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Research paper

Ohmic Heating Effect on Magneto Hydrodynamic Marangoni Mixed Convection Boundary Layer Nanofluid Flow

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Abstract

In surface driven flows, dissipative layers which occur along the surface of two immiscible fluids are known as marangoni boundary layers. Mixed connection takes place when buoyancy forces act beside marangoni effect. Consider a nanofluid flow along a flat surface experiencing marangoni convection with ohmic dissipation and magnetic field. Copper and Alumina are the nanoparticles with water as base fluid. The similarity equations are solved numerically by BVP solver 'bcp4c". The flow characteristics are analyzed graphically and discussed.

Keywords: *Heat transfer; Magneto hydrodynamics; Marangoni convection; Nanofluid; Ohmic heat*

1. Introduction

Study of nanofluids ignites interest due to its extensive usage in electronics, medicine, optical devices etc. The fluids formed by suspending the metallic nano sized particles, whose diameter is less than 100nm, in a base fluid are termed as nanofluids [1]. Later an extensive work on convective nanofluids has been done by many researchers [2]-[7]. The imbalance of surface tension gradients instigates marangoni convection. This convection is widely used in the areas of crystal growth, welding and semiconductor processing. An experimental work is carried out on marangoni convection [8]. Heat transfer effect with marangoni mixed convection is discussed in [9]. Double diffusion effects are coined by [10] in magneto hydrodynamics (MHD) marangoni convection flow over porous plate. It is found that Soret number significantly influences temperature and Sherwood number. Later a focus is put on MHD nanofluid flow subjected marangoni convection over a surface [11]. Copper species possess high heat and mass transfer rates compared to silver nanoparticles. Hydrodynamic nanofluid flow through a cone is examined by [12]. Marangoni convective flow with suction has been carried out by [13]. A rise in temperature exists when an electric current passes through a material, known as ohmic heat. This has a wide application in the process of etiolating, evaporation, fermentation of foods which will be used in long duration space missions. Ohmic heating explores the footprint of magnetic field over thermal transport. Ohmic heating consequences on mixed convection flow are addressed in [14]. Further [15] explored the significance of ohmic heating effect on momentum and heat transfer. [16] examined the reaction of Nusselt number over volume fraction and mixed convection .the characteristics are disused for both buoyancy –opposed and buoyancy favourable marangoni flows. A review on ohmic heating is addressed by [17].

Earlier researchers studied the problems with marangoni mixed convection in nanofluids without considering the experimental correlations addressed by [18] and ohmic heating effect. So this paper analyzes the consequences of ohmic heat on heat transfer and flow velocity and temperature profiles. The results are validated with [9]

2. Mathematical Formulation

Consider a two dimensional thermo capillary mixed convection boundary layer nanofluid flow along a surface. Assume the interface S is subjected to an external pressure gradient causing buoyancy effects. Fluid and species are assumed to be in thermal equilibrium with no – slip. Local co-ordinates axes are taken as x' and y'. A horizontal gravitational force g' is set with the interface which ignores the curvature of the interface. Further it is also assumed the fluid is non – viscous and incompressible. According to [19], coupled equations of motion are stated as follows

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}= u_e'\frac{du_e'}{dx'}+\nu_{nf}\frac{\partial^2 u'}{\partial y'^2}-\frac{\gamma B(x')}{\rho_{nf}}(u'-u_e')-\tau g\beta(t'-t_m) \tag{2}$$

$$u'\frac{\partial t'}{\partial x'}+v'\frac{\partial t'}{\partial y'}=\alpha_{nf}\frac{\partial^2 t'}{\partial y'^2}+\frac{\gamma B(x')}{(\rho c_p)_{nf}}(u'-u_e')^2 \tag{3}$$

Together with boundary conditions

$$v'=0, t'=t'_s(x'), \frac{\mu_{nf}}{\mu_f}\frac{\partial u'}{\partial y'}=\frac{\partial t'}{\partial x'} \text{ at } y'=0$$

