



Relative Study on the Incompressible Fluid Flow Past a Porous Plate with and Without the Radiation Effect

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Abstract

This paper focuses to compare the main flow velocity and temperature distribution in a three dimensional magneto hydrodynamic flow of a viscous incompressible fluid, past a vertical porous plate subjected to a periodic suction with and without the radiation effect. Governing equations are solved using perturbation technique and the results are discussed, on the results of the approximate solutions for velocity and temperature fields.

Keywords: Periodic suction, Porous plate, Radiation field, Magnetic field.

1. Introduction

Laminar flow plays a significant role in various disciplines of engineering applications. Analysis of the configurations of the suction holes and slits has wide range of the applications in ion propulsion, also the use of suction along with laminar flow controlled when the wall is being cooled is found to be important Singh et al [5]. Singh et al. [6]. Singh [4] studied the effect of magnetic field on the three dimensional flow of a viscous, incompressible fluid past a porous plate by applying transverse sinusoidal suction. Guria and Jana [2]. Guria, Kanch and R.N Jana [3] studied the “Hydromagnetic Effect on the Three Dimensional Flow Past a Vertical Porous Plate”. Guria, N.Ghara and R.N Jana [1] studied the “Radiation Effect on Three Dimensional MHD Flow Past a Vertical Porous Plate”. This paper focus to compare the flow of the viscous incompressible fluid flow past a vertical porous plate subjecting it to magnetic field with and without the Radiation effect.

2. Notations

| | | |
|-----------|--------|------------------------------------|
| ρ | -----> | Density |
| χ | -----> | Fluid pressure |
| G | -----> | Acceleration due to gravity |
| N | -----> | Coefficient of thermal expansion |
| δ | -----> | Coefficient of heat conduction |
| λ | -----> | Specific heat at constant pressure |
| σ | -----> | Electrical conduction |

\bar{G}_r -----> Grashof number

M -----> Magnetic parameter

\bar{P}_r -----> Prandtl number

\bar{T}_∞ -----> Temperature outside the boundary layer

\bar{P}_∞ -----> Pressure outside the boundary layer

ω -----> Radiation parameter

S -----> Suction parameter.

3. Formulation of the Problem

An incompressible fluid which is relatively highly resisting to flow which conducts electricity has been considered to flow past a semi infinite vertical porous plate. Vertical plate is placed along the \bar{x} - axis and the flow of the fluid is also taken along the vertical direction. \bar{y} - axis is taken perpendicular to this. \bar{z} - axis is taken perpendicular to the $\bar{x}\bar{y}$ - plane, along the \bar{z} - axis a steady magnetic field has been imposed. The periodic suction velocity distribution is

$$\bar{v} = -\bar{V}_0 [1 + \epsilon \cos \frac{u' \omega \bar{z}}{\gamma}] \tag{1}$$

Where ϵ strictly less than one is the amplitude of the suction velocity.

\bar{u} , \bar{v} , \bar{w} are denoted as velocity components in the \bar{x} , \bar{y} , \bar{z} axis respectively.

Case (1): Without Radiation Effect

The Governing equations

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{2}$$

$$\bar{V} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{W} \frac{\partial \bar{u}}{\partial \bar{z}} = \gamma \left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) + \text{GN}(\bar{T} - \bar{T}_\infty) - \frac{\sigma B_0^2 \bar{u}}{\rho}, \tag{3}$$

$$\bar{V} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{W} \frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{1}{\rho} \frac{\partial \chi}{\partial \bar{y}} + \gamma \left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right), \tag{4}$$

$$\bar{V} \frac{\partial \bar{w}}{\partial \bar{y}} + \bar{W} \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho} \frac{\partial \chi}{\partial \bar{z}} + \gamma \left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) - \frac{\sigma B_0^2 \bar{w}}{\rho}, \tag{5}$$

$$\bar{V} \frac{\partial \bar{T}}{\partial \bar{y}} + \bar{W} \frac{\partial \bar{T}}{\partial \bar{z}} = \frac{\delta}{\rho \lambda} \left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right), \tag{6}$$

