$\phi_2$ = $4a^2t^2$ (x+y) (10)

$\phi_0$= x + y

$\phi_1$= –2a(x+y)t

$\phi_2$ = $4a^2$(x+y)$t^2$

:

:

Therefore, explication of the burger standard obtained by homotopy perturbation method

$\phi$ (x, y, t) = $\sum_{i=0}^{n-1}\phi_i$(x,y,t)

$=\phi_0 + \phi_1+\phi_2+\ldots..$

$= x + y – 2a(x + y)t +4a^2(x+y)t^2–\ldots\ldots$

$=x + y(1–2at +4a^2t^2– \ldots\ldots\ldots)$ is become a GP series so

$=\frac{x+y}{1-2at}$

This is the exact solution of two dimensional Burger's models

## 4. Conclusion

The altered homotopy perturbation system is advised in view of this paper is an efficient system since it realizes the accurate clarification like the coupled Burgers equation, linear Burgers equation and the Heat equation. So, this system is capable mathematical application to solve any problem of partial differential equation

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