And

$$u'\to u'_e(x'), t'\to t_m \text{ as } y'\to\infty \tag{4}$$

where (u',v') denote fluid velocities along (x') and normal (y') to the interface. u'_e is free stream velocity, μ,β are dynamic viscosity, thermal expansion coefficient. A parameter τ assumes (+1) in case buoyancy forces are favourable and (-1) in case of buoyancy forces are opposing to marangoni flow. Further $\sigma_{t'}=-\frac{d\sigma}{dt'}>0$, arrives from the Boussinesq approximation

$$\sigma=\sigma_m-\sigma_{t'}(t'-t_m) \tag{5}$$

σ_m is the surface tension with respect to reference temperature $t_{m'}$

Let us consider the following dimensionless quantities

$$x=\frac{x'-l_0}{l}, y=\frac{\mathrm{Re}^{1/3}y'}{l}, u=\frac{u'}{u_r}, v=\frac{\mathrm{Re}^{1/3}v'}{u_r}, u_e(x)=\frac{u'_e(x')}{u_r}$$
$$u_r=\mathrm{Re}^{-1/3}u_m, t=\frac{t'-t_m}{\Delta t}, u_m=\frac{\sigma_{t'}\Delta t}{\mu_f}, \mathrm{Re}=\frac{u_m l}{\nu_f} \tag{6}$$

where the quantities Re, u_r, l_0, l connote Reynolds number, reference velocity associated with marangoni velocity u_m, location of origin, extension of the relevant interface S. Also according to [18] nanofluid effective quantities density, dynamic viscosity, thermal conductivity and heat capacitance are assumed to be

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \mu_{nf}=\Phi_1\mu_f, k_{nf}=\Phi_2 k_f$$
$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s \tag{7}$$

Where

$$\Phi_1=\begin{cases}1+39.11\phi+533.9\phi^2\ (\text{for } Al_2O_3)\\ \frac{1}{(1-\phi)^{2.5}}\ (\text{for } Cu)\end{cases} \quad \text{and}$$

$$\Phi_2=\begin{cases}1+7.47\phi\ (\text{for } Al_2O_3)\\ \frac{k_s+2k_f-2\phi(k_f-k_s)(1+R)^3}{k_s+2k_f+2\phi(k_f-k_s)(1+R)^3}\ (\text{for } Cu)\end{cases}$$

The ratio of nano layer thickness to nano particle radius is $R=0.1$. Equation (6) transforms equations (1) – (3) as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{1}{\phi_1}\left(\Phi_1\frac{\partial^2 u}{\partial y^2}-\frac{\gamma B(x)l}{\rho_f u_r}(u-u_e)\right)-\tau\lambda t \tag{9}$$

$$\Pr\phi_2\left(u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}\right)=\Phi_2\frac{\partial^2 t}{\partial y^2}+\frac{\gamma B(x)l}{\rho_f u_r}\Pr Ec(u-u_e)^2 \tag{10}$$

Further the boundary conditions (4) turn to

$$v=0, t=t_s(x), \Phi_1\frac{\partial u}{\partial y}=\frac{\partial t}{\partial x} \text{ at } y=0\ ;\ u\to u_e(x), t\to 0 \text{ as } y\to\infty \tag{11}$$

With $\phi_1=1-\phi+\phi\frac{\rho_s}{\rho_f}$ and $\phi_2=1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}$.

Assume the similarity solution [9]

$$u(x,y)=u_0x^m f'(\eta), \eta=x^p\frac{y}{l_0}, u_e(x)=u_0x^m, B(x)=(B_0x)^{m-p}$$
$$v(x,y)=-u_0l_0x^{m-p-1}\left[(m-p)f(\eta)+p\eta f'(\eta)\right], t=-t_0x^n\theta(\eta) \tag{12}$$

The constants m,n,p and constant scale factors u_0,l_0,t_0 are found upon substitution of (12) into the equations (8) – (11). The similarity solution endures for $m=3, n=5, p=1$. The coupled dimensionless non-linear ordinary differential equations for the flow can be obtained by choosing suitable scale factors [19] as follows

$$\frac{\Phi_1}{\phi_1}f'''+\frac{3}{2}\left(1-f'^2\right)+ff''+\tau\lambda\theta+\frac{H(1-f')}{\phi_1}=0 \tag{13}$$

$$\Phi_2\theta''+\phi_2\Pr\left(f\theta'-\frac{5}{2}f'\theta\right)-\frac{\chi(1-f')^2}{2}=0 \tag{14}$$

along with the transformed boundary conditions

$$f(0)=0, \Phi_1 f''(0)=-1, \theta(0)=1;\ f'(\infty)=1, \theta(\infty)=0 \tag{15}$$

where $\chi=\frac{MEc\ u_0x}{t_0}, H=\frac{\gamma B_0{}^2 l}{\rho_f u_r}, \lambda=\frac{g\beta\Delta_t}{u_r{}^2}$ represent ohmic heating parameter, magnetic number and marangoni mixed convection parameter respectively. Further σ^* and k^* respectively represent Steffen – Boltzmann constant and mean absorption coefficient. The dimensionless Nusselt number, measure of heat transfer rate, is obtained as $-\theta'(0)$

