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# An Application of Goal Programming in Data Envelopment Analysis

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**Abstract**

This research paper describes the transformation of the CCR fractional Programming Problem in to the CCR linear programming problem and the dual of the CCR problem. Here the Dual of CCR problem is viewed as BCC [1] formulation Obtained by axiomatic considerations. A goal programming method for finding common weights in DEA with an improved discriminating power for efficiency was proposed by A. Makui et.al [2]. Hamid Reza Babaee Asil and Sara Fanati Rashdi [3] investigated the relationship between resource allocation problems with the problem of weight control and target selection. M. Izadikhah, R. Roostaee et.al [4] proposed a method in which DEA problem can be solved by transforming it into MOLP formulation. The present research study explains the method of finding extremely efficient DMUs and their marveric indexes, the procedure to increase the discriminating power of DEA and the Process of obtaining a set of multiplier weights common to all the DMUs which leads to Global efficiency. In the following discussion a procedure to improve the discriminative power of DEA with goal Programming is proposed.

***Keywords***: *Decision Making Units (DMU), CCR, DEA, Marveric index, Global efficiency.*

## 1. Introduction

In Data envelopment analysis is a technique, first formulated by Charnes, Cooper and Rhodes [5] as a fractional programming problem. The fractional objective function can be interpreted as virtual output per unit of virtual input foregone. The numerator of the objective function is the weighted sum of all the outputs produced by the DMU and the denominator is the weighted sum of all the inputs combined by the DMU. The multiple inputs and outputs scenario is extended to the other DMUs, in that the virtual output to input ratio is forced not exceed unity. Fair enough, the DMU that evaluates its performance with input and output weights most favourable to it restricts its own performance ratio not to exceed unity. By Charnes and Cooper transformation the fractional programming problem can be transformed into linear programming problem. The linear programming problem so obtained admits constant returns to scale. As the constraints change from one linear programming problem to another and one linear programming is solved one time for one DMU, the input and output weights change from one DMU to another DMU. In many cases multiple optimal solutions occur for a DMU when its optimization problem is solved and the weights picked by the LP software belong to the first optimal solution, in majority cases. The Objective function is the sum of weighted outputs, in which one component is of output weight multiplied with output quantity that measures the contribution of that output to the total efficiency score. For the same DMU the contribution of that output to its total efficiency score is not unique as there exit multiple optimal solutions. It is hard to interpret the role of output component in determining efficiency score when their multiplier weights turn out to be zeroes.

## 2. The CCR Fractional Programming Problem

$$h_0 = Max \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

$$Subject\ to \quad \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \le 1,\ j = 1, 2, \ldots\ldots n \qquad (2.1)$$

The CCR linear programming problem is $h_0 = Max \sum_{r=1}^{s} y_{r0}$

$$Subject\ to \quad \sum_{i=1}^{m} v_i x_{i0} = 1,\ \sum u_r y_{rj} - \sum v_i x_{ij} \le 0$$

and

$$u_r, v_i \ge 0,\ \forall r,i \qquad (2.2)$$

The CCR linear programming problem as postulated above admits constant returns to scale. However, variable returns to scale can be modelled into CCR problem adding a constant to the objective function and to the individual constraints.

$$h_1 = Max\sum_{r=1}^{n} u_r y_{r0} + w, \text{ Subject to } \sum_{i=1}^{m} v_i x_{i0} = 1 \quad (2.3)$$

$$w + \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \text{ and } u_r, v_i \ge 0$$

Where, $w$ is unrestricted sign, $w > 0$ is decreasing returns to scale, $w < 0$ is increasing returns to scale and $w = 0$ is constant returns to scale. When a CCR problem is solved one time for DMU, it is likely that a considerable number of DMUs may attain an efficiency score of 100%, revealing that these DMUs are equally efficient. To further distinguish these DMUs peer counts may use.