The boundary conditions of the problem are

$$\bar{u}=0, \bar{v}=-\bar{V}_0 \left[1 + \epsilon \cos \frac{u'_\infty \bar{z}}{\gamma} \right], \bar{w}=0, \bar{T}=\bar{T}_\infty \text{ at } \bar{y}=0 \tag{7}$$

$$\bar{u}=0, \bar{v}=-\bar{V}_0, \bar{w}=0, \chi=\bar{P}_\infty \text{ at } \bar{y} \rightarrow \infty \tag{8}$$

Introducing the non-dimensional variables

$$\bar{y}' = \frac{\bar{u}'_\infty \bar{y}}{\gamma}, \bar{z}' = \frac{\bar{u}'_\infty \bar{z}}{\gamma}, p = \frac{\chi}{\rho u'_\infty}, \bar{u} = \frac{\bar{u}}{u'_\infty}, \tag{9}$$

$$\bar{v}' = \frac{\bar{v}}{u'_\infty}, \bar{\theta} = \frac{(\bar{T} - \bar{T}_\infty)}{\bar{T}_w - \bar{T}_\infty}, \tag{10}$$

Equation (2)-(6) becomes

$$\frac{\partial \bar{v}'}{\partial \bar{y}'} + \frac{\partial \bar{w}'}{\partial \bar{z}'} = 0, \tag{11}$$

$$\bar{V}' \frac{\partial \bar{u}'}{\partial \bar{y}'} + \bar{W}' \frac{\partial \bar{u}'}{\partial \bar{z}'} = \frac{\partial^2 \bar{u}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{u}'}{\partial \bar{z}'^2} + \bar{G}_r \bar{\theta}' - M \bar{u}', \tag{12}$$

$$\bar{V}' \frac{\partial \bar{v}'}{\partial \bar{y}'} + \bar{W}' \frac{\partial \bar{v}'}{\partial \bar{z}'} = -\frac{\partial p}{\partial \bar{y}'} + \left(\frac{\partial^2 \bar{v}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{v}'}{\partial \bar{z}'^2} \right), \tag{13}$$

$$\bar{V}' \frac{\partial \bar{w}'}{\partial \bar{y}'} + \bar{W}' \frac{\partial \bar{w}'}{\partial \bar{z}'} = -\frac{\partial p}{\partial \bar{z}'} + \left(\frac{\partial^2 \bar{w}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{w}'}{\partial \bar{z}'^2} \right) - M \bar{w}', \tag{14}$$

$$\bar{V}' \frac{\partial \bar{\theta}'}{\partial \bar{y}'} + \bar{W}' \frac{\partial \bar{\theta}'}{\partial \bar{z}'} = -\frac{1}{\text{Pr}} \left(\frac{\partial^2 \bar{\theta}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{\theta}'}{\partial \bar{z}'^2} \right), \tag{15}$$

Where $\bar{G}_r = \frac{\text{GN}(\bar{T}_w - \bar{T}_\infty)\gamma}{u'_\infty}$, $M = \frac{\sigma B_0^2 \gamma}{\rho u'_\infty}$ and $\bar{P}_r = \frac{\rho \gamma \lambda}{\delta}$. The corresponding boundary condition (7) becomes [3].

$$\bar{u}'=0, \bar{v}' = -S[1 + \epsilon \cos(\pi \bar{z}')], \bar{w}'=0, \bar{\theta}'=1 \text{ at } \bar{y}'=0$$

$$\bar{u}'=0, \bar{v}'=-S, \bar{w}'=0, \bar{\theta}'=0, \text{ at } \bar{y}' \rightarrow \infty \tag{16}$$

$S = \frac{\bar{V}_0}{u'_\infty}$, the suction parameter.

4. Solution

For (9)-(13), The assumed solution is:

$$\bar{u}' = \bar{u}'_0 + \epsilon \bar{u}'_1 + \epsilon^2 \bar{u}'_2 + \dots, \tag{17}$$

