International Journal of Engineering & Technology, 7 (4.10) (2018) 558-561

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Optimal Solution of Fuzzy Transshipment Problem Using Generalized Hexagonal Fuzzy Numbers

Kirtiwant P. Ghadle[1*], Sanjivani M. Ingle[2], Ahmed A. Hamoud[3]

[1,2,3] Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad-431004 (M.S.), India.

[3] *Department of Mathematics, Taiz University, Taiz, Yemen*.

**Corresponding author E-mail:* drkp.ghadle@gmail.com

Abstract

In the present paper, the generalized hexagonal fuzzy number has been newly introduced to deal fuzzy transshipment problem. We solved balance and unbalance fuzzy transshipment problem by various methods. Various methods are compared to find the best solution. The objective of our paper is to find the best method for solving transshipment problem. This procedure is illustrated with numerical examples.

Keywords: *BCM; Fuzzy transshipment problem; Generalized hexagonal fuzzy number; MAM'S method; MODI method; Zero suffix method.*

1. Introduction:

Transportation problem deals with shipments directly from the supply point to demand point. But in many situation shipments are allowed between supply points or between demand points. In such situation transshipment problem take an important role. The concept of transshipment problem is introduced by Orden (1956) [1].

The transshipment problem can be converted into a transportation problem. By solving transshipment problem we can minimize the total shipping costs or the total distance or time associated with each shipping route etc. Mohanapriya et al. [6] used trapezoidal fuzzy numbers in transshipment problem and solved by the modified procedure. Abirami [3] solved fuzzy transshipment problem by the proposed algorithm and advantage of this algorithm they do not use any artificial variable. Gayathri et al. [5] solved fuzzy transshipment by the max-min method.

In real life, the parameter of transshipment problem is not always exactly known and stable. For solving such transshipment problem we used generalized fuzzy numbers.

Many authors used the generalized hexagonal fuzzy number in transportation problem. But to the best of our knowledge till now no one has used the generalized hexagonal fuzzy number in transshipment problem.

In section 2 basic definitions are discussed. In section 3 we convert generalized hexagonal fuzzy transshipment problem to a crisp valued transshipment problem by using centroid ranking technique and then solved by various methods. In section 4 conclusion is discussed.

2. Preliminaries:

2.1 Transshipment Problem:

The fuzzy transshipment problem assumes that direct rout exists from each source to each destination. However, there is situation in which units may be shipped from one of the sources to other or to other destination before reaching their final destination. This is called a fuzzy transshipment problem. The purpose of the fuzzy transshipment problem with the distinction between a sources and destination is dropped so that the transportation problem with m sources and n destination gives rise to transshipment problem with m+n sources and m+n destination the basic feasible solution to which problem will involve [(m+n)+(m+n)-1] or 2m+2n-1 basic variables and if we omit the variable appearing in the (m+n) diagonal cells, we are left with (m+n-1) basic variables.

Thus fuzzy transshipment problem may be written as:

Maximize $\tilde{z} = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} \widetilde{c_{ij}}\widetilde{x_{ij}}$

Subject to

$\sum_{j=1, j\neq i}^{m+n} \widetilde{x_{ij}} - \sum_{j=1, j\neq i}^{m+n} \widetilde{x_{ji}} = \widetilde{a_i}$, i = 1, 2, ..., m.

$\sum_{i=1, i\neq j}^{m+n} \widetilde{x_{ij}} - \sum_{i=1, i\neq j}^{m+n} \widetilde{x_{ji}} = \widetilde{b_j}$,

j = m+1, m+2, m+3, ..., m+n,

where, $\widetilde{x_{ji}} \geq 0$, i, j= 1, 2, 3, ..., m+n, $j \neq i$ then the problem is balanced otherwise unbalanced.

The above formulation is a fuzzy transshipment model, the transshipment model is reduced to transportation model:

Maximize $\tilde{z} = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} \widetilde{c_{ij}}\widetilde{x_{ij}}$

Subject to,

$\sum_{j=1}^{m+n} \widetilde{x_{ij}} = \widetilde{a_i}$+T, i = 1, 2, 3,..., m.

$\sum_{j=1}^{m+n} \widetilde{x_{ij}} = T$, i = m+1, m+2, m+3,..., m+n.

$\sum_{i=1}^{m+n} \widetilde{x_{ij}} = T$, j = 1, 2, 3,..., m.
$\sum_{j=1}^{m+n} \widetilde{x_{ij}} = \widetilde{b_j} + T$, j = m+1, m+2, m+3, ..., m+n
Where,
$\widetilde{x_{ji}} \geq 0$, i, j= 1, 2, 3, ..., m+n, j≠ i.