The Dual of the CCR problem is $j_0 = Min\lambda$

Subject to $\sum_{j=1}^{n} \lambda_j x_{ij} \le \lambda x_{i0}$ for i=1,2,.......,m, and

$$\sum_{j=1}^{n} \lambda_j y_{rj} \le y_{r0}, \text{ for r=1,2,......s} \quad (2.4)$$

The dual of the CCR problem is viewed as BCC formulation [1] obtained by axiomatic considerations. The production possibility set of BCC formulation is based on the axioms of (i) convexity (ii) in efficiency (iii) ray unbounded ness and (IV) Minimum extrapolation. A decision making unit is inefficient if its efficiency score is less than unity and for efficient DMU efficiency score is 100% and all the slacks vanish.

Efficient DMU is $\sum_{j=1}^{n} \lambda_j^* x_{ij} = x_{i0}$

$$\sum_{j=1}^{n} \lambda_j^* y_{rj} = y_{r0}$$

$$\lambda_0^* = 1 \text{ and } \lambda_j^* = 0, \forall j \ne 0 \quad (2.5)$$

Inefficient DMU is $\sum_{j=1}^{n} \lambda_j^* x_{ij} \le x_{i0}$

$$\sum_{j=1}^{n} \lambda_j^* y_{rj} \ge y_{r0}, \lambda_0^* = 1 \text{ and } \lambda_j^* = 0, \quad (2.6)$$

for at least one j($\ne 0$)

For an inefficient DMU such DMU which correspond to non-zero $\lambda_j^*$ values are said to be its peers. Such peers are efficient DMUs viewed leaders and the efficient DMU is their follower. An efficient DMU that is peer to the largest number of inefficient DMU may be viewed as the most efficient DMU. However, ties often exit and further discrimination of efficient DMU comes to a halt. Another way to distinguish efficient DMU is computing their input and output super efficiencies [5]. For this purpose the DMU are classified into four groups (i) extremely efficient DMUs (ii) efficient but not extremely efficient DMUs (iii) Weakly efficient DMUs and (IV) inefficient DMUs. The super efficiently problems are solved for extremely efficient DMUs. Out of two extremely efficient DMUs the one with larger efficiency score is viewed as more efficient DMU. The efficiency problems often suffer from the problem of infeasibility if an extremely efficient DMU is removed from the reference technology, the production possibility set shrinks. If the removed DMU fails to have a reference DMU then its super efficiency problem is infeasible. It can be shown that if input super efficiency problem is infeasible, then output super efficiency is feasible and if output super efficiency problem is infeasible then the input super efficiency problem is feasible. Infeasibility of super efficiency problems is a setback in differentiating efficient DMUs. One objective of the proposed work is to get a common set of input and output eights for all the DMUs efficient and inefficient. A second objective is to see that these weights are positive for all the inputs and outputs. To accomplish these purposes 'cross efficiency' is taken as a basic tool. The efficiency score based on a common set of weights is called Global efficiency. The LPP solved for a DMU gives input and output weights to the best of its advantage. The efficiency of the DMU evaluated in this manner is called 'own' efficiency. Let DMU evaluates the efficiency of other DMUs with its weights. Consequently, the efficiency scores obtained in this manner are called as 'cross' efficiencies. An efficient DMU evaluated by the other DMU better is a better decision making unit. In aggressive formulation a DMU obtains its weights by minimizing the mean cross efficiency of its peers (all other DMUs expect the DMU in focus). Both the approaches many a times give (i) single set of multiplier weights for all the DMUs and (ii) non-zero multiplier weights. However, the variables dealt with are cross efficiencies but not own efficiencies. Cross efficiency is used to identify maverick DMUs. A maverick DMU enjoys the greatest relative increment when it shifts from peer appraisal to self- appraisal. If $DMU_k$ is efficient, its maverick index is defined as

$$M_k = \frac{h(k,k) - H_k}{H_k}$$

Where $h(k,k)$ is own efficiency of $DMU_k$. $H_k$ is mean cross efficiency of peer evaluation of $DMU^{j^{th}}$ by other DMUs. The higher $M_k$ is the greater maverick is $DMU_k$

## 3. Objectives

The proposed work is to deal with the following objectives.

a) To find efficient DMUs that is extremely efficient.
b) To compute maverick index for the extremely efficient DMUs.
c) To increase the discriminating power of DEA.
d) To obtain a set of multiplier weights common to all the DMUs which leads to global efficiency.