3.4. Results and Discussion

The set of non – liner differential equation (13) and (14) with boundary conditions (15) are solved numerically through BVP solver *bvp4c*, a residual control based numerical method [20]. Using the notations, $f_1=f, f_2=f', f_3=f'', f_4=\theta, f_5=\theta'$. Equation (13) and (14) are first transformed to a set of first order differential equations. For this set of equation an iterative fourth order approximate solution S (x) to y (x) is obtained in such a way that $\|y(x)-S(x)\|\le h^4$ where h is maximum of step sizes $h_n=x_{n+1}-x_n$. Secondly, an initial guess will be given using a specific subroutine *'bvpinit'*. Thirdly, we use *'solinit'* to compute the solution in the interval [0, 10]. Both relative and absolute error tolerances are taken up to 10^{-10}. The numerical results are validated through the numerical results obtained by [9]. The problem of marangoni mixed convection along with ohmic heating is considered. We consider both buoyancy – favourable marangoni flow $(\tau=-1)$ and buoyancy – opposed marangoni flow. To compare earlier work, we consider the range of particle volume fraction $0\le\phi\le 0.2$ along with the physical properties of nanoparticles [18]. The Prandtl number for water $\Pr=6.2$ is taken. Table 1 shows, the validity of existing values with present work.

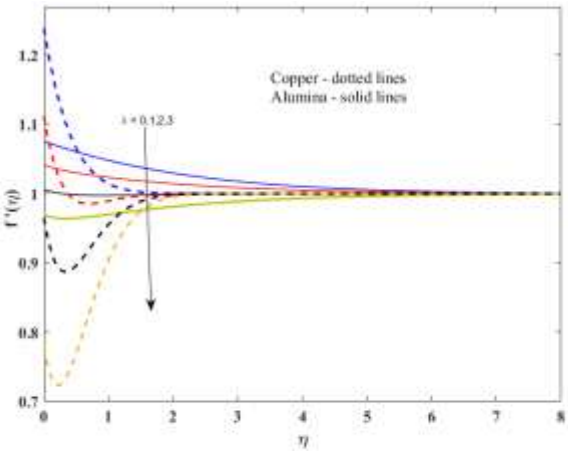

Fig. 1: Velocity against marangoni number for $H = \chi = 0, \phi = 0.2, \tau = -1$

Table 1: Comparison of $f'(0)$ and $\theta'(0)$ with earlier work for $\lambda = 1$ when $u_e \frac{du_e}{dx} = 0, \tau = -1$

Pr	$f'(0)$ [9]	$f'(0)$ (present)	$\theta'(0)$ [9]	$\theta'(0)$ (present)
2.8	0.6993	0.69926	2.090	2.0918
3.0	0.7122	0.71218	2.2095	2.20916
5.0	0.7689	0.76913	3.1119	3.11243

Fig. 1 and Fig. 2 depict the impact of marangoni mixed convection on boundary layers in a buoyancy – favourable flow. Velocity decreases with an increase in convection parameter. .Both copper and alumina species exhibits same pattern. But momentum boundary layer thickness abruptly decreases in copper species. Further temperature rises with convection parameter.

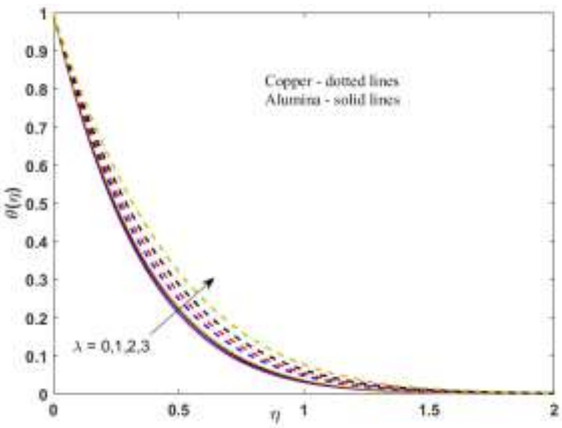

Fig. 2: Temperature against marangoni number ($H = \chi = 0, \phi = 0.2, \tau = -1$)

