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Research paper

Inverse Connected and Disjoint Connected Domination Number of a Jump Graph

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Abstract

Let D be the minimum connected dominating set of a jump graph $J(G)$. If $V-D$ of $J(G)$ contains a connected dominating set D', then D' is called the inverse connected dominating set of the jump graph $J(G)$. The minimum cardinality of an inverse connected dominating set is the inverse connected domination number of the jump graph, denoted by$\gamma_c{}^{-1}[J(G)]$. The disjoint connected domination number,$\gamma_c\,\gamma_c$ of the jump graph $J(G)$, is the minimum cardinality of the union of two disjoint connected dominating set of $J(G)$. In this paper we have established bounds, exact values of $J(G)$and graph theoretic relations between the inverse connected domination number of the jump graph with other parameters of G.

Keywords: *Domination number of a jump graph, Inverse domination number of a jump graph, connected domination number of a jump graph , Inverse connected dominating set and Inverse connected Domination number of a jump graph, Well dominating number of a jump graph, Disjoint connected dominating set of a jump graph.*

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1. Introduction

Let G(p, q) be a finite, simple, connected, undirected graph with p vertices and q edges. For all notations and terminology we refer to [1,2,3].

The line graph L(G) of G, is a graph whose vertices are the edges of G. Any two vertices in L(G) are adjacent if and only if their corresponding edges are adjacent in G. The complement of line graph L(G) is called as the **jump graph of G**, denoted as $J(G)$.Thus, the jump graph $J(G)$of a graph G is the graph defined on the edge set E of G, where two vertices are adjacent if and only if their corresponding edges are not adjacent in G. The isolated vertices of the graph G (if any) plays no part in both line graph L(G) and jump graph $J(G)$,as both the line and jump graphs are defined on the edges set of G.

A subset $D \subseteq V$(G) is a dominating set if every vertex in$V-D$ is adjacent to some vertex in D. The domination number,$\gamma(G)$ of G is the minimum cardinality of the dominating set of G. Let D be the minimum dominating set of the graph G. If $V-D$ contains a dominating set $D'$of G, then D' is called an inverse dominating set of G with respect to D. The minimum cardinality of an inverse dominating set is the inverse domination number,$\gamma^{-1}(G)$.

A subset $D \subseteq V[J(G)]$ is called the dominating set of$J(G)$, if every vertex not in D is adjacent to a vertex in D. The domination number, $\gamma[J(G)]$ is the minimum cardinality of adominating set in $J(G)$.Let $D \subseteq V[J(G)]$ be the minimum dominating set of $J(G)$ of the graph G. If,$V-D$ contain a dominating set D',then D' is called the inverse dominating set of the jump graph $J(G)$ with respect to the set D of $J(G)$. The inverse domination number,$\gamma^{-1}[J(G)]$ of the jump graph, is the minimum cardinality of the inverse dominating set of$J(G)$[5,6]

A graph G is said to be a well dominated graph if the cardinality of all the minimal dominating set of G are equal.

The disjoint domination number,$\gamma\gamma(G)$ of G is the minimum cardinality of two disjoint dominating sets in G[9,10].

In this paper, we investigate two parameters, inverse connected domination number of a jump graph and the disjoint connected domination number of a jump graph.

2. Basic Definitions

Definition: 2.1.Let $D \subseteq V[J(G)]$ be the minimum connected dominating set of a jump graph $J(G)$. If$V-D$of $J(G)$ contains a connected dominating set D', then D' is called the inverse connected dominating set of the jump graph $J(G)$ with respect to D.The inverse connected domination number $\gamma_c^{-1}[J(G)]$, is the minimum cardinality of an inverse connected dominating set of $J(G)$.

Definition:2.2.The inverse connected dominating set of $J(G)$ with maximum cardinality is said to be the upper inverse connected domination number of$J(G)$, denoted by $\Gamma_c{}^{-1}[J(G)]$

3. Inverse Connected Domination Number of a Jump Graph

Theorem: 3.1.

Exact values of standard graphs

(i). For any Path P_p with $p \geq 6, \gamma_c^{-1}[J(G)] = 2$.

(ii). For any cycle C_p with $p \geq 6$, $\gamma_c^{-1}[J(G)] = 2$.