The above mathematical model represents standard transportation problem with (m+n) origin and m+n destination T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount goods can be transshipped at each point, T should be large enough to take care of all transshipment. It is clear that volume of goods transshipped at any point cannot exist the amount produced or received and hence we take T= $\sum_{i=1}^{m} \widetilde{a_i}$ or $\sum_{j=1}^{m} \widetilde{b_j}$.

2.2 Fuzzy Number:

A fuzzy number is generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function [9, 10].

A fuzzy number is a convex normalized fuzzy set on the real line R such that:
1) There exist at least one $x \in R$ $with$ $\mu_{\tilde{A}}(x) = 1$.
2) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

2.3 Generalized Fuzzy Number:

A fuzzy set A is defined on universal set of real numbers is said to be generalized fuzzy number if its membership function has the following attributes:
(i) $\mu_A(x)$: R→ [0, 1] is continuous;
(ii) $\mu_A(x)$: 0 for all $x \in A$ (-∞, a] U [d, ∞);
(iii) $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$;
(iv) $\mu_A(x) = w$ for all $x \in$ [b, c], where $0 < w \leq 1$.

2.4 Generalized Hexagonal Fuzzy Number:

A fuzzy number A is said to be a generalized hexagonal fuzzy number denoted by ,
$A = (a_1, a_2, a_3, a_4, a_5, a_6; w)$, where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and w is its maximum membership degree. Its membership function μ_A is given by:

$$\mu_A(x) = \begin{cases} \frac{w}{2}\frac{(x-a_1)}{(a_2-a_1)}, & for\ a_1 \leq x \leq a_2 \\ \frac{w}{2} + \frac{w}{2}\frac{(x-a_2)}{(a_3-a_2)}, & for\ a_2 \leq x \leq a_3 \\ w, & for\ a_3 \leq x \leq a_4 \\ w - \frac{w}{2}\frac{(x-a_4)}{(a_5-a_4)}, & for\ a_4 \leq x \leq a_5 \\ \frac{w}{2}\frac{(a_6-x)}{(a_6-a_5)}, & for\ a_5 \leq x \leq a_6 \\ 0, & otherwise \end{cases}$$

2.5 Ranking of Generalized Hexagonal Fuzzy Number [2]:

The ranking function of generalized hexagonal fuzzy number $\widetilde{A_H}$= $(a_1, a_2, a_3, a_4, a_5, a_6; w)$ which maps the set of all fuzzy numbers to a set of real numbers.

$$R(\widetilde{A_H}) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18} \times \frac{5w}{18}\right) \quad (1)$$

3. Numerical Examples:

3.1 Balanced Generalized Fuzzy Transshipment Problem:

	$\widetilde{D_1}$	$\widetilde{D_2}$
$\widetilde{D_1}$	(0,0,0,0,0,0)	(0,1,2,3,4,5)
$\widetilde{D_2}$	(3,6,9,12,15,18)	(0,0,0,0,0,0)

	$\widetilde{O_1}$	$\widetilde{O_2}$
$\widetilde{O_1}$	(0,0,0,0,0,0)	(1,2,3,4,5,6)
$\widetilde{O_2}$	(1,5,9,14,15,16)	(0,0,0,0,0,0)

	$\widetilde{D_1}$	$\widetilde{D_2}$	Supply
$\widetilde{O_1}$	(1,3,4, 6,8,10)	(2,4,6, 8,10,12)	(6,8,10, 12,15,19)
$\widetilde{O_2}$	(1,2,6, 7,9,11)	(-1,1,3, 5,6,8)	(5,7,10, 13,18,24)
Demand	(5,6,8, 10,12,15)	(9,11,13, 15,19,23)	

	$\widetilde{O_1}$	$\widetilde{O_2}$	Supply
$\widetilde{D_1}$	(1,4,6,8,10,12)	(0,3,5,7,9,11)	(8,10,12,14,17,19)
$\widetilde{D_2}$	(0,3,5,7,9,11)	(1,2,4,7,8,9)	(7,9,11,13,15,17)
Demand	(7,9,11,13,15,17)	(5,8,11,14,19,24)	

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	Supply
$\widetilde{O_1}$	(0,0,0, 0,0,0)	(1,2,3, 4,5,6)	(1,3,4, 6,8,10)	(2,4,6, 8,10,12)	(6,8,10, 12,15,19)
$\widetilde{O_2}$	(1,5,9, 14,15,16)	(0,0,0, 0,0,0)	(1,2,6, 7,9,11)	(-1,1,3, 5,6,8)	(5,7,10, 13,18,24)
$\widetilde{D_1}$	(1,4,6, 8,10,12)	(0,3,5, 7,9,11)	(0,0,0, 0,0,0)	(0,1,2, 3,4,5)	(8,10,12, 14,17,19)
$\widetilde{D_2}$	(0,3,5, 7,9,11)	(1,2,4, 7,8,9)	(3,6,9, 12,15,18)	(0,0,0, 0,0,0)	(7,9,11, 13,15,17)
Demand	(7,9,11, 13,15,17)	(5,8,11, 14,19,24)	(5,6,8, 10,12,15)	(9,11,13, 15,19,23)	