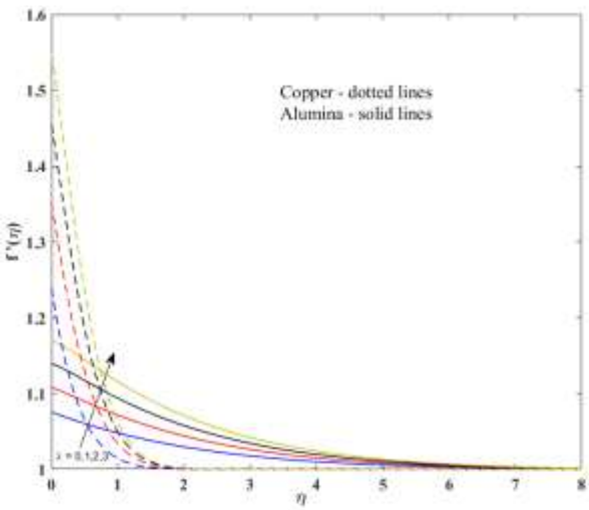

Fig. 3: Velocity against marangoni number for $H = \chi = 0, \phi = 0.2, \tau = 1$

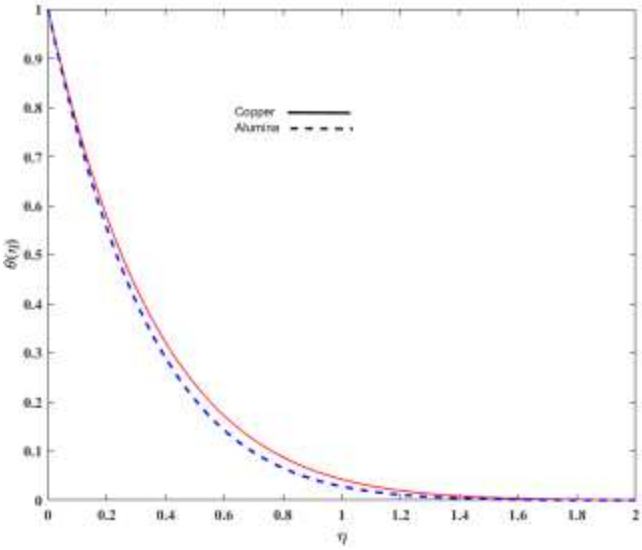

Fig. 4: Temperature against marangoni number ($H = \chi = 0, \phi = 0.2, \tau = 1$)

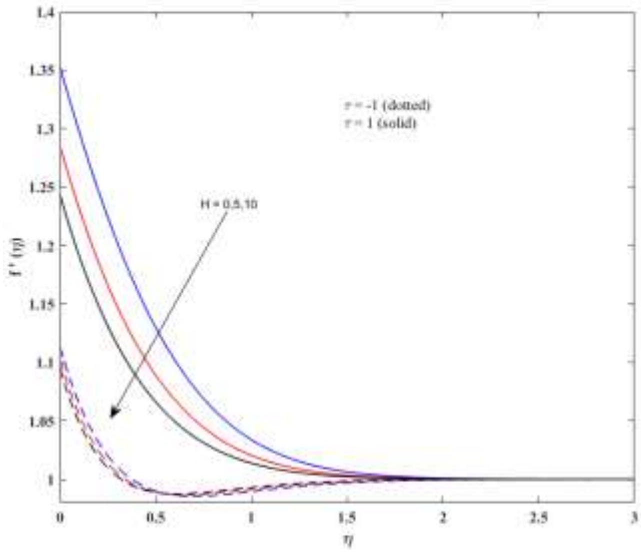

Fig. 5: Velocity against Hartmann number in copper nanofluid $\lambda = 1, \chi = 0, \phi = 0.2$

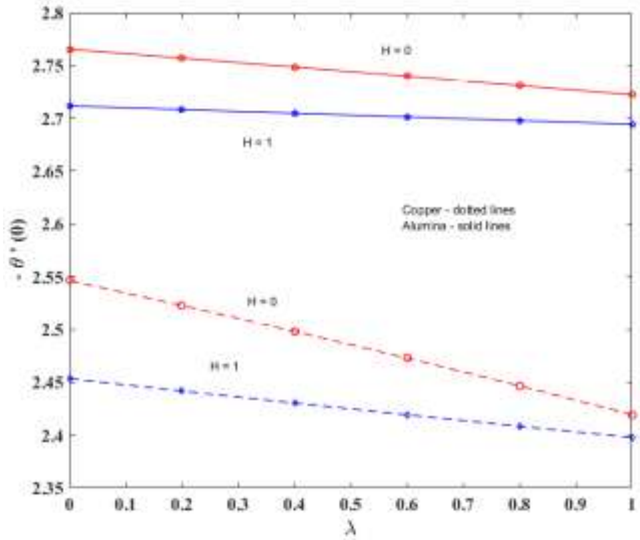

Fig. 6: Heat transfer against marangoni number for $\chi = 0.2, \phi = 0.2, \tau = -1$