To find the first objective the CCR problem with constant returns to scale is solved for each DMU. To compute the maverick index the cross efficiency based objective function is solved as a secondary goal. The estimates of cross efficiency based multiplier weights are used to compute maverick index for extremely efficient DMUs. The primary goal in the approach is to identify extremely efficient DMUs. To improve the discriminative power of DEA a goal programming problem is postulated. The efficiencies obtained in this manner, let us call 'global efficiencies'. A DEA multi objective fractional problem may be expressed as,

$$Max\left\{\frac{\sum u_r y_{rj}}{\sum v_i x_{ij}}\right\}, \text{ for j=1, 2, ………….. n}$$

$$Subject \text{ to } \frac{\sum u_r y_{rj}}{\sum v_i x_{ij}} \le 1 \text{ and } u_r, v_i \ge 0 \quad (3.1)$$

The problem (3.1) deals with n objectives. It is a fractional programming problem. Optimization of (3.1), one time for one DMU gives the efficiency point (3.1), goal programming problem with n objectives.

Let $h_j^* = \dfrac{\sum u_r^* y_j^*}{\sum v_i^* x_{ij}}$

Optimal efficiency point $(h_1^*, h_2^*, h_3^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots, h_n^*)$. As a secondary goal we solve the following fractional programming problem:

$$Min\ d^- \quad Subject\ to \frac{\sum u_r y_{rj}}{\sum v_i x_{ij}} + d_j = h_j^* \text{ and } (z - d_j) \geq 0, u_r, v_i \geq 0 \tag{3.2}$$

The objective is minimization of mean deviation. $d_j$ is $j^{th}$ deviational variable that measured the departure of efficient score of $j^{th}$ DMU from $h_j^*$.The variable z may be interpreted as the largest value of all the deviations. Another variation of secondary goal is,

$$Subject\ to \frac{\sum u_r y_{rj}}{\sum v_i x_{ij}} + d_j = h_j^*$$

$$(z - d_j) \geq 0, u_r, u_i, d_j \geq 0 \tag{3.3}$$

(3.1) and (3.2) are fractional programming problems that are to be solved by non-linear methods. However, the multi-criteria problem can be reformulated and expressed as a linear programming.

**Goal (1)**:
For each DMU solve the following multiplier problem

$$h_0^* = Max \sum u_r y_{r0}$$

$$Subject\ to \sum vix_{i0} = 1 \text{ and} \sum u_r y_{rj} - \sum v_i x_{ij} \leq 0 \text{ for}$$

j=1,2,……………….,n (3.4)

**Goal (2):**

**(i)** $Min\ d^-$

Subject to $\sum vix_{i0} = 1$

$$\sum u_r y_{rj} - h_j^* \sum v_i x_{ij} + d_{ij} = 0 \text{ and}$$

$$z + \sum u_r y_{rj} - h_j^* \sum v_i x_{ij} \geq 0, where\ z, u, v_i \geq 0 \tag{3.5}$$

$$(ii) Min\ z^-$$

$$Subject\ to \sum vix_{i0} = 1, h_j^* \sum u_r y_{rj} - \sum v_i x_{ij} \leq 0 \tag{3.6}$$

and

$$u_r, v_0, z, d_j \geq 0$$

## 4. Conclusion

In the above research study a goal programming problem has been suggested to increase the discriminating power of DEA and in that manner global efficiency has been obtained. In addition to these the CCR fractional and linear programming problems are proposed and its dual has been derived. Moreover classification of DMU's is also presented in this research article.

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