Using ranking function (1) we get,

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	**Supply**
$\widetilde{O_1}$	0	0.97	1.47	1.94	3.19
$\widetilde{O_2}$	2.87	0	1.68	1.03	3.47
$\widetilde{D_1}$	1.91	1.64	0	0.69	3.69
$\widetilde{D_2}$	1.64	1.45	2.92	0	3.33
Demand	3.33	3.69	2.56	4.10	

By applying Best candidate method the allocations are obtained as follows,

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	**Supply**
$\widetilde{O_1}$	**3.19** 0	0.97	1.47	1.94	3.19
$\widetilde{O_2}$	2.87	**3.47** 0	1.68	1.03	3.47
$\widetilde{D_1}$	**0.14** 1.91	**0.22** 1.64	**2.56** 0	**0.77** 0.69	3.69
$\widetilde{D_2}$	1.64	1.45	2.92	**3.33** 0	3.33
Demand	3.33	3.69	2.56	4.10	

Transportation cost by using Best candidate method is given by,

$0.14\times1.91 + 0.22\times1.64 + 0.77\times0.69 = 1.16$

Comparison table is given below,

Example	Modi Method [6]	Zero suffix Method [8]	Best candidate method [4]	Monalisha,s Approximation Method [7]
1	1.16	1.16	1.16	1.16

3.2 Unbalanced generalized fuzzy transshipment problem:

	$\widetilde{D_1}$	$\widetilde{D_2}$
$\widetilde{D_1}$	(0,0,0, 0,0,0)	(5,10,15, 20,25,30)
$\widetilde{D_2}$	(4,9,14, 19,24,29)	(0,0,0, 0,0,0)
	$\widetilde{O_1}$	$\widetilde{O_2}$
$\widetilde{O_1}$	(0,0,0, 0,0,0)	(1,2,3, 4,5,6)
$\widetilde{O_2}$	(3,6,9, 12,15,18)	(0,0,0, 0,0,0)

	$\widetilde{D_1}$	$\widetilde{D_2}$	**Supply**
$\widetilde{O_1}$	(2,4,6,8,10,12)	(0,2,5,6,7,8)	(2,4,8,11,14,17)
$\widetilde{O_2}$	(0,2,5,7,9,11)	(11,12,13,14,15,16)	(2,3,5,6,7,9)
Demand	(3,6,9,12,15,18)	(1,2,4,5,6,8)	

	$\widetilde{O_1}$	$\widetilde{O_2}$	**Supply**
$\widetilde{D_1}$	(6,7,8,9,10,11)	(1,3,5,7,9,11)	(3,4,5,6,8,9)
$\widetilde{D_2}$	(1,4,6,7,8,9)	(2,4,6,9,11,13)	(1,4,5,7,8,10)
Demand	(0,1,2,3,4,5)	(3,4,5,6,7,8)	

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	**Supply**
$\widetilde{O_1}$	(0,0,0, 0,0,0)	(1,2,3, 4,5,6)	(2,4,6, 8,10,12)	(0,2,5, 6,7,8)	(2,4,8, 11,14,17)
$\widetilde{O_2}$	(3,6,9, 12,15,18)	(0,0,0, 0,0,0)	(0,2,5, 7,9,11)	(11,12,13 14,15,16)	(2,3,5 6,7,9)
$\widetilde{D_1}$	(6,7,8, 9,10,11)	(1,3,5, 7,9,11)	(0,0,0, 0,0,0)	(5,10,15, 20,25,30)	(3,4,5, 6,8,9)
$\widetilde{D_2}$	(1,4,6, 7,8,9)	(2,4,6 9,11,13)	(4,9,14, 19,24,29)	(0,0,0, 0,0,0)	(1,4,5, 7,8,10)
Demand	(0,1,2, 3,4,5)	(3,4,5, 6,7,8)	(3,6,9, 12,15,18)	(1,2,4, 5,6,8)	

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	**Dummy**	**Supply**
$\widetilde{O_1}$	(0,0,0,	(1,2,3,	(2,4,6,	(0,2,5,	(0,0,0,	(2,4,8,

