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Research paper

Synchronous and Asynchronous Boundary Temperature Modulations on Triple-Diffusive Convection in Couple Stress Liquid Using Ginzburg-Landau Model

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Abstract

A nonlinear study of synchronous and asynchronous boundary temperature modulations on the onset of triple-diffusive convection in couple stress liquid is examined. Two cases of temperature modulations are studied: (a) Synchronous temperature modulation ($\varphi = 0$) and (b) Asynchronous temperature modulation ($\varphi \neq 0$). It is done to examine the influence of mass and heat transfer by deriving Ginzburg-Landau equation. The resultant Ginzburg-Landau equation is Bernoulli equation and it is solved numerically by means of Mathematica. The influence of solute Rayleigh numbers and couple stress parameter is studied. It is observed that couple stress parameter increases the mass and heat transfer whereas solute Rayleigh numbers decreases the mass and heat transfer.

Keywords: *Triple-diffusive convection; temperature modulation; couple stress parameter; Ginzburg-Landau model.*

1. Introduction

In Rayleigh-Bénard problem, when the instability is due to the difference in temperature between the upper and lower surfaces, then it is termed as single component convection and when the instability is due to two differing density components, then it is termed as two-component convection or double diffusive convection. Correspondingly, when the instability is due to three different diffusivities then the physical and mathematical state becomes increasingly rich. Such problems in literature are termed as three component convection or triple-diffusive convection. The problems of three component convection have been considered by Pearlstein et al. [1], Lopez et al. [2], Rionero [3], Sumithra [4] and recently by Sameena and Pranesh [5] and Sameena [6].

In the group of non-Newtonian liquids, couple stress has exclusive features such as polar effect and ability to possess huge viscosity. Couple stress liquid was established by Stokes [7] and has focused to the development of several notions of liquid with microstructure. The influence of couple stresses in liquid have no microstructure; therefore, the angular momentum and kinematic energy of spin density are not taken into consideration and are entirely determined by the velocity field. Hence, couple stresses in liquid results in the equation analogous to that of Navier-Stokes equation. The problems with couple stress liquid in Rayleigh-Bénard convection have been studied by Shivakumara et al. [8] and Siddeshwar and Pranesh [9].

One of the ways to regulate convection is by sustaining nonlinear temperature gradient and it is only space-dependent. On the other hand, in real-world circumstances, nonlinear temperature gradients get its source in cooling or heating at the surfaces, therefore the profile of temperature depends explicitly on time and position. This is termed as **temperature modulation**. This can be used as a way of controlling the convective flow by appropriate tuning of frequency and amplitude of modulation. Temperature modulation has a range of applications like, during solidification of metallic alloys in crystal growth; the time dependent temperature gradient influences the mass and heat transfer procedure and controls the formation and also the quality of the resultant solid. Temperature modulation may also be used in material processing as the means to attain higher efficiency of solid and also to enhance heat, mass and momentum transfer. The notable work in temperature modulation is due to Venezian [10]. Many authors Bhadauria [11], Pranesh and Sangeetha [12] and Siddheshwar and Pranesh [13] have considered the influence of temperature modulation under different conditions to study its effect on the onset of convection.

The problem under study has numerous applications like in petroleum reservoirs, solidification of alloys, oceanography, material processing, crystal growth, space crafts and so on. In all these it is important to examine the mass and heat transfer and since linear study only provides the stability condition, we have done a nonlinear study of the problem which helps to examine the mass and heat transfer and this is done by using Ginzburg-Landau model. The advantage of using Ginzburg-Landau model is that it gives nonlinear solution in the form of a series whose convergence is definite. Therefore, the foremost objective of this article is to study synchronous and asynchronous boundary temperature modulations on triple-diffusive convection in couple stress liquid using Ginzburg-Landau model.

2. Mathematical Formulation

Consider horizontal layer of couple stress fluid of thickness ‘d’ restricted between two parallel plates. A Cartesian system is taken with origin in the lower boundary and z-axis vertically upward (see Fig. 1). Let $\Delta T, \Delta S_1$ and ΔS_2 be the differences in temperature and solute concentrations, respectively of the liquid between the lower and upper plates.

