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Research paper

Effect of Gravity Modulation on Single Component Convection in a Couple Stress Fluid with Maxwell-Cattaneo Law

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Abstract

The outset of convection in a thin layer of couple stress fluid is analyzed using the linear stability analysis when the fluid is heated from below. In order to assimilate the inertial effects Maxwell-Cattaneo law is used in lieu of the classical Fourier's heat conduction law. The normal mode analysis is used to arrive at the eigenvalues of the perturbed state and a regular perturbation method to find the analytical solutions. The effect of Cattaneo number, couple stress parameter and Prandtl number is discussed and it is concluded that gravity modulation can delay or advance the onset of convection.

Keywords: *Couple stress fluid; Gravity modulation; Maxwell-Cattaneo law; Rayleigh-Bénard convection;*

1. Introduction

The best-known model for heat transfer by diffusion is Fourier's law. However Fourier's model does not take into account inertial effects, i.e., if there is a sudden perturbation in the temperature field, it predicts that the perturbation will be felt instantaneously and everywhere. It is also inadequate for describing phenomena that involve heat transport at very high frequencies such as scattering of light in gases and that of neutron in liquids, ultrasound propagation, heat transfer at low temperatures or when the relaxation time of the fluxes is longer as in the case of polymer solutions and suspensions, super-fluids and superconductors [4]. To circumvent these anomalies, Cattaneo [3] proposed a concept of wave nature of heat propagation by introducing a heat flux relaxation term which enables the transfer of heat by thermal waves with finite speed. Straughan [10] and Bissell [2] considered the problem of thermal convection with Maxwell-Cattaneo law and have shown that thermal relaxation time has profound effect on thermal convection.

Stokes [9] proposed a theory of micro-continuum fluid which characterizes the field of rotation in terms of the velocity field. It is relatively mathematically simple compared to many other specialised theories of complex fluids. This has encouraged researchers to apply the couple stress fluid model to understand the theoretical implications of flow problems in fluid mechanics. Pranesh and Sangeetha [7] analysed dielectric couple stress fluid under the influence of temperature modulation. Shivakumara and Naveen [8] studied convection in a couple stress fluid layer with three solvents.

The study of the effect of time-periodic body force has attracted a number of investigators because of the need to understand, model and analyse fluid mechanism under microgravity conditions on spacecraft. When a system with density gradient undergoes fluctuations or vibrations, it results in body forces which will have a significant impact on the flow. It has been shown that gravity modulation stabilizes a Newtonian fluid layer heated from below and there has been a growing interest in understanding the stability of such flows in various fluids and under various systems([11]). Bhadauria et al. [1] studied couple stress fluid under modulated convection with a solute. Kiran [5] investigated Darcy convection with vertical through-flow and gravity modulation. The aim of the present work is to analyse the impact of modulation in gravity on convection in a couple stress by employing Maxwell-Cattaneo law.

2. Mathematical Formulation

Consider a physical configuration as in Figure 1. The system is subjected to a periodically varying vertical gravity field, given by

$$\vec{g}\,(t) = -g_0\,(1 + \varepsilon\cos(\omega t))\,\hat{k} \tag{1}$$

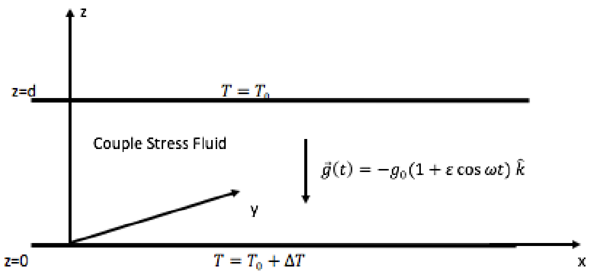

Fig. 1: Schematic representation of the problem

Under the Boussinesq approximation, the equations that govern the system are given by

$$\nabla.\vec{q} = 0, \quad (2)$$
$$\rho_0\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \rho\,\vec{g}\,(t) + \mu\nabla^2\vec{q} - \mu'\nabla^4\vec{q}, \quad (3)$$
$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = -\nabla.\vec{Q}, \quad (4)$$
$$\tau\left[\frac{\partial \vec{Q}}{\partial t} + \overrightarrow{\omega_1}\times\vec{Q}\right] = -\vec{Q} - \kappa\nabla T, \quad (5)$$
$$\rho = \rho_0[1-\alpha(T-T_0)]. \quad (6)$$