	0,0,0)	4,5,6)	8,10,12)	6,7,8)	0,0,0)	11,14,17)
$\widetilde{O_2}$	(3,6,9, 12,15,18)	(0,0,0, 0,0,0)	(0,2,5, 7,9,11)	(11,12,13 14,15,16)	(0,0,0, 0,0,0)	(2,3,5 6,7,9)
$\widetilde{D_1}$	(6,7,8, 9,10,11)	(1,3,5, 7,9,11)	(0,0,0, 0,0,0)	(5,10,15, 20,25,30)	(0,0,0, 0,0,0)	(3,4,5, 6,8,9)
$\widetilde{D_2}$	(1,4,6, 7,8,9)	(2,4,6 9,11,13)	(4,9,14, 19,24,29)	(0,0,0, 0,0,0)	(0,0,0, 0,0,0)	(1,4,5, 7,8,10)
Demand	(0,1,2, 3,4,5)	(3,4,5, 6,7,8)	(3,6,9, 12,15,18)	(1,2,4, 5,6,8)	(1,2,3, 4,5,6)	

Using ranking function (1) we get,

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	**Dummy**	**Supply**
$\widetilde{O_1}$	0	0.97	1.94	1.34	0	2.59
$\widetilde{O_2}$	2.92	0	1.59	3.75	0	1.48
$\widetilde{D_1}$	2.36	1.67	0	4.86	0	1.60
$\widetilde{D_2}$	1.67	2.08	4.58	0	0	1.64
Demand	0.69	1.53	2.92	1.20	0.97	

By applying Best candidate method the allocations are obtained as follows:

	$\widetilde{O_1}$	$\widetilde{O_2}$	$\widetilde{D_1}$	$\widetilde{D_2}$	**Dummy**	**Supply**
$\widetilde{O_1}$	**0.69** 0	**0.05** 0.97	**1.32** 1.94	1.34	**0.53** 0	2.59
$\widetilde{O_2}$	2.92	**1.48** 0	1.59	3.75	0	1.48
$\widetilde{D_1}$	2.36	1.67	**1.60** 0	4.86	0	1.60
$\widetilde{D_2}$	1.67	2.08	4.58	**1.20** 0	**0.44** 0	1.64
Demand	0.69	1.53	2.92	1.20	0.97	

Transportation cost by using Best candidate method is given by,

$0.05\times0.97+1.32\times1.94 = 2.6$

Comparison table is given below,

Example	Modi method	Zero suffix method	Best candidate method	Monalisha,s Approximation method
2	2.6	2.6	2.6	2.6

4. Conclusion:

We have solved the generalized fuzzy hexagonal transshipment problems for balanced as well as unbalanced cases by various methods. The methods Modi method, Zero suffix method, Monalisha's Approximation Method, Best candidate method all are compared. We find that Best candidate method is easy to solve and take less computational time and it minimizes the iteration. Therefore it is best from other methods.

References:

[1] A. Orden, Transshipment Problem, Management Science, Vol.2, No.3, (1956), pp.276-285.

[2] A. Thamaraiselvi and R. Santhi, Solving fuzzy transportation problem with generalized hexagonal fuzzy numbers, IOSR Journal of Mathematics, Vol.11, No.5 (2015), pp.8-13.

[3] B. Abirami and R. Sattanathan, Fuzzy transshipment problem, International Journal of Computer Applications, Vol.46, No.17, (2012), pp.40-45.

[4] M.S. Annie Christi and Malini, Solving transportation problems with hexagonal fuzzy numbers using the best candidates method and different ranking techniques, International Journal of Engineering Research and Applications, Vol.6, No.2, (2016), pp.76-81.

[5] P. Gayathri, K. Kannan and D. Sarala, Max-Min method to solve fuzzy transshipment problem, Applied Mathematical Sciences, Vol.9, No.7, (2015), pp.337-343.

[6] S. Mohanapriya and V. Jeyanthi, Modified Procedure to solve fuzzy transshipment problem by using trapezoidal fuzzy number, International Journal of Mathematics and Statistics Invention, Vol.4, No,4, (2016), pp.30-34.

[7] S. Vimala and S. Krishna Prabha, Fuzzy transportation problem through Monalisha’s approximation method, British Journal of Mathematics and computer, Vol.17, No.2, (2016), pp.1-11.

[8] S. Kumaraguru, B. Sathisheesh Kumar and M. Revathy, Comparative study of various methods for solving transportation problem, International Journal of Scientific Research, Vol.3, No.9, (2014), pp. 244-246.

[9] A.A. Hamoud and K.P. Ghadle, Modified Adomian decomposition method for solving fuzzy Volterra-Fredholm integral equations, J. Indian Math. Soc. Vol.85, No.(1-2) (2018), pp.52-69.

[10] A.A. Hamoud, A.D. Azeez and K.P. Ghadle, A study of some iterative methods for solving fuzzy Volterra-Fredholm integral equations, Indonesian J. Elec. Eng. & Comp. Sci. Vol.11, No.3, (2018), pp.1228-1235.