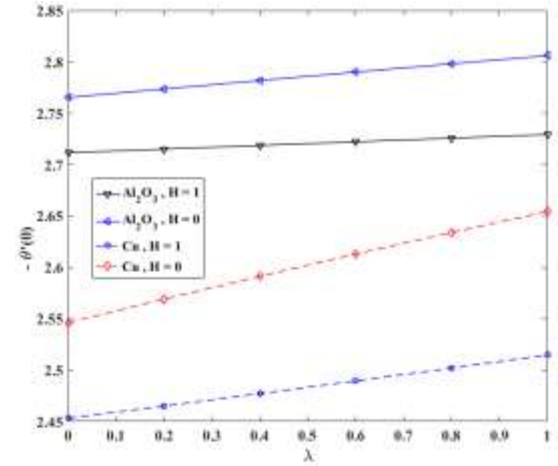

Fig. 7: Heat transfer against marangoni number for $\chi = 0.2, \phi = 0.2, \tau = 1$

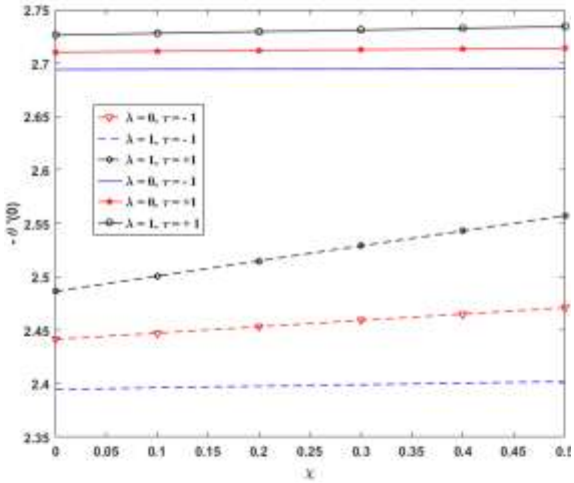

Fig. 8: Heat transfer with ohmic dissipation $H = 1\chi = 0.2, \phi = 0.2$

Fig. 3 witnesses the effect of mixed convection on velocity. It shows that the velocity increases with increasing convection parameter. Fig. 4 addresses the temperature profiles for both Copper and Alumina nanoparticles in presence of buoyancy – opposing marangoni flow without magnetic field. Temperature distribution in Alumina is lower than that of Copper particles. Fig. 5 depicts the effects of velocity across Hartmann number. It is found that velocity descent occurs with increase in magnetic number in buoyancy – opposing marangoni flow. It is quite interesting to observe the phenomena in buoyancy – favourable case. It is further near the wall $\eta \le 0.6$, the velocity decreases with increase of Hartmann number and away from the wall that is $\eta > 0.6$, flow reversal takes place. The magnetic effect produces a drag force which sets velocity lower. But in buoyancy – favourable marangoni favourable flow marangoni convection nullifies the impact of this drag force on the flow, away from the wall. From fig. 6 and fig. 7, it is observed that in case of buoyancy–favourable flow, increase in mixed convection parameter lowers the heat transfer rates. But this trend reverses in buoyancy – opposing flow. Copper species have low heat transfer rate compared to Alumina species. Further due to restoring force generated by magnetic field lowers the heat transfer rate. The effect of ohmic heat on heat transfer is witnessed in fig. 8. A steady growth in Nusselt number is found with ohmic dissipation parameter. This growth is significant in copper species in case of buoyancy – opposing marangoni flow.

3. Conclusion

The problem of marangoni mixed convection parameter with ohmic heating is considered. A nanofluid flow with uniform magnetic field is considered. Buoyancy – favourable and buoyancy – opposing marangoni flow cases are dealt. It is observed that in favourable case, increasing convection reduces velocity and enhances temperature profiles. The reversal takes place in opposing in opposing flow case. Ohmic heating phenomenon enhances heat transfer in both nanofluids.

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