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Research paper

A Stochastic Model for Three Species

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Abstract

The present work is related to a three species ecosystem including a mutualism interaction between two species and a predator, where the predator is depending on both the mutual species. All three species in this model are considered in limited resources. The sustainability of the system (local stability) is discussed through the perturbed technique at the possible existing each equilibrium points. Using Lyapunov's technique the global stability of the system is also described. Further the nature of the system is observed by introducing the stochastic process to the species and the numerical simulations are studied to know the interaction among the species.

Keywords: *Global stability; Local stability;Lyapunov's technique; Mutual species;predator;Stochastic process.*

1. Introduction

Investigation of biological community in scientific demonstrating is to comprehend the systems that impact the development of the species and their reality and steadiness. Research in the field of hypothetical biology was first started by Lotka [8], Volterra [24]. There after a few mathematicians and hypothetical biologists have added to the development of this region of information as announced in the treatises of Meyer [9], Chusing [2], Freedman [3], Haberman [5], Paul Colinvaux [15], Pielou [16], Kapur [6, 7], Gause [4], Thompson [23]. Later Mukharjee [10, 11 has considered security investigation of a stochastic model for Prey-Predator framework with disease in the prey species. The work related to host- mortal commensal eco system with a host steady reaping rate and furthermore a monad display with commensal association between species were exhibited by Seshagiri rao etl. [17-20]. Suresh etl.,[21] Paparao etl. [12-14] explored a three species environmental model with a prey, predator and contender show with time delay and furthermore built up a model for three species types natural model with prey, predator and a contender to both the prey and predator.Srinivas etl. [22] decided optimal harvesting strategy and stochastic examination for two species commensaling framework.

In the present article, a three species food-web system consisting of mutualism interaction between two species and a predator which is depending on both the mutual species are considered. It is governed by a system of three first order non-linear O.D.E in x_1, x_2 and y. The equation for y is non-linear but de-coupled with those of two mutual species. We observed the eight non-negative equilibrium points for the system and the stability criteria is discussed for all extinct, partially washed out and co-existence equilibrium points. The global stability is also discussed by using Lyapunov's function. The stochastic nature of the model is studied. In the last we presented the numerical solutions of the basic model equations keeping all parameters constants expect some parameters for observing the nature of dynamical system.

2. Model Formulation

Now, the following schematic diagram (Fig. 1) shows the different (two) types interactions among the three species

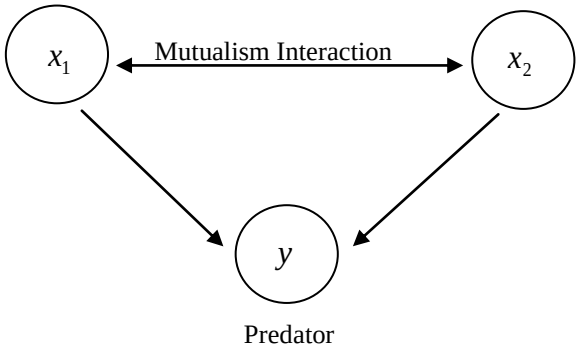


Fig.1: Schematic diagram for three species food-web system

Let x_1 and x_2 are the species having mutual interaction between them and the species y is a predating on both the mutual species with natural growth rates $a_i\,(i=1,2,3)$ and the carrying capacities $\varepsilon_i\,(i=1,2,3)$ respectively. The coefficients Δ_i, δ_i and $\eta_i\,(i=1,2)$ are the mutual, inhibition coefficients of

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mutual species due to the predator and the consumption coefficients of the predator over the mutual species. Based on the interactions among species, this eco-system is governed by the following set of non-linear ordinary differential equations:

$$\frac{dx_1}{dt} = a_{11}x_1\left(\varepsilon_1 - x_1 + \Delta_1 x_2 - \delta_1 y\right)$$

$$\frac{dx_2}{dt} = a_{22}x_2\left(\varepsilon_2 - x_2 + \Delta_2 x_1 - \delta_2 y\right)$$

$$\frac{dy}{dt} = a_{33}y\left(\varepsilon_3 - y + \eta_1 x_1 + \eta_2 x_2\right) \quad (1)$$