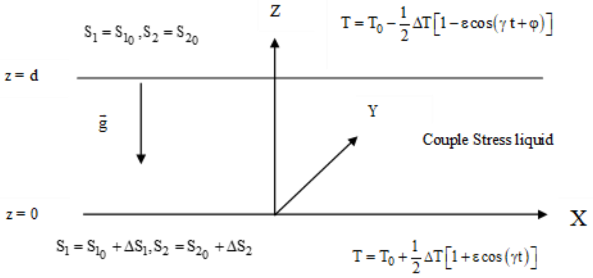

Fig. 1: Physical configuration

The basic governing equations for three component convection in a couple stress fluid under Boussinesquian approximation are:

$$\nabla.\vec{q} = 0\,, \tag{1}$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{q} - \mu'\nabla^4\vec{q}\,, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi\nabla^2 T\,, \tag{3}$$

$$\frac{\partial S_1}{\partial t} + (\vec{q}.\nabla)S_1 = \chi_{S1}\nabla^2 S_1\,, \tag{4}$$

$$\frac{\partial S_2}{\partial t} + (\vec{q}.\nabla)S_2 = \chi_{S2}\nabla^2 S_2\,, \tag{5}$$

$$\rho = \rho_0\left[1 - \alpha_t(T - T_0) + \alpha_{S1}(S_1 - S_{10}) + \alpha_{S2}(S_2 - S_{2_0})\right], \tag{6}$$

where, $\vec{q}$ is velocity, p is pressure, ρ_0 is the constant density, ρ is density, $\vec{g}$ is gravitational force, μ is viscosity, μ' is couple stress viscosity, T is temperature, S_1 is solute1, S_2 is solute2, χ is the thermal diffusivity, χ_{S1} is solute1 diffusivity, χ_{S2} is solute2 diffusivity, α_t is coefficient of thermal expansion to determine how fast the density decreases with temperature, α_{S1} is the coefficient of solute1 expansion and α_{S2} is the coefficient of solute2 expansion.

The externally imposed time dependent boundary temperatures are given as:

$$T(0,t) = T_0 + \frac{1}{2}\Delta T\left[1 + \varepsilon\cos(\gamma t)\right], \tag{7}$$

$$T(d,t) = T_0 - \frac{1}{2}\Delta T\left[1 - \varepsilon\cos(\gamma t + \varphi)\right], \tag{8}$$

where, ε is the amplitude of modulation, γ is the frequency of modulation and φ is the phase angle. The boundary temperature modulations arise in the existing problem through the boundary conditions (7) and (8).

The basic state of the fluid is at rest and is given by:

$$\left.\begin{array}{l}\vec{q}_b = (0,0,0), p = p_b(z), \rho = \rho_b(z), \\ T = T_b(z), S_1 = S_{1b}(z), S_2 = S_{2_b}(z)\end{array}\right\}, \tag{9}$$

where, the subscript ‘b’ represents the basic state.

Substituting equation (9) in equation (1) to (6), we get,

$$\frac{dp_b}{dz} + \rho_b g = 0\,, \tag{10}$$

$$\frac{\partial T_b}{\partial t} = \chi\frac{\partial^2 T_b}{\partial z^2}\,, \tag{11}$$

$$\frac{d^2 S_{1_b}}{dz^2} = 0\,, \tag{12}$$

$$\frac{d^2 S_{2_b}}{dz^2} = 0\,, \tag{13}$$

$$\rho_b = \rho_o\left[\begin{array}{r}1 - \alpha_t(T_b - T_0) + \alpha_{S1}(S_{1_b} - S_{1_0}) \\ +\alpha_{S2}(S_{2_b} - S_{2_0})\end{array}\right]. \tag{14}$$

Solving equation (11) with the boundary conditions (7) and (8) is

$$T_b(z,t) = T_0 + \frac{\Delta T}{2}\left(1 - \frac{2z}{d}\right) + \varepsilon f(z,t)\,, \tag{15}$$

where,

$$f(z,t) = \mathrm{Re}\left\{\left[\left[A(\lambda)e^{\frac{\lambda z}{d}} + A(-\lambda)e^{\frac{-\lambda z}{d}}\right]e^{-i\gamma t}\right\}\right.,$$

$$\lambda = (1 - i)\sqrt{\frac{\gamma}{2}}\,,$$

and $A(\lambda) = \frac{\Delta T}{2}\left[\frac{e^{-i\phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right]$.

The stability of the basic state is examined by presenting the subsequent perturbation:

$$\left.\begin{array}{l}\vec{q} = \vec{q}_b + \vec{q}', p = p_b + p', \rho = \rho_b + \rho', \\ T = T_b + T', S_1 = S_{1_b} + S_1', S_2 = S_{2_b} + S_2'\end{array}\right\}, \tag{16}$$

where, the prime quantities represent the infinitesimal perturbations.