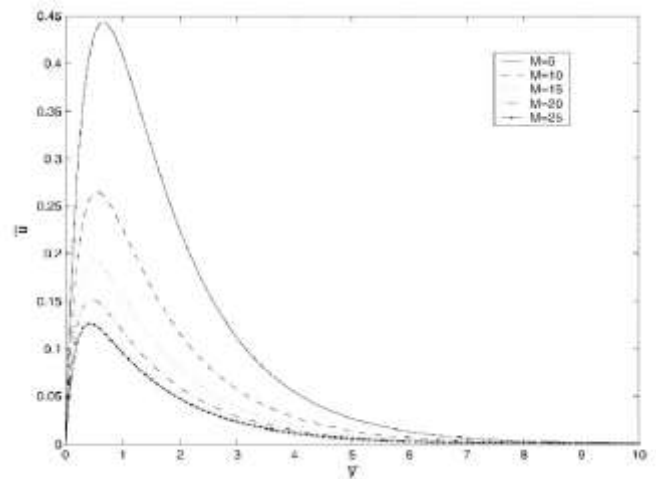
$$\bar{v}' = \bar{v}'_0 + \epsilon \bar{v}'_1 + \epsilon^2 \bar{v}'_2 + \dots, \tag{18}$$

$$\bar{w}' = \bar{w}'_0 + \epsilon \bar{w}'_1 + \epsilon^2 \bar{w}'_2 + \dots, \tag{19}$$

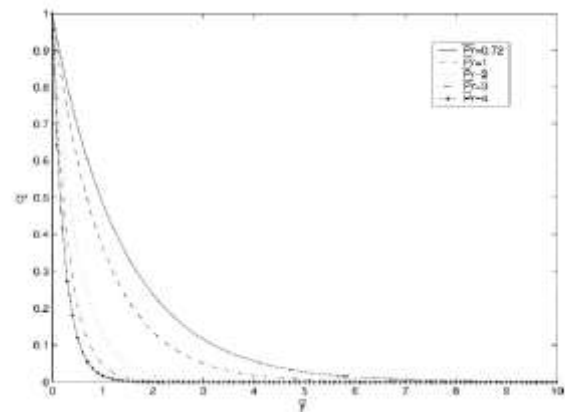
$$p = p_0 + \epsilon p_1 + \epsilon^2 p_2 + \dots, \tag{20}$$

$$\bar{\theta}' = \bar{\theta}'_0 + \epsilon \bar{\theta}'_1 + \epsilon^2 \bar{\theta}'_2 + \dots, \tag{21}$$

Substituting (15) in (9)-(13), and comparing the corresponding terms and for its corresponding boundary conditions, solutions are obtained.



Graph 1: Main flow velocity u' for $\bar{G}_r=5.0, S=1.0, \bar{P}_r=0.72, \bar{z}'=0.0, \epsilon=0.2$



Graph 2: Temperature profile $\bar{\theta}'$ for $S=1.0, M=10.0, \bar{z}'=0.2, \epsilon=0.2$

5. Result and Discussion

The non dimensional velocity \bar{u}' for $\bar{G}_r=5.0, \bar{P}_r=0.72, S=1.0, \epsilon=0.2$ against \bar{y}' for different values of magnetic parameter M is given in graph 1. It is shown that when the magnetic parameter increases the main flow velocity \bar{u}' decreases.

*Magnetic field affects the velocity.

Graph 2 shows, when the prandtl number increases, the temperature decreases.

Thermal boundary layer thickness decreases with the increase of prandtl number.

*Prandtl number affects the temperature profile.

Case (2): With the Radiation Effect

The Governing equations

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{16}$$

$$\bar{V} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = \gamma \left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) + GN(T - \bar{T}_\infty) - \frac{\sigma B_0^2 \bar{u}}{\rho}, \tag{17}$$

$$\bar{V} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{1}{\rho} \frac{\partial \chi}{\partial \bar{y}} + \gamma \left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right), \tag{18}$$

$$\bar{V} \frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho} \frac{\partial \chi}{\partial \bar{z}} + \gamma \left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) - \frac{\sigma B_0^2 \bar{w}}{\rho}, \tag{19}$$

$$\bar{V} \frac{\partial T}{\partial \bar{y}} + \bar{w} \frac{\partial T}{\partial \bar{z}} = \frac{\delta}{\rho \lambda} \left(\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\partial^2 T}{\partial \bar{z}^2} \right) - \frac{1}{\rho \lambda} \frac{\partial q^*}{\partial \bar{y}}, \tag{20}$$

The boundary conditions of the problem are

$$\bar{u}=0, \bar{v}=-\bar{V}_0 [1+\epsilon \cos(\pi \bar{z}')] , \bar{w}=0, T=\bar{T}_w \text{ at } \bar{y}=0 \tag{21}$$

$$\bar{u}=0, \bar{v}=-\bar{V}_0, \bar{w}=0, \chi=\bar{P}_\infty, T=\bar{T}_\infty \text{ at } \bar{y} \rightarrow \infty$$

