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Research paper

Magneto-Convective Boundary-Layer Flow of a Nanofluid Past Impulsive Vertical Plate with Inclined Magnetic Field.

A.G. Vijaya Kumar[1,*] , Rahul Gopal Innani [2] and Yash R. Pawar [2]

1Department of Mathematics, SAS, VIT , Vellore-632 014, TN, INDIA
2 Department of Mechanical Engineering, SMEC, VIT, Vellore-632 014, TN, INDIA.
Corresponding Author E-mail: vijayakumarag@vit.ac.in

Abstract

The following paper is a study of unsteady hydro-magnetic flow on a boundary layer over an impulsive nanofluid vertical plate in the sight of transverse magnetic field inclined at an angle. The considered fluid is a non-scattering medium and pressure gradient is described by Boussinesq's approximation and radiative heat flux in the equation of energy. Three types of nanofluids containing Al_2O_3 engine oil, Boron Engine oil and Graphite engine oil are considered by us. The partial differential equations are converted to dimensionless with particular dimensional quantities and then Laplace transform technique is used to get a closed form solution for velocity and temperature fields without any limits. From the obtained graphs, we studied the changes according to given values of nanofluids on the temperature, transverse velocity, skin-friction and the heat transfer rate at the surface of the wall.

Keywords: *Engine oil; Hydrodynamic flow; Inclined magnetic field; Laplace transform; Nano fluids*

1. Introduction

Nanomaterials are currently used in various fields due to their attractive properties and applications. They have massive contribution in fields like electronics, material sciences and medicine. Choi [2] defined nanofluid as a base fluid with strong nanoparticles. Nanofluids are better than the liquids used in heat transfer applications because they have high thermal conductivity and hence have the ability for slight improvement in heat transfer. Due to various applications in fins designs, manufacturing processes, steel rolling and various aircraft devices, investigation of MHD has attracted many researches. Das [5] studied the flow of natural convection magneto-nanofluid on a vertical flat surface. Satya Narayana et al. [8] and Turkyilmazoglu [10] analyzed the unsteady hydro magnetic radiative nanofluid flow on a flat surface. Chamkha [1] made a study fluid containing nanoparticles with vertical surface within the vision of magnetic field. Chio et al. [3] observed a two times rise in the thermal conductivity of nanofluid having Al_2O_3-water and TiO_2-water nanoparticles past a small temperature increase from 21-51℃. The size and concentration were varied form 0%-5% and 25nm-100nm for Au-water nanofluid to study the concentration and variation of size effects of the nanoparticles.

In this paper, we have considered three types of nanofluids: Aluminium Engine oil, Boron Engine oil and Graphite Engine oil. We aim to study the hydro magnetic flow on a boundary layer in nanofluids on an upright plate in inclined transverse field of magnet. The dimensionless leading attached uses Laplace transform technique to solve linear partial differential equations.

2. Mathematical Analysis

By Considering impulsive vertical flat plate having an unsteady flow of a nanofluid with in the sight of thermal radiation affected by the inclined transverse magnetic field, where field of magnetism is kept constant with respect to flowing material to with respect to the steady plate. On *x*-axis movement in upward direction is present while y axis is perpendicular to it. Initially, both the surface and adjacent fluid have exact same temperature with exact same species concentration in the static condition for every one of the focuses in whole stream area. At the time t > 0 vertical motion of plate starts with velocity against to the gravitational field. At the same time a constant temperature is raised near the wall. A strength of magnetism is at alpha angle to the vertical. qr is the radiative heat flux acting against the flow of fluid. We have considered three Alumnium oxide, boron, graphite various significance of nanofluids containing. Material phase and the suspended nanoparticles are assumed to be in thermal equilibrium. Nanofluids representing thermophysical properties shown in following table.