(iii). For the complete graph K_p,

$$\gamma_c^{-1}[J(G)] = 3, \ if \ p \geq 6$$

(iv). For any complete bipartite graph k_{p_1,p_2}

$$\gamma_c^{-1}[J(G)] = \begin{cases} 4, & p_1 = 2, p_2 \geq 3 \\ 3, & 3 \leq p_1 \leq p_2 \end{cases}$$

(v). For wheel graph W_p,

$$\gamma_c^{-1}[J(G)] = \begin{cases} 4, & p = 5 \\ 3, & p = 6 \\ 2, & p \geq 7 \end{cases}$$

(vi). For Corona graph of two graphs,

a). $G = C_p \circ K_1$. $\quad \gamma_c^{-1}[J(G)] = 2, \ p \geq 4$

b). $G = P_p \circ K_1$. $\quad \gamma_c^{-1}[J(G)] = 2, p \geq 4$

(vii).For Petersen graph, G =(10, 15), $\gamma_c^{-1}[J(G)] = 2$.

Remark:3.2.

If the graph G contains atleast an edge e, such that deg(e) =q−1, where q is the size of G, the corresponding jump graph $J(G)$ of G will have more than one component. (i.e) $J(G)$of G is a disconnected graph.

Remark: 3.3.

If there exist an edge e in G such that deg(e) =q−2, then the corresponding jump graph $J(G)$ of G will have a pendent vertex.

Remark : 3.4.

Inverse connected domination number of a jump graph does not exist for all graphs G .

Proposition: 3.5. [7] For any connected graph G,

$\gamma_c^{-1}[J(G)] \geq 2$.

Proof: Let G be a simple connected graph, then the jump graphs $J(G)$ will be of order, $|V[J(G)]| = |E(G)| = q$.Hence, if $\gamma_c^{-1}[J(G)]$ exist then it has to be greater than or equal to 2.

Theorem:3.6. **[7]**For any connected graph G with size q, $2 \leq \gamma_c^{-1}[J(G)] \leq \lfloor q/2 \rfloor$. Bound is sharp for $P_6$and C_6.

Proof: The lower bound is attained since it is a connected set and the upper bound is obvious by Ore ,as, q is the vertex set of $J(G)$.

Let the graph be either a path or a cycle on six vertices , then, equality holds.

Observation: 3.7. Let G(p, q) be a connected graph, then

$$\gamma_c[J(G)] + \gamma_c^{-1}[J(G)] \leq q - 1 .$$

For the path graph P_6 the boundis sharp.

Proof: Since the γ_c and γ_c^{-1}- sets of the jump graph has its value greater than or equal to 2, we must have, $|V[J(G)]| = |E(G)| \geq 4$. Suppose q = 4, then $\gamma_c^{-1}[J(G)]$- set does not exist for G. Thus $|E(G)| = q \geq 5$.

Path graph P_6 satisfies the equation.

Observation: 3.8.Let $J(G)$be the jump graph of a connected graph G, then

$$\gamma^{-1}[J(G)] + \gamma_c^{-1}[J(G)] \leq q - 1.$$

Further, equality holds if G= P_6 .

Theorem : 3.9. [7]For any connected graph G,

$\gamma[J(G)] \leq \gamma_c^{-1}[J(G)] \leq \left\lfloor \frac{q}{2} \right\rfloor$.

Bound is sharp for P_6.

Proof: Since every inverse connected dominating set of the jump graph $J(G)$ is the dominating set of $J(G)$,we have, $\gamma[J(G)] \leq \gamma_c^{-1}[J(G)]$. Also, $|E(G)| = |V[J(G)]|$, by ore,$\gamma_c^{-1}[J(G)] \leq \left\lfloor \frac{q}{2} \right\rfloor$.

Theorem :3.10. For any connected graph G with $diam(G) \leq 2$, we have,

$\gamma_c^{-1}[J(G)] \leq 1 + \gamma[J(G)]$.

Bound is sharp, when G =$k_{2,p}$, $p \geq 4$.