With the non-negative initial conditions:

$$x_1(0) \geq 0\,,\, x_2(0) \geq 0\,,\, y(0) \geq 0 \quad (2)$$

3. Local Stability Analysis

In this section, the local stability of the existing positive equilibrium points is discussed. The Jacobean matrix for each equilibrium point $E(x_1, x_2, y)$ is given as:

$$J_E = \begin{bmatrix} a_{11}(\varepsilon_1 - 2x_1 + \Delta_1 x_2 - \delta_1 y) & a_{11}\Delta_1 x_1 & -a_{11}\delta_1 x_1 \\ a_{22}\Delta_2 x_2 & a_{22}(\varepsilon_2 - 2x_2 + \Delta_2 x_1 - \delta_2 y) & -a_{22}\delta_2 x_2 \\ a_{33}\eta_1 x_3 & a_{33}\eta_2 y & a_{33}(\varepsilon_3 - 2y + \eta_1 x_1 + \eta_2 x_2) \end{bmatrix} \quad (3)$$

(i). The trial equilibrium point $E_1 = (0,0,0)$ always exists. The eigen values at E_1 are $a_{11}\varepsilon_1$, $a_{22}\varepsilon_2$, $a_{33}\varepsilon_3$, all three are positive and hence is unstable.

(ii).The axial equilibrium point $E_2 = (0,0,\varepsilon_3)$ always exists. The eigen values at E_2 are $a_{11}(\varepsilon_1 - \delta_1\varepsilon_3)$, $a_{22}(\varepsilon_2 - \delta_2\varepsilon_3)$, $-a_{33}\varepsilon_3$. By observing different possibilities, the point E_2 is locally asymptotically stable if $\varepsilon_1 < \delta_1\varepsilon_3$ and $\varepsilon_2 < \delta_2\varepsilon_3$. Otherwise it will be unstable or saddle point in other cases. The system admits transcritical bifurcation corresponding to zero eigen values if $\varepsilon_1 = \delta_1\varepsilon_3$ and $\varepsilon_2 = \delta_2\varepsilon_3$

(iii). The second axial equilibrium point $E_3 = (0,\varepsilon_2,0)$ always exists. The corresponding eigen values at E_3 are $a_{11}(\varepsilon_1 + \Delta_1\varepsilon_2)$, $-a_{22}\varepsilon_2$ and $-a_{33}(\varepsilon_3 + \eta_2\varepsilon_2)$, E_2 is a saddle point and then sytem is unstable manifold in x_1-direction and stable manifold in x_2 as well as in y.

(iv). The third axial equilibrium point $E_4 = (\varepsilon_1,0,0)$ also exists always. The eigen values at E_4 are $-a_{11}\varepsilon_1$, $a_{22}(\varepsilon_2 + \Delta_2\varepsilon_1)$ and $a_{33}(\varepsilon_3 + \eta_1\varepsilon_1)$, hence E_4 is a saddle and having stable manifold in x_1- direction and unstable manifold in x_2 as well as in y

(v). The first boundary equilibrium point $E_5 = (0, \overline{x_2}, \overline{y})$ in plane $x_2 - y$ is given as:

$$\overline{x_2} = \frac{\varepsilon_2 - \delta_2\varepsilon_3}{1 + \eta_2\delta_2}\,,\ \overline{y} = \frac{\eta_2\varepsilon_2 + \varepsilon_3}{1 + \eta_2\delta_2}$$

The equilibrium point exists if $\varepsilon_2 > \delta_2\varepsilon_3$. The eigen values of E_5 obtained as $\lambda_1 + \lambda_2 = -\left(a_{22}\overline{x_2} + a_{33}\overline{y}\right)$

$$\lambda_1\lambda_2 = \left(1 + \eta_2\delta_2\right)a_{22}a_{33}\overline{x_2}\,\overline{y}$$

$$\lambda_3 = a_{11}\left(\varepsilon_1 + \Delta_1\overline{x_2} - \delta_1\overline{y}\right)$$

Clearly E_5 is stable point in (x_2,y) plane since roots of the characteristic equation are positive.

(vi). The second boundary equilibrium point $E_6 = (\overline{x_1}, 0, \overline{y})$ in the plane $x_1 - y$ is given as:

$$\overline{x_1} = \frac{\varepsilon_1 - \delta_1\varepsilon_3}{1 + \eta_1\delta_1}\ ,\ \overline{y} = \frac{\eta_1\varepsilon_1 + \varepsilon_3}{1 + \eta_1\delta_1}$$

The equilibrium point exists if $\varepsilon_1 > \delta_1\varepsilon_3$.

The eigen values of E_6 are

$$\lambda_1 + \lambda_2 = -\left(a_{11}\overline{x_1} + a_{33}\overline{y}\right)$$

$$\lambda_1\lambda_2 = (1 + \eta_1\delta_1)a_{11}a_{33}\overline{x_1}\,\overline{y}$$

$$\lambda_3 = a_{22}\left(\varepsilon_2 + \Delta_2\overline{x_1} - \delta_2\overline{y}\right)$$

One can observed that the point E_6 has the same stability in the interior of the (x_1, y) plane.