Substituting equation (16) in equations (1) to (6), using basic state solutions (10)-(15) and by nondimensionlising using the subsequent definitions:

$$\left.\begin{array}{l}(x^*, y^*, z^*) = \left(\frac{x'}{d}, \frac{y'}{d}, \frac{z'}{d}\right), t^* = \frac{t'}{d^2/\chi}, q^* = \frac{q'}{\chi/d}, \\ p^* = \frac{d^2}{\mu\chi}p', \theta = \frac{T'}{\Delta T}, \phi_{S1} = \frac{S_1'}{\Delta S_1}, \phi_{S2} = \frac{S_2'}{\Delta S_2}\end{array}\right\}, \tag{17}$$

We obtain the following dimensionless equations by eliminating the pressure term and also by introducing stream function ψ (after dropping the asterisk):

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix} \begin{bmatrix} \psi \\ \theta \\ \phi_{S1} \\ \phi_{S2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2\psi\right) + \frac{1}{Pr}\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial\left(x,z\right)} \\ -\frac{\partial\theta}{\partial t} + \varepsilon f\frac{\partial\psi}{\partial x} + \frac{\partial\left(\psi,\theta\right)}{\partial\left(x,z\right)} \\ -\frac{\partial\phi_{S1}}{\partial t} + \frac{\partial\left(\psi,\phi_{S1}\right)}{\partial\left(x,z\right)} \\ -\frac{\partial\phi_{S2}}{\partial t} + \frac{\partial\left(\psi,\phi_{S2}\right)}{\partial\left(x,z\right)} \end{bmatrix}, \qquad (18)$$

Where, $Pr = \frac{\mu}{\rho_0\chi}$ is Prandtl number,

$C = \frac{\mu'}{\mu d^2}$ is couple stress parameter,

$Ra = \frac{\rho_0\alpha_t g\Delta T d^3}{\mu\chi}$ is Rayleigh number,

$R_{S1} = \frac{\rho_0\alpha_{S1} g\Delta S_1 d^3}{\mu\chi}$ is solute Rayleigh number1,

$R_{S2} = \frac{\rho_0\alpha_{S2} g\Delta S_2 d^3}{\mu\chi}$ is solute Rayleigh number2,

$\tau_1 = \frac{\chi_{S1}}{\chi}$ is ratio of diffusivity of solute1 and heat diffusivity and

$\tau_2 = \frac{\chi_{S2}}{\chi}$ is ratio of diffusivity of solute2 and heat diffusivity.

3. Nonlinear Analysis

In this section, a local nonlinear stability investigation of triple diffusive convection is performed using Ginzburg-Landau model to show that the modulation can be used to augment or reduce the mass and heat transfer. To make this study we introduce following perturbation:

$$\left.\begin{array}{l} Ra = Ra_0 + \delta^2 Ra_2 + \delta^4 Ra_4 + ... \\ \psi = \delta\psi_1 + \delta^2\psi_2 + \delta^3\psi_3 + ... \\ \theta = \delta\theta_1 + \delta^2\theta_2 + \delta^3\theta_3 + ... \\ \phi_{S1} = \delta\phi_{S11} + \delta^2\phi_{S12} + \delta^3\phi_{S13} + ... \\ \phi_{S2} = \delta\phi_{S21} + \delta^2\phi_{S22} + \delta^3\phi_{S23} + ... \end{array}\right\}, \qquad (19)$$

where, Ra_0 is Rayleigh number for steady case, δ is the expansion parameter and subscripts are the series representation of perturbation.

Substituting equation (19) in equation (18) and comparing the like powers of δ on both sides, we get solutions of various modes.

At the lowermost order, we have,

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \theta_1 \\ \phi_{S11} \\ \phi_{S2_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (20)$$

The solution to the lowermost order of the system is,

$$\left.\begin{array}{l} \psi_1 = A\left(\tau\right)\sin\left(ax\right)\sin\left(\pi z\right) \\ \theta_1 = -\frac{a}{k^2}A\left(\tau\right)\cos\left(ax\right)\sin\left(\pi z\right) \\ \phi_{S1_1} = -\frac{a}{k^2\tau_1}A\left(\tau\right)\cos\left(ax\right)\sin\left(\pi z\right) \\ \phi_{S2_1} = -\frac{a}{k^2\tau_2}A\left(\tau\right)\cos\left(ax\right)\sin\left(\pi z\right) \end{array}\right\}, \qquad (21)$$

Where, $\psi_1, \theta_1, \phi_{S1}$ and ϕ_{S2} are the first order solutions of perturbation series, $A\left(\tau\right)$ is the finite amplitude of minimal representation of Fourier series and a is the wave number. Substituting equation (21) into equation (20), we attain the expression for Rayleigh number as:

$$Ra_0 = \frac{R_{S1}}{\tau_1} + \frac{R_{S2}}{\tau_2} + \frac{k^6\left(1 + Ck^2\right)}{a^2}, \qquad (22)$$

where,

$k^2 = \pi^2 + a^2$.