Proof: Since $\mathrm{diam}(G) \leq 2$, the induced subgraph,<D'>of the jump graph $J(G)$with respect to the $\gamma_c[J(G)]$- set will be a disconnected graph. Choose a vertex $v' \in V[J(G)] - D$, Such that $d(v', v'_i) = 1$, for any vertex $v'_i$of D' .Then the resulting induced subgraph$< D' \cup \{v'\} >$ will be the minimum inverse connected dominating set of $J(G)$. Thus,$\gamma_c^{-1}[J(G)] \leq 1 + \gamma[J(G)]$.

Theorem :3.11. [7]$\gamma_c^{-1}[J(T)] = 2$,where T is a tree whose diameter is not less than or equal to three.

Proof: For any graph T with diameter less than or equal to three, the jump graph $J(T)$ will not contain inverse connected dominating set, since $J(T)$ will have more than one component. Hence, assume $diam(T) > 3$, then for a $\gamma_c[J(G)]$ set of $J(T)$ we can find a maximum length $(u' - v')$path in T whose induced graph, for the edges adjacent to $u' \ and \ v'$ will form a minimum inverse connected dominating set of $J(G)$.Thus $\gamma_c{}^{-1}[J(T)] = 2$.

Theorem: 3.12. [7]$\gamma_c^{-1}[J(G)] \leq q - \Delta'(G)$, where G is a connected graph with q edges and $\Delta'(G)$ is the maximum edge degree of G.

Proof: Let e_i be an edge of G with maximum edge degree among all other edges $(e_1, e_2, \dots \dots e_q)$. Let E_1be the set of edges adjacent to e_i in G. Then for $D = \gamma_c[J(G)]$ of $J(G)$ we find vertices in $V[J(G)] - D$ which belong to $(E - E_1)$ of G forming a minimal connected dominating set. Thus ,$\gamma_c{}^{-1}[J(T)] \leq q - \Delta'(G)$

Theorem: 3.13 [6]. Let the minimum vertex degree of a connected graph be denoted by $\delta(G)$then,

$$\gamma_c{}^{-1}[J(G)] \leq \delta(G) + 1.$$

Bound is sharp for $k_{2,p}$, $p \geq 4$.

Note:3.14. For the complete graphK_5, $\gamma_c{}^{-1}[J(G)]$ –set does not exists with respect to$\gamma_c[J(G)]$-set.

Theorem:3.15. For the complete graph K_5 ,

$\Gamma_c[J(G)] = \gamma_c{}^{-1}[J(G)] = 5$.

Proof: Consider the complete graph K_5, with the edge set$(e_1, e_2, \dots \dots e_{10})$. Let $e_1 and \ e_2$ be any two edges that are non-adjacent to each other .Choose edgese_i,e_j,ande_k , such that these edges are adjacent to any one of e_1 or e_2 . Then the vertices v'_1, v'_2, v'_i, v'_j and v'_k, corresponding to the edges, e_1,e_2,e_i,e_j,ande_k in the jump graph$J(G)$ will form a dominating$D$whose induced subgraph is connected with maximum cordinality. The remaining vertices of the jump graph forms a connected dominating set with respect to D. Thus,$\Gamma_c[J(G)] = \gamma_c{}^{-1}[J(G)] = 5$.

4. Well Domination in Connected and Inverse Connected Dominating Set of a Jump Graph

Definition:4.1.

The jump graph J(G) of a connected graph G is said to be well dominated if ,$\gamma_c[J(G)] = \Gamma_c[J(G)]$.

Graph with Well domination number :4.2.

1. For P_p ,$p \geq 8$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 2$

2. For a C_p ,$p \geq 6$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 2$

3. For K_p ,$p \geq 6$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 3$

4. For K_{p_1,p_2} ,$p = p_1p_2$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 3$, $p \geq 10$

5. For W_p , $p \geq 7$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 2$

6. Corona Graph G of graphs

(i).For G =$C_p \circ K_1$,$p \geq 5$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 2$

(ii).For G =$P_p \circ K_2$,$p \geq 2$,

$\gamma_c[J(G)] = \Gamma_c[J(G)] = \gamma_c^{-1}[J(G)] = \Gamma_c^{-1}[J(G)] = 2$

7. Jump graph of Petersen graph is also well dominated with$\gamma_c^{-1}[J(G)] = 2$

5. Disjoint Connected Domination Number of a Jump Graph of a Graph

Definition: 5.1.