The quiescent basic state is expressed as

$$\overrightarrow{q_b}(z) = \vec{0},\ p = p_b(z),\ \rho = \rho_b(z),\ \vec{Q} = \vec{Q}_b(z),\ T = T_b(z) \quad (7)$$

Substituting (7) in (2)-(6), we obtain:

$$\frac{\partial p_b}{\partial z} = -\rho_b g_0\,(1+\varepsilon\cos(\omega t));\ \overrightarrow{Q_b} = -\kappa\frac{\partial T_b}{\partial z};\quad \rho_b = \rho_0[1-\alpha(T_b-T_0)]. \quad (8)$$

In order to study the stability of the system we superpose infinitesimal perturbations on the system, given by

$$\vec{q} = \overrightarrow{q_b} + \overrightarrow{q'},\ p = p_b + p',\ T = T_b + T',\ \rho = \rho_b + \rho',\ \vec{Q} = \overrightarrow{Q_b} + \overrightarrow{Q_b} \quad (9)$$

Substituting (9) in (2)-(6), along with the basic state equations, gives the linearized equations for the infinitesimally small perturbations. These equations are then non-dimensionalised using:

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right);\ \overrightarrow{q^*} = \frac{\overrightarrow{q'}}{\left(\frac{\kappa}{d}\right)};\ t^* = \frac{t}{\left(\frac{d^2}{\kappa}\right)};\quad T^* = \frac{T'}{\Delta T};\ \omega^* = \frac{\omega}{\left(\frac{\kappa}{d^2}\right)}; \quad (10)$$

to obtain, ignoring the asterisks:

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 W) = \nabla^4 W - C\nabla^6 W + R\,(1+\varepsilon\cos(\omega t))\,\nabla_1^2 T, \quad (11)$$

$$\left(1+2M\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} = \left(1+2M\frac{\partial}{\partial t}\right)W - M\,\nabla^2 W + \nabla^2 T. \quad (12)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$.

The boundaries considered are stress-free, isothermal, couple-stress vanishing. Hence (11) and (12) are solved for

$$W = \frac{\partial^2 W}{\partial z^2} = T = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (13)$$

W is eliminated from (11) and (12) to obtain

$$\{X_1 R\,(1+\varepsilon\,\mathrm{f})\,\nabla_1^2 + X_2\,X_3\}T = 0. \quad (14)$$

with the boundary conditions

$$T = \frac{\partial^2 T}{\partial z^2} = \frac{\partial^4 T}{\partial z^4} = \frac{\partial^6 T}{\partial z^6} = 0 \text{ at } z = 0,1. \quad (15)$$

where

$$\mathrm{f} = \cos(\omega t)$$

$$X_1 = \left[1+2M\frac{\partial}{\partial t} - M\nabla^2\right];\quad X_2 = \left[\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 - \nabla^4 + C\,\nabla^6\right];\quad X_3 = \left[\nabla^2 - \left(1+2M\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right]. \quad (16)$$

3. Solution

A solution of (13) in the form

$$(T,R) = (T_0,R_0) + \varepsilon(T_1,R_1) + \varepsilon^2(T_2,R_2) + \cdots \quad (17)$$

is considered where T_0 is the eigenfunction and R_0 is eigenvalue for the system in the absence of modulation. T_i, for $i > 0$, is the correction to T_0 and R_i, $i > 0$, is that to R_0 in the presence of modulation. Substituting (17) in (14), we obtain:

$$LT_0 = 0, \quad (18)$$
$$LT_1 = -(R_1 + R_0 f)X_1\nabla_1^2 T_0, \quad (19)$$
$$LT_2 = -(R_1 + R_0 f)X_1\nabla_1^2 T_1 - (R_2 + R_1 f)X_1\nabla_1^2 T_0, \quad (20)$$

where

$$L = R_0\,X_1\nabla_1^2 + X_2\,X_3. \quad (21)$$

The marginal stability solution for the problem is

$$T_0 = \sin(\pi z)\exp[i(lx+my)] \quad (22)$$

Substituting (22) in (18), we obtain

$$R_0 = \frac{k_1^6 + C\,k_1^8}{a^2\,(1+Mk_1^2)} \quad (23)$$

where $k_1^2 = \pi^2 + a^2$.