(vii). The third boundary equilibrium point $E_7 = (\overline{x_1}, \overline{x_2}, 0)$ in the plane $x_1 - x_2$ is given as:

$$\overline{x_1} = \frac{\varepsilon_1 + \Delta_1\varepsilon_2}{1 - \Delta_1\Delta_2}\ ,\ \overline{x_2} = \frac{\varepsilon_1\Delta_2 + \varepsilon_2}{1 - \Delta_1\Delta_2}$$

The equilibrium point exists if $1 > \Delta_1\Delta_2$.

The eigen values at E_7 are

$$\lambda_1 + \lambda_2 = -\left(a_{11}\overline{x_1} + a_{22}\overline{x_2}\right)$$

$$\lambda_1\lambda_2 = \left(1 - \Delta_1\Delta_2\right)a_{11}a_{22}\overline{x_1}\,\overline{x_2}$$

$$\lambda_3 = a_{33}\left(\varepsilon_3 + \eta_1 a_{11}\overline{x_1} + \eta_2 a_{22}\overline{x_2}\right)$$

Clearly E_7 has the same stability in the interior of the (x_1, x_2) plane.

(viii). The interior equilibrium point $E_8 = (\overline{x_1}, \overline{x_2}, \overline{y})$ is obtained as:

$$\overline{x_1} = \frac{\varepsilon_1 + \Delta_1\varepsilon_2}{1 - \Delta_1\Delta_2}\ ;\ \overline{x_2} = \frac{\varepsilon_1\Delta_2 + \varepsilon_2}{1 - \Delta_1\Delta_2}\ ;\ \overline{y} = k^* \text{ where}$$

$$k^* = \frac{\left(\varepsilon_3 + \eta_1\varepsilon_1 + \eta_2\varepsilon_2 + \eta_1\Delta_1\varepsilon_2 + \eta_2\Delta_2\varepsilon_1\right) - \Delta_1\Delta_2\varepsilon_3}{1 - \Delta_1\Delta_2}$$

E_8 is feasible under the following parametric restrictions:

$$1 > \Delta_1\Delta_2\ ,$$

$$\left(\varepsilon_3 + \eta_1\varepsilon_1 + \eta_2\varepsilon_2 + \eta_1\Delta_1\varepsilon_2 + \eta_2\Delta_2\varepsilon_1\right) > \Delta_1\Delta_2\varepsilon_3$$

The Jacobian at $E_8 = (\overline{x_1}, \overline{x_2}, \overline{y})$ is

$$J_{E_8}=\begin{bmatrix} -a_{11}\bar{x}_1-\lambda & a_{11}\Delta_1\bar{x}_1 & -a_{11}\delta_1\bar{x}_1 \\ a_{22}\Delta_2\bar{x}_2 & -a_{22}\bar{x}_2-\lambda & -a_{22}\delta_2\bar{x}_2 \\ a_{33}\eta_1\bar{y} & a_{33}\eta_2\bar{y} & -a_{33}\bar{y}-\lambda \end{bmatrix}$$

The characteristic equation is $\lambda^3+k_1\lambda^2+k_2\lambda+k_3=0$, with

$$k_1=a_{11}\bar{x}_1+a_{22}\bar{x}_2+a_{33}\bar{y}$$

$$k_2=a_{11}a_{22}\bar{x}_1\bar{x}_2(1-\Delta_1\Delta_2)+a_{11}a_{33}\bar{x}_1\bar{y}(1+\delta_1\eta_1)+a_{22}a_{33}\bar{x}_2\bar{y}$$

$$k_3=a_{11}a_{22}a_{33}\bar{x}_1\bar{x}_2\bar{y}(1-\Delta_1\Delta_2+\Delta_1\delta_2\eta_1+\Delta_2\delta_1\eta_2+\delta_1\eta_1)$$

The necessary and sufficient conditions for stability of the system by Routh-Hurwitz criterion are $k_1>0$, $k_3>0$, $k_1k_2>k_3$. Here it is clear that $k_1>0$, $k_3>0$ and $k_1k_2>k_3$ and hence, the equilibrium point E_8 is stable.