Physical properties	Base fluid	Boron	Al_2O_3
ρ (kg/m^3)	751	2500	3970
c_p (J/kg. K)	2.06	1105	765
k (W/m. K)	0.1164	27.6	40
$\beta \times 10^{-5}(K^{-1})$	21	0.70	0.85
σ $(\Omega.m)^{-1}$	0.05	2×10^{-7}	1×10^{-10}

Based on these assumptions, the fluid flow can be represented by the following system of governing equations:

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma_{nf}B_0^2 u\,,$$

B0=Bsin(alpha) (1)

$$(\rho c_p)_{nf}\frac{\partial T}{\partial t}=k_{nf}\frac{\partial^2 T}{\partial t^2}-\frac{\partial q_r}{\partial y} \quad (2)$$

if the field of magnetism adjusted as for the plate, equation of momentum(1) is replaced by (see [Cramer and Pai [4])

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma_{nf}B_0^2\left[u-u_0 f(t')\right] \quad (3)$$

With (1) and (3)

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma_{nf}B_0^2\left[u-K u_0 f(t')\right] \quad (4)$$

here $K=\begin{cases}0 \text{ if } B_0 \text{ is fixed relative to the fluid}\\ 1 \text{ if } B_0 \text{ is fixed relative to the plate}\end{cases}$

the implusive motion has been given to the plate, $f(t')=\exp(a_0't')$, where a_0' represents dimensionless accelerating parameter.

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\ (\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\phi(\rho c_p)_s,$$

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\ \sigma=\frac{\sigma_s}{\sigma_f},$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s,$$

$$\sigma_{nf}=\sigma_f\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right] \quad (5)$$

$$k_{nf}=k_f\left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right] \quad (6)$$

In view Eqs. (1) to (6), respectively.

The given initial and end boundary conditions are given below:

$$t\le 0: u=0,\quad T=T_\infty,\quad \text{for all}\quad y\ge 0$$
$$t>0: u=u_0 f(t'),\quad T=T_w\quad \text{at}\quad y=0$$
$$u\to 0,\quad T\to T_\infty,\quad \text{as}\quad y\to\infty$$

Radiative heat flux can be obtained through following way:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (7)$$

where $\sigma^*(=5.67\times10^{-8}W/m^2K^4)$ thus:

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)---- \quad (8)$$

Ignoring superior order terms in Eq. (8),

$$T^4\cong T_\infty^4+4T_\infty^3(T-T_\infty)$$
$$T^4\cong T_\infty^4+4TT_\infty^3-4T_\infty^4$$
$$T^4\cong 4TT_\infty^3-3T_\infty^4 \quad (9)$$

Equations (7) and (9) is simplified as

$$\frac{\partial T}{\partial t}=\frac{1}{(\rho c_p)_{nf}}\left(k_{nf}+\frac{16\sigma^*T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \quad (10)$$

Introducing non-dimensional variables

$$\eta=\frac{u_0 y}{v_f},\ \tau=\frac{u_0^2 t}{v_f},\ u_1=\frac{u}{u_0},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ t=\frac{t'u_0^2}{v},\ a_0=\frac{a'v}{u_0^2} \quad (11)$$

Substituting from equation (11) into the equations (4) and (10) becomes

$$\frac{\partial u_1}{\partial\tau}=a_1\frac{\partial^2 u_1}{\partial\eta^2}+Gra_2\theta-M^2a_3(u_0-Ke^{a_0t}) \quad (12)$$

$$\frac{\partial\theta}{\partial\tau}=a_4\frac{\partial^2\theta}{\partial\eta^2} \quad (13)$$

where

$$x_1=\left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right],$$

$$x_2=\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right],\ x_3=\left[(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right],$$

$$x_4=\left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right],$$

$$x_5=\left[1+\frac{3(\frac{\sigma_s}{\sigma_f}-1)\phi}{(\frac{\sigma_s}{\sigma_f}+2)-(\frac{\sigma_s}{\sigma_f}-1)\phi}\right],\ a_1=\frac{1}{(1-\phi)^{2.5}x_1},$$

$$a_2=\frac{x_2}{x_1},\ a_3=\frac{x_5}{x_1},\ a_4=\frac{1}{x_3\,\text{Pr}}(x_4+Nr) \quad (14)$$

and $M^2=\frac{\sigma_f B_0^2 v_f}{\rho_f u_0^2}$, is a magnetic parameter, $Nr=\frac{16\sigma^*T_\infty^3}{3k_f k^*}$ the radiation parameter $\text{Pr}=\frac{\mu_f c_p}{k_f}$, Prandtl number and $Gr=\frac{g\beta_f v_f(T_w-T_\infty)}{u_0^3}$ the Grashof number.