The disjoint connected domination number of the jump graph of a graph G, denoted by $\gamma_c\gamma_c[J(G)]$, is the minimum cardinality taken over the union of two disjoint connected dominating sets of J(G). Thus, the $\gamma_c\gamma_c[J(G)]-$ pair of J(G) is those two disjoint connected dominating sets whose union has the cardinality $\gamma_c\gamma_c[J(G)]$.

Example: 5.2. The graphs, P_5 *and* C_4 does not have $\gamma_c\gamma_c[J(G)]$.

Remarks: 5.3. From the definition of disjoint connected dominating sets of the Jump graph J(G), it is clear that not all graph has $\gamma_c\gamma_c[J(G)]$–pair.

Exact Value of some Graphs:**5.4**.

1. ForP_p , $p \geq 6$, $\gamma_c\gamma_c[J(G)] = 4$.

2. ForC_p , $p \geq 6$, $\gamma_c\gamma_c[J(G)] = 4$.

3. ForK_p , $p \geq 6$, $\gamma_c\gamma_c[J(G)] = 6$

4. ForK_{p_1,p_2} , $p = p_1p_2$,

$\gamma_c\gamma_c[J(G)] = 6$,$p \geq 8$

5. For W_p, $\gamma_c\gamma_c[J(G)] = \begin{cases} 6, & p = 5 \\ 4, & p = 10 \end{cases}$

6. For $C_p \circ K_{1,}\gamma_c\gamma_c[J(G)] = 4$,$p \geq 4$

$P_p \circ K_{2,}\gamma_c\gamma_c[J(G)] = 4$,$p \geq 2$

7. For Peterson graph, $\gamma_c\gamma_c[J(G)] = 4$

Theorem: 5.5.[8] For any connected graph G, with $\gamma_c^{-1}[J(G)]$

$$\gamma_c\gamma_c[J(G)] = \gamma_c[J(G)] + \gamma_c^{-1}[J(G)].$$

Proof :Since,$\gamma_c[J(G)]$ – set and $\gamma_c^{-1}[J(G)]$- set of the jump graph is the minimum minimal sets of$J(G)$. Thus the theorem.

Theorem:5.6. Let $J(G)$ be the jump graph of a connected graph G for which$\gamma_c^{-1}[J(G)]$ exists, then

$$\gamma_c\gamma_c[J(G)] \leq q - 1.$$

The equality holds when G≅ P_6.

Proof: The theorem follows from theorem 3.7.

Theorem: 5.7.[8] Let G be a connected graph whose jump graph J(G) has at least a pair of minimum connected dominating set $\gamma_c[J(G)]$, then

$$2\gamma_c[J(G)] \leq \gamma_c\gamma_c[J(G)].$$

The bound is sharp for C_p *and* $P_p (p \geq 6)$,

$C_p \circ K_1$ $(p \geq 4)$ *and* $P_p \circ K_2$ $(P \geq 2)$

Proof: Proof of the theorem is obvious from the definition of jump graph $J(G)$.

Bounds equality can be observed from the exact values.

Definition 5.8.

If$\gamma_c\gamma_c[J(G)] = 2\gamma_C[J(G)]$, then the jump graph $J(G)$ is, $\gamma_c\gamma_c[J(G)]$-minimum. $P_p, C_p (p \geq 6)$, and $C_p \circ P_1 (p \geq 4)$, $P_p \circ K_2$ are some graphs for which its jump graphs is$\gamma_c\gamma_c[J(G)] -$ minimim.

Definition 5.9.

If $\gamma_c\gamma_c[J(G)] = q - 1$, then the $J(G)$ is called $\gamma_c\gamma_c - maximum$.

The path graph $P_6$is the graph whose jump graph is $\gamma_c\gamma_c$ −maximum .

Proposition 5.10.[8] Let J(G) be the jump graph of G then,

$\gamma\gamma[J(G)] \leq \gamma_c\gamma_c[J(G)]$.

Proof: Every disjoint connected dominating set of a jump graph is a disjoint dominating set.

Proposition 5.11.[8]

$\gamma\gamma[J(G)] = \gamma_c\gamma_c[J(G)] = 2\gamma_c[J(G)] = 4$, only if G is isomorphic to any one ofP_p, C_p (p ≥ 6), $C_p \circ K_1 (p \geq 4)$,$P_p \circ K_2$ $(p \geq 2)$.

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