At the second order, we have,

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_2 \\ \theta_2 \\ \phi_{S12} \\ \phi_{S2_2} \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \\ R_{24} \end{bmatrix}, \qquad (23)$$

where,

$R_{21} = 0$,

$R_{22} = -\frac{a^2\pi}{2k^2}A^2\left(\tau\right)\sin\left(2\pi z\right)$,

$R_{23} = -\frac{a^2\pi}{2k^2\tau_1}A^2\left(\tau\right)\sin\left(2\pi z\right)$ and

$R_{24} = -\frac{a^2\pi}{2k^2\tau_2}A^2\left(\tau\right)\sin\left(2\pi z\right)$.

The second order solutions are obtained as follows:

$$\left.\begin{array}{l} \psi_2 = 0 \\ \theta_2 = -\frac{a^2}{8\pi k^2}A^2\left(\tau\right)\sin\left(2\pi z\right) \\ \phi_{S12} = -\frac{a^2}{8\pi k^2\tau_1^2}A^2\left(\tau\right)\sin\left(2\pi z\right) \\ \phi_{S22} = -\frac{a^2}{8\pi k^2\tau_2^2}A^2\left(\tau\right)\sin\left(2\pi z\right) \end{array}\right\}, \qquad (24)$$

The average Nusselt number Nu and Sherwood numbers Sh_1 and Sh_2 are given by:

$$Nu(\tau)=1+\frac{a^2}{4k^2}A^2(\tau)\,, \tag{25}$$

$$Sh_1(\tau)=1+\frac{a^2}{4k^2\tau_1^2}A^2(\tau)\,, \tag{26}$$

$$Sh_2(\tau)=1+\frac{a^2}{4k^2\tau_2^2}A^2(\tau)\,. \tag{27}$$

At the third order, we have,

$$\begin{bmatrix} -\nabla^4+C\nabla^6 & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix}\begin{bmatrix} \psi_3 \\ \theta_3 \\ \phi_{S13} \\ \phi_{S23} \end{bmatrix}=\begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \\ R_{34} \end{bmatrix}, \tag{28}$$

where,

$$\left.\begin{aligned} R_{31} &= -Ra_2\frac{\partial\theta_1}{\partial x}-\frac{1}{Pr}\frac{\partial}{\partial\tau}\left(\nabla^2\psi_1\right) \\ R_{32} &= -\frac{\partial\theta_1}{\partial\tau}+\varepsilon_2 f\left(z,\tau\right)\frac{\partial\psi_1}{\partial x}+J\left(\psi_1,\theta_2\right) \\ R_{33} &= -\frac{\partial\phi_{S11}}{\partial\tau}+J\left(\psi_1,\phi_{S12}\right) \\ R_{34} &= -\frac{\partial\phi_{S21}}{\partial\tau}+J\left(\psi_1,\phi_{S22}\right) \end{aligned}\right\}. \tag{29}$$

It is presumed that modulations are of minor amplitude and hence $\varepsilon=\delta^2\varepsilon_2$ and $\tau=\delta^2 t$.

For the existence of third order solution of the system we apply the solvability condition given by:

$$\int_{z=0}^{1}\int_{x=0}^{2\pi/a}\begin{bmatrix} \psi_1 R_{31}+Ra\theta_1 R_{32}-R_{S1}\phi_{S1}R_{33} \\ -R_{S2}\phi_{S2}R_{34} \end{bmatrix}dxdz=0\,, \tag{30}$$

Substituting $\psi_1,\theta_1,\phi_{S11},\phi_{S21},\theta_2,\phi_{S12}$ and ϕ_{S22} into equation (30), we obtain the expressions for R_{31},R_{32},R_{33} and R_{34} and arrive at the non-autonomous Ginzburg-Landau equation in the form:

$$A_1A'(\tau)-A_2A(\tau)+A_3A^3(\tau)=0\,, \tag{31}$$

where,

$$A_1=\frac{k^2}{Pr}+\left(-Ra_0+\frac{R_{S1}}{\tau_1^2}+\frac{R_{S2}}{\tau_2^2}\right)\frac{a^2}{k^4},$$

$$A_2=\left(Ra_2+2Ra_0\varepsilon_2 I\right)\frac{a^2}{k^2},$$

$$A_3=\left(-Ra_0+\frac{R_{S1}}{\tau_1^3}+\frac{R_{S2}}{\tau_2^3}\right)\frac{a^4}{8k^4},$$

$$I=\int_0^1 f\left(z,\tau\right)\sin^2\left(\pi z\right)dz\,.$$

The Ginzburg-Landau equation in equation (31) is Bernoulli equation and it’s difficult to get analytical solution for the same. In view of this, it has been solved numerically using Mathematica, with respect to initial condition $A(0)=a_0$, where, a_0 is the preferred initial amplitude of convection. Without loss of generality, it is assumed that $Ra_2=Ra_0$ in the calculations and this is done to retain the parameters to a minimum.