associated conditions in dimensionless form are shown below:

$$t\le 0: u_1=0,\quad \theta=0,\quad \text{for all}\quad \eta\ge 0$$
$$t>0: u_1=\exp(a_0 t),\quad \theta=1,\quad \text{at}\quad \eta=0$$
$$u_1\to 0,\quad \theta\to 0,\quad \text{as}\quad \eta\to\infty \quad (15)$$

Equation (12) and (13) are solved by using laplace,

$$\theta(\eta,\tau) = erfc\left(\frac{\eta}{2\sqrt{a_4\tau}}\right) \tag{16}$$

$$u_1(\eta,\tau) = \frac{(1-a_9)e^{(a_0\tau)}}{2}\left[\exp(y\sqrt{b_0})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_0a_1\tau}\right)+\exp(-y\sqrt{b_0})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_0a_1\tau}\right)\right]$$

$$+\frac{a_8}{2}\left[\exp(y\sqrt{b_1})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_1a_1\tau}\right)+\exp(-y\sqrt{b_1})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_1a_1\tau}\right)\right]$$

$$-\frac{a_8e^{(a_7\tau)}}{2}\left[\exp(y\sqrt{b_2})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_2a_1\tau}\right)+\exp(-y\sqrt{b_2})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_2a_1\tau}\right)\right]$$

$$+a_9\exp(-a_5\tau)erfc(\frac{\eta}{2\sqrt{a_1\tau}}) - a_8erfc(\frac{\eta}{2\sqrt{a_4\tau}})$$

$$+a_9\exp(a_0\tau) + a_9\exp(-a_5\tau)$$

$$+\frac{a_8e^{(a_7\tau)}}{2}\left[\exp(y\sqrt{b_3})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_3a_1\tau}\right)+\exp(-y\sqrt{b_3})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_3a_1\tau}\right)\right] \tag{17}$$

Where

$$a_5 = a_3M^2,\ a_6 = \frac{-Gra_2a_4}{a_1-a_4},\ a_7 = \frac{a_5a_4}{a_1-a_4},\ a_8 = \frac{a_6}{a_7},$$

$$a_9 = \frac{a_5K}{a_0+a_5},\ b_0 = \frac{a_0+a_5}{a_1},\ b_1 = \frac{a_5}{a_1},\ b_2 = \frac{a_5+a_7}{a_1},\ b_3 = \frac{a_7}{a_4}$$

Skin-friction:

We study the rate of change of transverse velocity at the surface of the wall which is given in dimensionless form as follows:

$$\tau_x = -\frac{1}{(1-\phi)^{2.5}}\left[\frac{\partial u_1}{\partial\eta}\right]_{\eta=0}$$

(18)

In view Eqs. (17) and (18), we get:

$$\tau_x = (1-a_9)e^{(a_0\tau)}\left(\sqrt{b_0}erf\left(\sqrt{b_0a_1\tau}\right)+\frac{e^{-a_1b_0\tau}}{\sqrt{\pi a_1\tau}}\right)$$

$$+a_8\left(\sqrt{b_1}erf\left(\sqrt{b_1a_1\tau}\right)+\frac{e^{-a_1b_1\tau}}{\sqrt{\pi a_1\tau}}\right)$$

$$-a_8\left(\sqrt{b_2}erf\left(\sqrt{b_2a_1\tau}\right)+\frac{e^{-a_1b_2\tau}}{\sqrt{\pi a_1\tau}}\right) - a_9\frac{e^{-a_5\tau}}{\sqrt{\pi a_1\tau}}$$

$$+a_8\frac{1}{\sqrt{\pi a_4\tau}} + a_8e^{(a_7\tau)}\left(\sqrt{b_3}erf\left(\sqrt{b_3a_4\tau}\right)+\frac{e^{(-a_4b_3\tau)}}{\sqrt{\pi a_4\tau}}\right) \tag{19}$$

Nusselt Number:

Change of temperature at the surface of the wall:

$$Nu = -\frac{k_{nf}}{k_f}\left[\frac{\partial\theta}{\partial\eta}\right]_{\eta=0} = -x_4\left[\frac{\partial\theta}{\partial\eta}\right]_{\eta=0} \tag{20}$$

In view Eqs. (16) and (20)

$$Nu = x_4\left[\frac{1}{\sqrt{a_4\pi\tau}}\right] \tag{21}$$

3. Results and Discussion

Boundary condition (15) is obtained from equation (12) and (13) by using Laplace transform. To check if the obtained result is correct or not it's been compared with previously found result of Das and Jana where field of magnet is considered to be constant with respect to the flowing fluid as K=0. Regardless of this original research we have obtained much more precise results for magnetic field at certain angles with varying magnetic field. The obtained result is then represented Graphically with various deciding factors like parameter of magnet parameter of radiation (Nr), time parameter of volume fraction and thermal Grashof number (Gr) and using this Graphs are been plot with change in velocity, skin fraction, Prandtl number, and radiative coefficient shown in Figures 2 9.

The nature of fluid temperature when subjected to various values of nanofluid is shown in fig 2 to 3. Now as we move on to figure 2 and 3 we see results related to Nr and the angles at which magnetic field is applied and is been observed that as increasing the Nr values of Al_2O_3-engine oil temperature distribution increases and with C0 its decreases . It is seen that the fluid temperature increases as Nr increment respecting to the conduction effect of the nanofluid increases in the existence of thermal radiation. As per observation increment in values of Nr higher heat flux is also generated and that's help in generation in temperature inside the boundary layer of thickness

Now moving on to next graphs (fig 4- 5) are the graphs between the variation in velocity with respect to all three nanofluids and variation parameters mentioned above. From Figure 4 it is seen that the change in velocity distribution for Al2O3-engine oil and B-engine oil and C-engine oil, Al2O3-engine oil having least of variation while C-engine oil having highest variation in velocity. Now with Figure 5 depicts that the velocity variation with various magnetic parameter. It is understood that as magnetic parameter increases the velocity also increases for both k=0 and k=1. It is due to the application of inclined transverse magnetic field will result a Lorentz force, which has one component in direction of fluid flow and helps in increment of velocity.

Figure 6 shows the variation in Nusselt number due to all three nanofluids.as shown in the graph variation in maximum in Al2O3-engine oil and lesser for boron and graphite. Figure 7 is the description of change in C0, as increase in value increase in Nusselt number Moving on to next curve we see results related to skin fraction drawn against the dimensionless time as shown in figure 8 skin fraction is maximum for Al2O3-engine oil. And lesses for boron and graphite with respect to Al2O3-engine oil. From Figure 9, we can obtained the result as due to variation of angles at which magnetic field is applied.

The unsteady flow of boundary-layer past impulsive upright flat plate of a nanofluid in the company of thermal radiation in the existence of an inclined transverse magnetic field. The result shown for different fluid characteristics like the temperature, transverse velocity, friction at skin and the rate of transfer of heat at the surface of the wall are also under the impression. The obtained results are good in agreement with those studied by Das and Jana. With the above results the following conclusions are made:

- Effect of volume fraction or radiation parameter on fluid is it enhances the temperature.
- With increasing Pr Nano fluid temperature decreases.
- Effect on velocity is it decreases with increase in Nr.
- The friction at skin at the surface of the wall for Al2O3-engine oil nanofluid is found to be lesser than other particles.

The following study has various applications involving heat transfer and different applications, for example, heat exchangers, cooling of a metallic plate, coating detergency, and suspensions, geothermal and oil recovery, materials processing exploiting and paper production.