Then (19) for T_1 becomes

$$T_1 = -a^2(R_1 + R_0 f)(1+Mk_1^2)T_0 \quad (24)$$

The condition for solubility in the power integral technique demands the time independent part of the RHS of (24) to be orthogonal to T_0. As f is a sinusoidal function of time, the only time independent term is $a^2R_1(1+Mk_1^2)T_0$. Hence $R_1 = 0$ and it follows that in (17), $R_3 = R_5 = \ldots = 0$. RHS of (24) is expanded using Fourier series and by the inversion of the operator L we obtain

$$T_1 = a^2R_0(1+Mk_1^2)Re\left\{\sum_{n=1}^{\infty}\frac{e^{-i\omega t}}{L_1(\omega,n)}\sin(n\pi z)\right\}, \quad (25)$$

where

$$k_n^2 = n^2\pi^2 + a^2 \quad (26)$$

$$L_1(\omega,n) = [(-k_n^2 + 2\omega^2 M)(-k_n^4 - C\,k_n^6) - \frac{\omega^2 k_n^2}{Pr} - a^2R_0(Mk_n^2+1)] + i\omega[-k_n^4 - C\,k_n^6 + \frac{k_n^2}{P_r}(-k_n^2 + 2\omega^2 M) + 2Ma^2R_0] \quad (27)$$

(20) for T_2 becomes

$$LT_2 = R_0\,a^2(1+Mk_n^2 - i2M\omega)\,f\,T_1 + a^2R_2(1+Mk_1^2)T_0, \quad (28)$$

We use (28) to determine R_2.

$$R_2 = -\frac{a^2\,R_0^2}{4}\sum_{n=1}^{\infty}\left\{\frac{[N(\omega,n) + N^*(\omega,n)]}{|L_1(\omega,n)|^2}\right\} \quad (29)$$

where $*$ denotes a complex conjugate and

$$N(\omega, n) = L_1^*(\omega, n)(1 + Mk_n^2 - i2M\omega) \tag{30}$$

4. Results and Discussion

This paper analyses the onset of convection in couple stress fluid employing Maxwell-Cattaneo law under modulated gravity. The modulating amplitude is taken to be small with respect to the mean of the gravity. Also the convective currents are assumed to be weak and hence non-linear effects may be neglected. These assumptions cannot be neglected as it would bring in significant change in the results. This is because the method of perturbation enforces the requirement that the amplitude of εW_1 should not transcend that of W_0 i.e., $\omega > \varepsilon$. Thus the results depend on the values taken by the frequency of modulation, ω. If $\omega \ll 1$, the modulation period is large and it influences the fluid layer. The effect of modulation disappears for very large frequencies because the buoyant force attains a mean value resulting in the equilibrium state of the system with no modulation. Thus we consider only small to moderate values of ω ([6]).

Figures 2, 3 and 4 show the change in R_{2c} with respect to ω for the governing parameters. These figures show that the graph of R_{2c} is in the positive quadrant for all values of ω. This implies that time-periodic body force or gravity modulation stabilizes the system in comparison with the system with no modulation i.e., convection occurs at a later point.

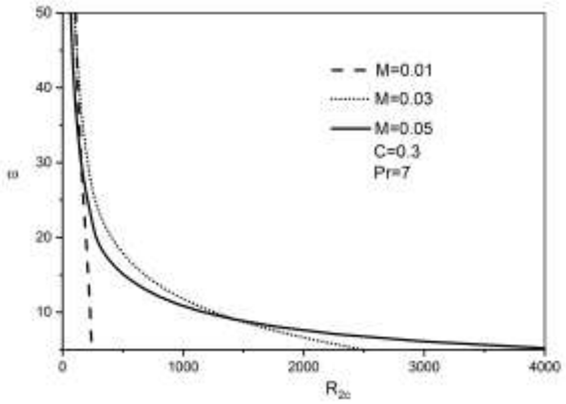

Fig. 2: Change in R_{2C} in relation to ω for various values of Cattaneo number

Figure 2 shows the deviation of R_{2c} with the modulating frequency ω for Cattaneo number, M. Cattaneo number has stabilizing effects for small values of the frequency. As Cattaneo number characterizes the non-Fourier effects in terms of the relaxation time, the temperature propagation will decrease with increase in relaxation time. However this effect reverses and becomes destabilizing for moderate values as it leads to narrowing of the convective cells. The value of ω at which stabilizing effect reverses is dependent on the values of the other governing parameters.