4. Global Stability of the System by Lyapunov's Function

Here, we discussed the system's global stability of the state of co-existence and investigate thouroughly the global stability to the dynamical system (2.1). It has been already noted that this system has a unique, stable non-trial co-existent equilibrium state at E_8:

$$\bar{x}_1=\frac{\varepsilon_1+\Delta_1\varepsilon_2}{1-\Delta_1\Delta_2}\ ;\ \bar{x}_2=\frac{\varepsilon_1\Delta_2+\varepsilon_2}{1-\Delta_1\Delta_2}\ ;\ \bar{y}=k^*$$

We define a Liapunov's function

$$V(x_1,x_2,y)=x_1-\bar{x}_1-\bar{x}_1\log\left(\frac{x_1}{\bar{x}_1}\right)+l_1\left\{x_2-\bar{x}_2-\bar{x}_2\log\left(\frac{x_2}{\bar{x}_2}\right)\right\}+l_2\left\{y-\bar{y}-\bar{y}\log\left(\frac{y}{\bar{y}}\right)\right\} \tag{4}$$

where l_1 and l_2 are the desirable constants which are to be found in the following steps .

The time derivative of V along the solution of (2.1) is

$$\frac{dV}{dt}=\left(\frac{x_1-\bar{x}_1}{x_1}\right)\frac{dx_1}{dt}+l_1\left(\frac{x_2-\bar{x}_2}{x_2}\right)\frac{dx_2}{dt}+l_2\left(\frac{y-\bar{y}}{y}\right)\frac{dy}{dt} \tag{5}$$

$$\frac{dV}{dt}=\left(\frac{x_1-\bar{x}_1}{x_1}\right)\left[a_{11}x_1(\varepsilon_1-x_1-\Delta_1x_2-\delta_1y)\right]+l_1\left(\frac{x_2-\bar{x}_2}{x_2}\right)\left[a_{22}x_2(\varepsilon_2-x_2+\Delta_2x_1-\delta_2y)\right]+l_2\left(\frac{y-\bar{y}}{y}\right)\left[a_{33}y(\varepsilon_3-y+\eta_1x_1+\eta_2x_2)\right] \tag{6}$$

$$=\left(x_1-\bar{x}_1\right)\left[a_{11}\varepsilon_1-a_{11}x_1-a_{11}\Delta_1x_2-a_{11}\delta_1y\right]+l_1\left(x_2-\bar{x}_2\right)\left[a_{22}\varepsilon_2-a_{22}x_2+a_{22}\Delta_2x_1-a_{22}\delta_2y\right]+l_2\left(y-\bar{y}\right)\left[a_{33}\varepsilon_3-a_{33}y+\eta_1a_{33}x_1+\eta_2a_{33}x_2\right] \tag{7}$$

$$\frac{dV}{dt}=(x_1-\bar{x}_1)\left[-a_{11}(x_1-\bar{x}_1)+a_{11}\Delta_1(x_2-\bar{x}_2)-a_{11}\delta_1(y-\bar{y})\right]+l_1\left(x_2-\bar{x}_2\right)\left[-a_{22}\left(x_2-\bar{x}_2\right)+a_{22}\Delta_2\left(x_1-\bar{x}_1\right)-a_{22}\delta_2\left(y-\bar{y}\right)\right]+l_2\left(y-\bar{y}\right)\left[-a_{33}\left(y-\bar{y}\right)+\eta_1a_{33}\left(x_1-\bar{x}_1\right)+\eta_2a_{33}\left(x_2-\bar{x}_2\right)\right] \tag{8}$$

$$\frac{dV}{dt}=-a_{11}\left(x_1-\bar{x}_1\right)^2-l_1a_{22}\left(x_2-\bar{x}_2\right)^2-l_2a_{33}\left(y-\bar{y}\right)^2-a_{11}\delta_1\left(x_1-\bar{x}_1\right)\left(y-\bar{y}\right)-l_1a_{22}\delta_2\left(x_2-\bar{x}_2\right)\left(y-\bar{y}\right)+a_{11}\Delta_1\left(x_1-\bar{x}_1\right)\left(x_2-\bar{x}_2\right)+l_1a_{22}\Delta_2\left(x_1-\bar{x}_1\right)\left(x_2-\bar{x}_2\right)+l_2\eta_1a_{33}\left(x_1-\bar{x}_1\right)\left(y-\bar{y}\right)+l_2\eta_2a_{33}\left(x_2-\bar{x}_2\right)\left(y-\bar{y}\right) \tag{9}$$

For $l_1=\frac{a_{33}\eta_2}{a_{22}\delta_2}l_2$, $l_2=\frac{a_{11}\delta_1}{a_{33}\eta_1}$ and

$a_{11}\left(x_1-\bar{x}_1\right)^2+l_1a_{22}\left(x_2-\bar{x}_2\right)^2+l_2a_{33}\left(y-\bar{y}\right)^2>a_{11}\Delta_1\left(x_1-\bar{x}_1\right)\left(x_2-\bar{x}_2\right)+l_1a_{22}\Delta_2\left(x_1-\bar{x}_1\right)\left(x_2-\bar{x}_2\right)$

Then (9) becomes, $\frac{dV}{dt}<0$ and hence E_8 is stable globally.