4. Results and Discussions

In this article, the influence of temperature modulations on triple-diffusive convection in couple stress liquid heated and added solutes from below is made. The behaviour of various parameters like couple stress parameter C, solute Rayleigh number1 R_{S1} and solute Rayleigh number2 R_{S2} are analyzed. To have a better comprehension of the influence of temperature modulations on mass and heat transfer, we need to choose a distinctive time interval. The interval $\left[0,2\pi\right]$ appears to be a suitable interval to analyze mean Nusselt and Sherwood numbers. The mean Nusselt number $\overline{Nu}$, and the mean Sherwood numbers $\overline{Sh_1}$ and $\overline{Sh_2}$ are defined as:

$$\overline{Nu}=\frac{\gamma}{2\pi}\int_0^{2\pi/\gamma} Nu(\tau)d\tau\,, \tag{32}$$

$$\overline{Sh_1}=\frac{\gamma}{2\pi}\int_0^{2\pi/\gamma} Sh_1(\tau)d\tau\,, \tag{33}$$

$$\overline{Sh_2}=\frac{\gamma}{2\pi}\int_0^{2\pi/\gamma} Sh_2(\tau)d\tau\,. \tag{34}$$

The subsequent dual categories of modulations can be considered for nonlinear study:

(a) Synchronous temperature modulation, where $\varphi=0$.

(b) Asynchronous temperature modulation, where $\varphi\neq 0$.

The mass and heat transfer results for synchronous and asynchronous temperature modulations are presented in Figures 2 to 4. Fig. 2 depicts the effect of φ on (a) $\overline{Nu}$ for $\gamma=10$ (b) $\overline{Nu}$ for $\gamma=100$ (c) $\overline{Sh_1}$ for $\gamma=10$ (d) $\overline{Sh_1}$ for $\gamma=100$ (e) $\overline{Sh_2}$ for $\gamma=10$ and (f) $\overline{Sh_2}$ for $\gamma=100$ for different values of couple stress parameter C. From the figure, it is found that the increase in C, increases $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$, indicating enhancement in mass and heat transfer. This is due to the existence of couple stress which augments the viscosity of the liquid and therefore advanced heating is essential to have the instability with an increasing value of C.

Fig. 3 depicts the effect of φ on (a) $\overline{Nu}$ for $\gamma=10$ (b) $\overline{Nu}$ for $\gamma=100$ (c) $\overline{Sh_1}$ for $\gamma=10$ (d) $\overline{Sh_1}$ for $\gamma=100$ (e) $\overline{Sh_2}$ for $\gamma=10$ and (f) $\overline{Sh_2}$ for $\gamma=100$ for different values of solute Rayleigh number1 R_{S1}. From the figure, it is found that the increase in R_{S1}, decreases $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$, indicating reduction in mass and heat transfer. Fig. 4 depicts the effect of φ on (a) $\overline{Nu}$ for $\gamma=10$ (b) $\overline{Nu}$ for $\gamma=100$ (c) $\overline{Sh_1}$ for $\gamma=10$ (d) $\overline{Sh_1}$ for $\gamma=100$ (e) $\overline{Sh_2}$ for $\gamma=10$ and (f) $\overline{Sh_2}$ for $\gamma=100$ for different values of solute Rayleigh number2 R_{S2}. From the figure, it is found that the increase in R_{S2}, decreases $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$, indicating reduction in mass and heat transfer. This is because when the solutes are added from below, the concentration of solutes settles at the bottom boundary without disturbing the system. We can also draw following conclusions from the Figures 2 to 4:

(i) In the interval $\left[0,\pi\right]$, $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ increases with increase in the phase angle φ.

(ii) In the interval $\left[\pi,2\pi\right]$, $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ decreases with increase in the phase angle φ.

(iii) The maximum values of $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ is at the value $\varphi \approx \pi$.

(iv) As frequency of modulation γ increases, the $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ curve becomes flatter and flatter, indicating that the modulation effect vanishes for large frequencies.