The influence of couple stress parameter on R_{2c} is shown in Figure 3. It can be seen that the stability of the system is enhanced by the couple stress parameter. C represents the concentration of the suspended particles. Therefore R_c rise with a rise in C and hence stabilize the system.

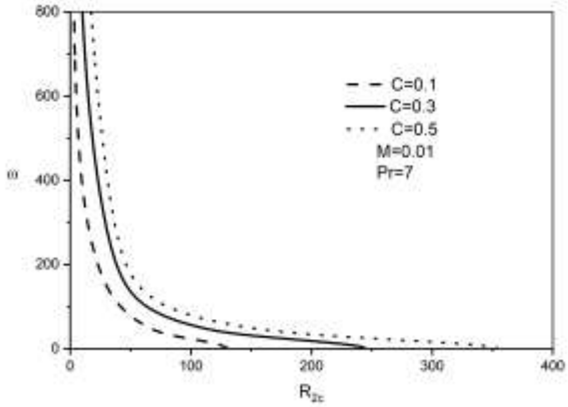

Fig. 3: Change in R_{2C} in relation to ω for various values of couple stress parameter

Figure 4 demonstrates the influence of Prandtl number on the system. An increase in the Prandtl number reduces the magnitude of the correction Rayleigh number. This indicates that Pr inhibits the stabilizing effect of the modulation. This impact is significant for moderate values of the modulating frequency.

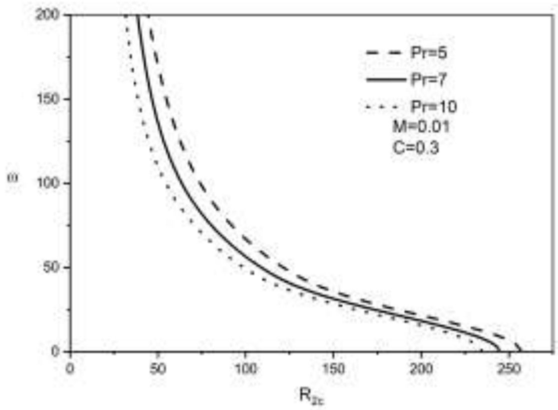

Fig. 4: Change in R_{2C} in relation to ω for various values of Prandtl number

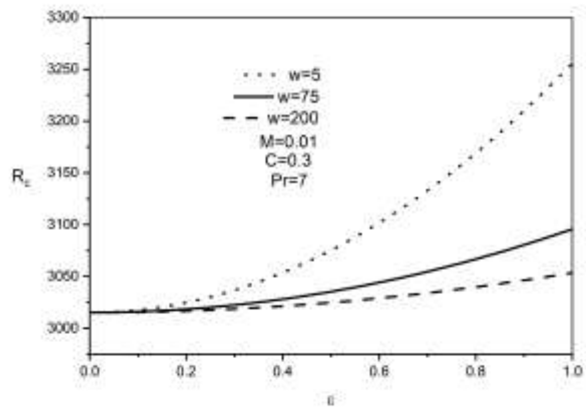

Fig. 5: Change in R_C in relation to ε for various values of ω

Figure 5 displays the change in critical Rayleigh number with the amplitude of modulation for a range of values of ω. We observe that for small and moderate values of ω, as ε increases R_c also increases. However as ω increases R_c decreases. Thus amplitude of modulation stabilizes the system for small and moderate values of ω. This confirms the earlier discussion that in the presence of non-Fourier effects moderate values of ω destabilize the system.

5. Conclusion

The onset of convection in a thin layer of couple stress fluid is examined using Maxwell-Cattaneo law when is system is acted on by a fluctuating gravitational field and the following conclusions are drawn:

- Cattaneo number is stabilizing for small frequencies and destabilizing for moderate frequencies. The point of transition from stabilization to destabilization depends on the values of the other governing parameters.
- Stability is stepped up by the couple stress parameter.
- Stability of the system is diminished by Prandtl number.

The influence of low frequency time-periodic body force on the system is significant. The problem gives insight into external means of controlling convection.

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