Introducing the non-dimensional variables

$$\bar{y}' = \frac{\bar{u}'_\infty \bar{y}}{\gamma}, \bar{z}' = \frac{\bar{v}'_\infty \bar{z}}{\gamma}, t = ct^*, p = \frac{\chi}{\rho u'_{\infty 2}}, \tag{22}$$

$$\bar{u}' = \frac{\bar{u}}{u'_{\infty}}, \bar{v}' = \frac{\bar{v}}{v'_{\infty}}, \bar{w}' = \frac{\bar{w}}{u'_{\infty}}, \bar{\theta} = \frac{(T - T_\infty)}{T_w - T_\infty},$$

Equation (16)-(20) becomes

$$\frac{\partial \bar{v}'}{\partial \bar{y}'} + \bar{w}' \frac{\partial \bar{w}'}{\partial \bar{z}'} = 0, \tag{23}$$

$$\bar{V}' \frac{\partial \bar{u}'}{\partial \bar{y}'} + \bar{w}' \frac{\partial \bar{u}'}{\partial \bar{z}'} = \frac{\partial^2 \bar{u}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{u}'}{\partial \bar{z}'^2} + \bar{G}_r \bar{\theta} - M \bar{u}', \tag{24}$$

$$\bar{V}' \frac{\partial \bar{v}'}{\partial \bar{y}'} + \bar{w}' \frac{\partial \bar{v}'}{\partial \bar{z}'} = -\frac{\partial p}{\partial \bar{y}'} + \left(\frac{\partial^2 \bar{v}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{v}'}{\partial \bar{z}'^2} \right), \tag{25}$$

$$\bar{V}' \frac{\partial \bar{w}'}{\partial \bar{y}'} + \bar{w}' \frac{\partial \bar{w}'}{\partial \bar{z}'} = -\frac{\partial p}{\partial \bar{z}'} + \left(\frac{\partial^2 \bar{w}'}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{w}'}{\partial \bar{z}'^2} \right) - M \bar{w}', \tag{26}$$

$$\bar{V}' \frac{\partial \bar{\theta}}{\partial \bar{y}'} + \bar{w}' \frac{\partial \bar{\theta}}{\partial \bar{z}'} = -\frac{1}{Pr} \left(\frac{\partial^2 \bar{\theta}}{\partial \bar{y}'^2} + \frac{\partial^2 \bar{\theta}}{\partial \bar{z}'^2} \right) - \omega \bar{\theta}, \tag{27}$$

Where $\bar{G}_r = \frac{GN(T_w - T_\infty)\gamma}{u'_{\infty 2}}$, $M = \frac{\sigma B_0^2 \gamma}{\rho u'_{\infty 2}}$ and $\bar{P}_r = \frac{\rho \gamma \lambda}{\delta}$. The corresponding boundary condition (21) becomes [1].

$$\bar{u}'=0, \bar{v}'=-S[1+\epsilon \cos(\pi \bar{z}')], \bar{w}'=0, \bar{\theta}=1 \text{ at } \bar{y}'=0$$

$$\bar{u}'=0, \bar{v}'=-S, \bar{w}'=0, \bar{\theta}=0, \text{ at } \bar{y}' \rightarrow \infty \tag{28}$$

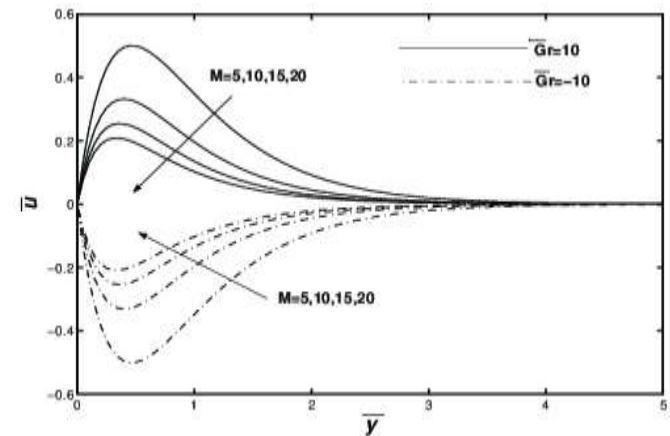
$S = \frac{\bar{V}_0}{u'_{\infty}}$, the suction parameter.