5. Conclusion

The following observations are made from the study:

1. Increase in couple stress parameter increases mean Nusselt and Sherwood numbers whereas increase in solute Rayleigh numbers decreases mean Nusselt and Sherwood numbers.
2. In the interval $\left[0,\pi\right]$, $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ increases with increase in the phase angle φ and in the interval $\left[\pi,2\pi\right]$, $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ decreases with increase in the phase angle φ.
3. The maximum values of $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ is at the value $\varphi \approx \pi$.
4. As frequency of modulation γ increases, the $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ curve becomes flatter and flatter, indicating that the modulation effect vanishes for large frequencies.

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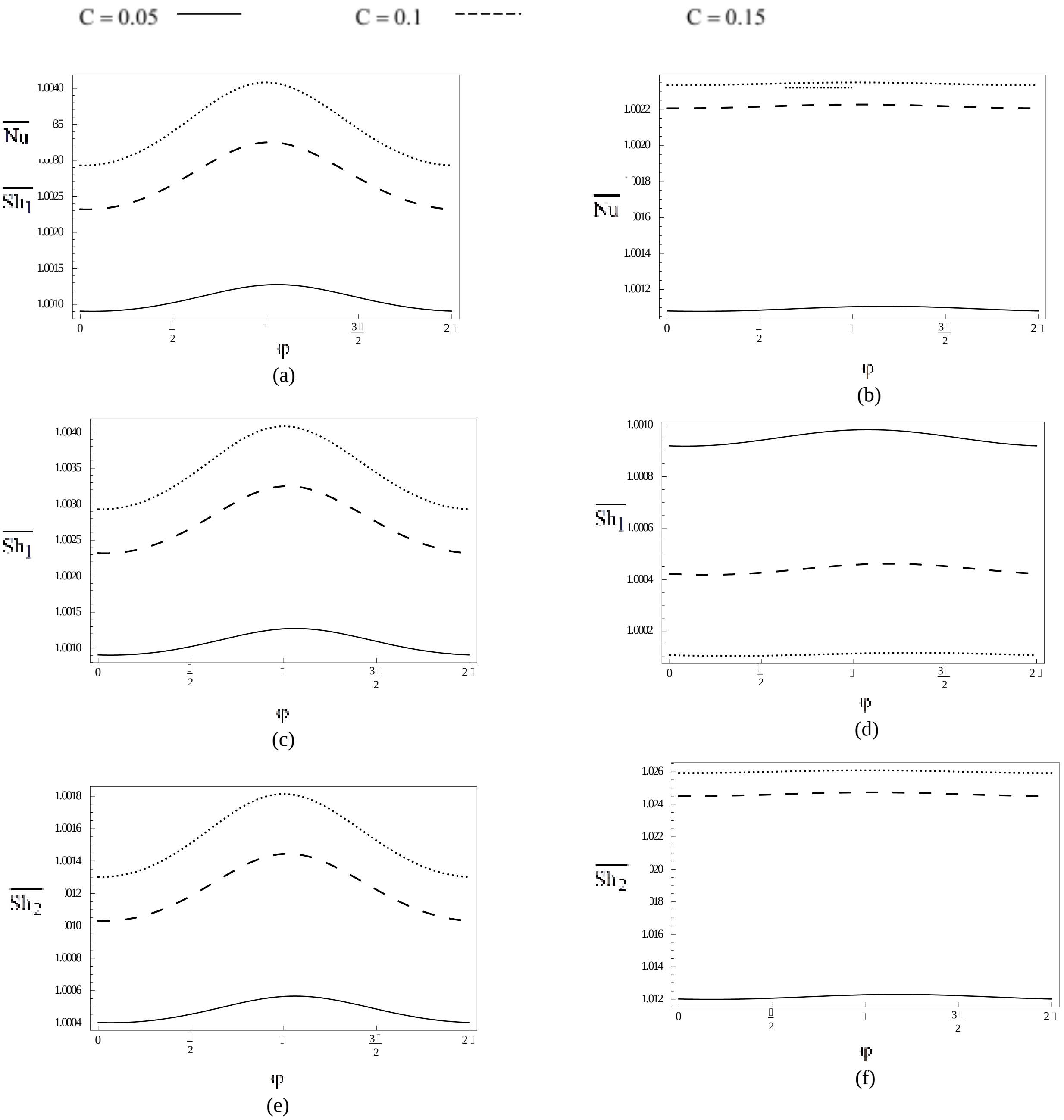

(a) (b) (c) (d) (e) (f)

Figure 2: Effect of φ (a) $\overline{Nu}$ for $\gamma = 10$ (b) $\overline{Nu}$ for $\gamma = 100$ (c) $\overline{Sh_1}$ for $\gamma = 10$ (d) $\overline{Sh_1}$ for $\gamma = 100$ (e) $\overline{Sh_2}$ for $\gamma = 10$ and (f) $\overline{Sh_2}$ for $\gamma = 100$ for different values of C with $R_{S1} = 25, \tau_1 = 0.2, R_{S2} = 50, \tau_2 = 0.3, Pr = 5, \varepsilon = 0.3$