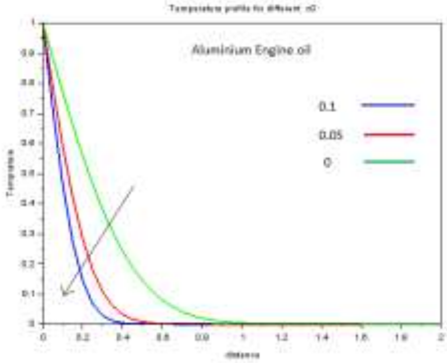

Fig. 2: Temperature profile for different C_0

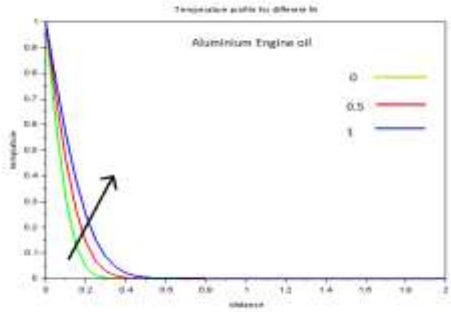

Fig. 3: Temperature profile for different *Nr*

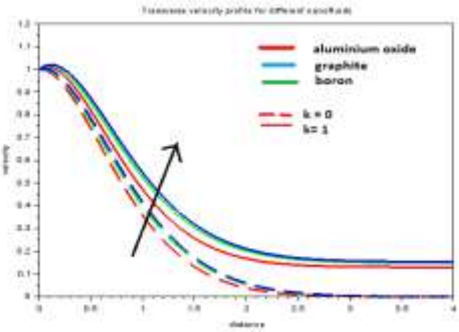

Fig. 4: Transverse velocity profile nanofluids

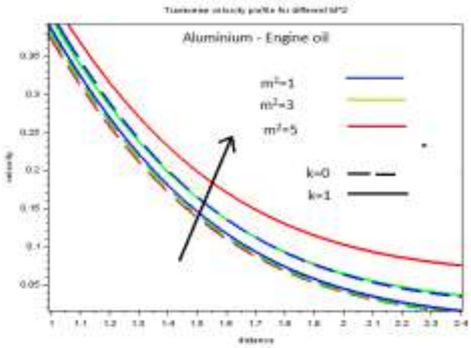

Fig. 5: Transverse velocity profile for different M^2

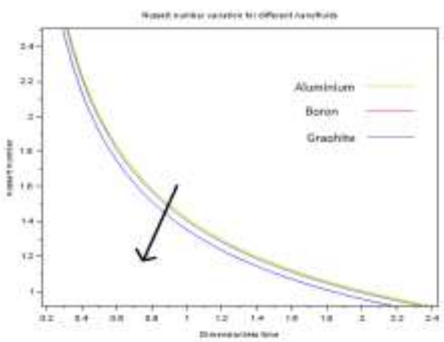

Fig. 6: Nusselt number variation for different nanofluids

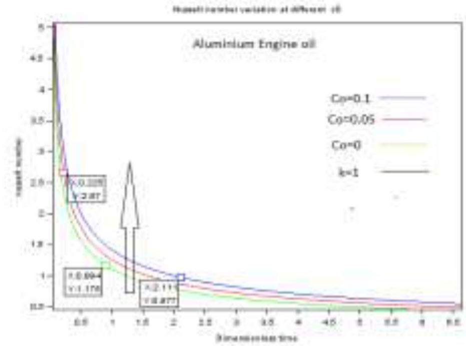

Fig. 7: Nusselt number variation at different C_0

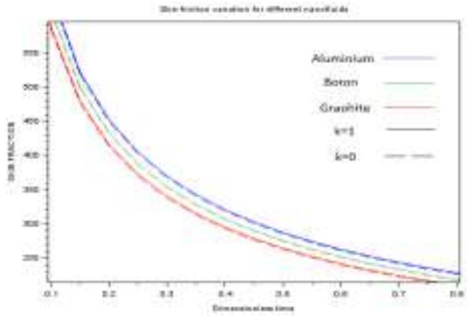

Fig. 8: Skin-friction variation for different nanofluids

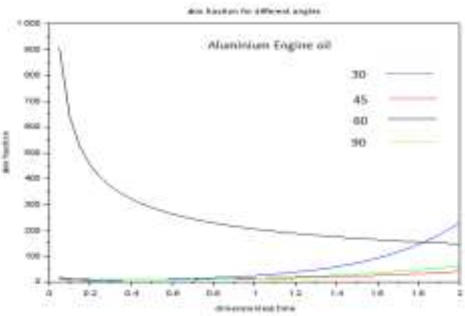

Fig. 9: Skin-friction variation for different values of alpha.

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