6. Solution

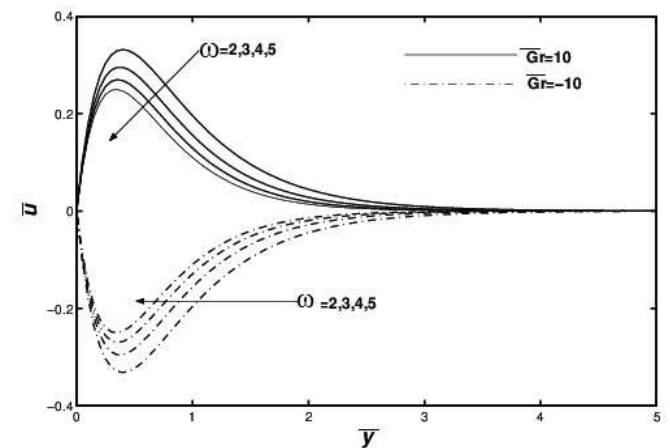
For (23)-(27), The assumed solution is:

$$\begin{aligned} \bar{u}' &= \bar{u}'_0 + \epsilon \bar{u}'_1 + \epsilon^2 \bar{u}'_2 + \dots, \\ \bar{v}' &= \bar{v}'_0 + \epsilon \bar{v}'_1 + \epsilon^2 \bar{v}'_2 + \dots, \\ \bar{w}' &= \bar{w}'_0 + \epsilon \bar{w}'_1 + \epsilon^2 \bar{w}'_2 + \dots, \\ p &= p_0 + \epsilon p_1 + \epsilon^2 p_2 + \dots, \\ \bar{\theta} &= \bar{\theta}_0 + \epsilon \bar{\theta}_1 + \epsilon^2 \bar{\theta}_2 + \dots, \end{aligned} \tag{15}$$

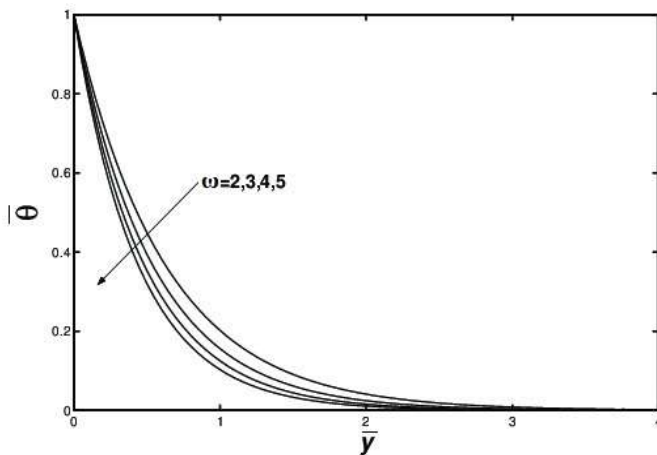
and for the corresponding boundary conditions, the solutions are obtained.



Fig(a): Variation of main flow velocity for $\bar{P}_r=0.17, \omega=2.0, S=1.0, \epsilon=0.05, \bar{z}'=0.0$



Fig(b): Variation of main flow velocity for $\bar{P}_r=0.71, M=5.0, S=1.0, \epsilon=0.05, \bar{z}'=0.0$.



Fig(c): Variation of temperature profile for $\bar{P}_r=0.71$, $M=5.0$, $S=1.0$, $\epsilon=0.05$, $\bar{z}'=0.0$.

7. Result and Discussion

\bar{G}_r is considered for both positive and negative values. The values greater than zero corresponds to the cooled plate and the value less than zero corresponds to the hot plate $\bar{p}_r = 0.71$ corresponds to the air.

Fig (a): We consider the impact of the increase in magnetic number indicates a decrease in the main flow velocity when plate is cooled and an increase when the plate is heated.

Fig (b): We understand the impact of the increase in the radiation parameter ω indicates decrease in the main flow velocity when the plate is cooled and an increase when the plate is heated.

Fig (c): We understand the impact of the increase in the radiation parameter ω shows a rapid decrease in the temperature.

8. Conclusion

The main flow velocity is affected by the magnetic field and the flow characterization and the temperature distribution is controlled by the radiation field.

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