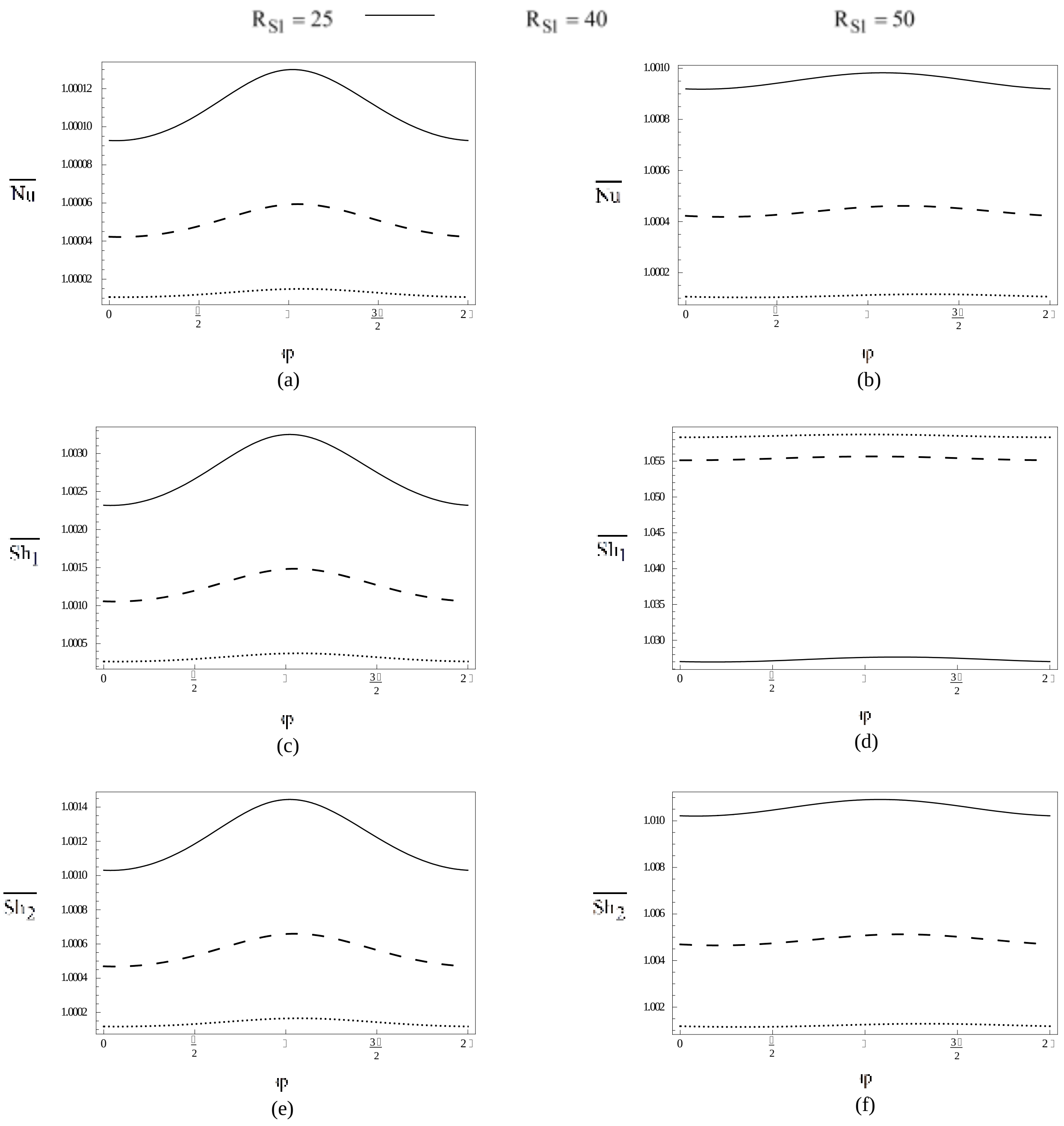

Figure 3: Effect of φ (a) $\overline{Nu}$ for $\gamma = 10$ (b) $\overline{Nu}$ for $\gamma = 100$ (c) $\overline{Sh_1}$ for $\gamma = 10$ (d) $\overline{Sh_1}$ for $\gamma = 100$ (e) $\overline{Sh_2}$ for $\gamma = 10$ and (f) $\overline{Sh_2}$ for $\gamma = 100$ for different values of R_{S1} with $\tau_1 = 0.2, R_{S2} = 50, \tau_2 = 0.3, C = 0.1, Pr = 5, \varepsilon = 0.3$

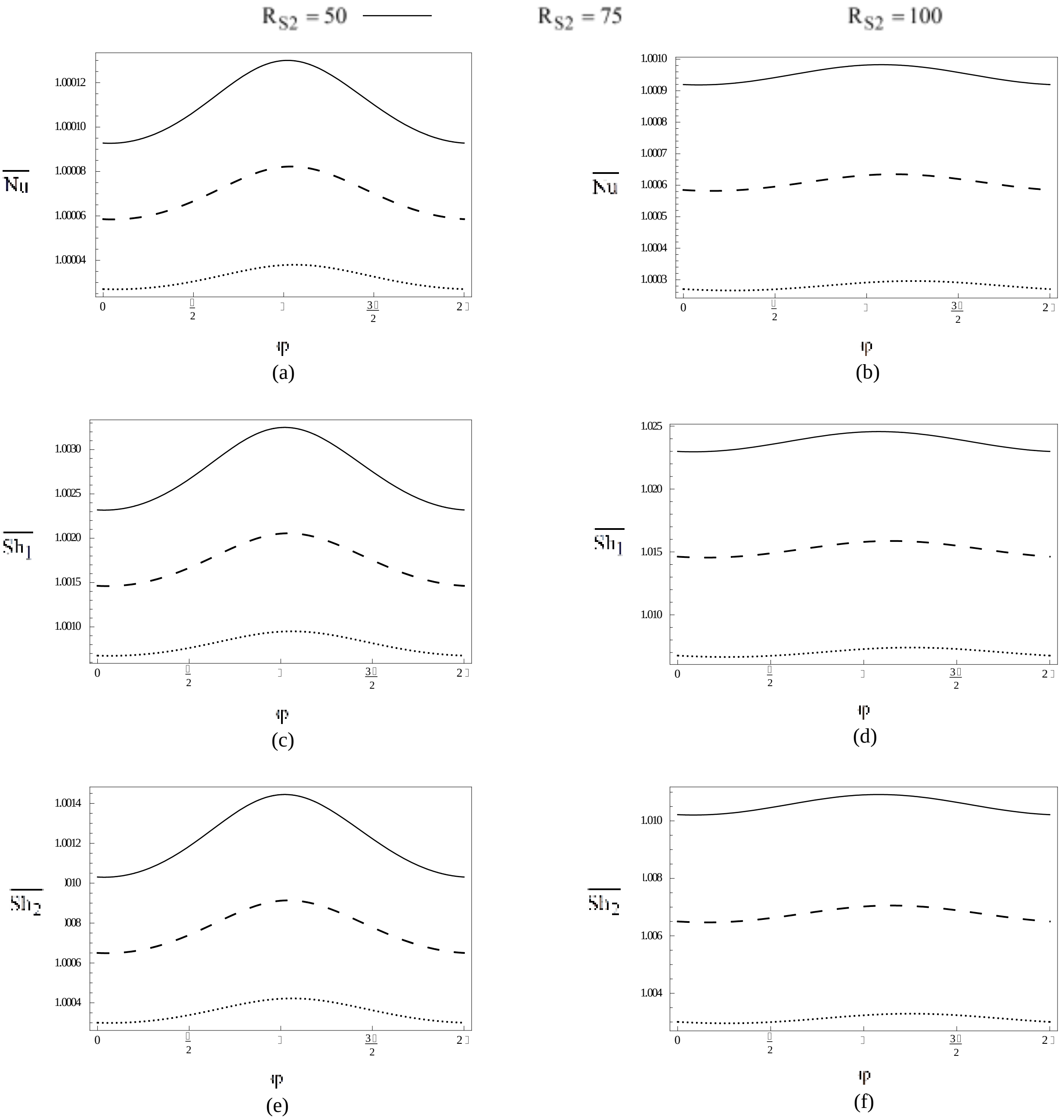

Figure 4: Effect of φ (a) $\overline{Nu}$ for $\gamma = 10$ (b) $\overline{Nu}$ for $\gamma = 100$ (c) $\overline{Sh_1}$ for $\gamma = 10$ (d) $\overline{Sh_1}$ for $\gamma = 100$ (e) $\overline{Sh_2}$ for $\gamma = 10$ and (f) $\overline{Sh_2}$ for $\gamma = 100$ for different values of R_{S2} with $R_{S1} = 25, \tau_1 = 0.2, \tau_2 = 0.3, C = 0.1, Pr = 5, \varepsilon = 0.3$