5. The Stochastic Model of the System

The stochastic perturbations were acquainted with the main parameters involved in this model. In the present work, we permit stochastic perturbations to the variables x_1, x_2 and y in the neighborhood of their values at the equilibrium point E_8, when it is viable and locally asymptotically stable. Further one can observe that the equilibrium point E_8 is local stability by following the existence conditions. And hence, in the model (1) we allowed the stochastic perturbations to the variables around their values at E_8 are of white noise type, which are proportional to the distances of x_1, x_2 and y from values $\bar{x}_1$, $\bar{x}_2$ and $\bar{y}$. So that the system (1) becomes

$$\begin{aligned} dx_1&=a_{11}x_1\left[\varepsilon_1-x_1+\Delta_1x_2-\delta_1y\right]+\sigma_1(x_1-\bar{x}_1)d\xi_t^1\\ dx_2&=a_{22}x_2\left[\varepsilon_2-x_2+\Delta_2x_1-\delta_2y\right]+\sigma_2(x_2-\bar{x}_2)d\xi_t^2\\ dy&=a_{33}y\left[\varepsilon_3-y+\eta_1x_1+\eta_2x_2\right]+\sigma_3(y-\bar{y})d\xi_t^3 \end{aligned} \tag{10}$$

where σ_i are real constants, $\xi_t^i=\xi_i(t),\ i=1,2,3$ are independent from each other by standard Wiener processes.

Stochastic stability for the equilibrium point E_8

By introducing the following transformation around the positive equilibrium E_8 the stochastic differential system (10) becomes

$$u_1 = x_1 - \bar{x}_1, u_2 = x_2 - \bar{x}_2, u_3 = y - \bar{y} \tag{11}$$

Now, the linearize stochastic differential equations around E_8 is taken in the form

$$d(u(t)) = f(u(t))dt + g(u(t))d\xi(t) \tag{12}$$

where $u(t) = col(u_1(t), u_2(t), u_3(t))$ and

$$f(u(t)) = \begin{bmatrix} -a_{11}\bar{x}_1 & a_{11}\Delta_1\bar{x}_1 & -a_{11}\delta_1\bar{x}_1 \\ a_{22}\Delta_2\bar{x}_2 & -a_{22}\bar{x}_2 & -a_{22}\delta_2\bar{x}_2 \\ a_{33}\eta_1\bar{y} & a_{33}\eta_2\bar{y} & -a_{33}\bar{y} \end{bmatrix} u(t) \tag{13}$$

$$g(u) = \begin{bmatrix} \sigma_1 u_1 & 0 & 0 \\ 0 & \sigma_2 u_2 & 0 \\ 0 & 0 & \sigma_3 u_3 \end{bmatrix} \tag{14}$$

It is clear that the trial solution for the system (12) around the positive equilibrium is $u(t) = 0$.

Now, let us define a set U such that $U = (t \geq t_0) \times R^n$, where $t_0 \in R^+$. So that the set $V(t,u) \in C_2^0(U)$ is twice continuously differentiable function of u and is also a continuous functions with respect to t. Hence by Afanas'ev et.al [1], the following theorem holds. It can be observe that

$$LV(t,u) = \frac{\partial V(t,u)}{\partial t} + f^T(u)\frac{\partial V(t,u)}{\partial u} + \frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 V(t,u)}{\partial u^2}g(u)\right]$$

where

$$\frac{\partial V}{\partial u} = col\left(\frac{\partial V}{\partial u_1}, \frac{\partial V}{\partial u_2}, \frac{\partial V}{\partial u_3}\right), \qquad \frac{\partial^2 V}{\partial u^2} = \frac{\partial^2 V}{\partial u_j u_i},$$

$i, j = 1,2,3$ and T is a transposition.

Theorem 1:

Suppose that $\frac{1}{2}\left(\sigma_1^2 w_1 + K_1 w_1 + K_2 w_2\right) < w_1 a_{11}\bar{x}_1$

$\frac{1}{2}\left(\sigma_2^2 w_2 + K_1 w_1 + K_2 w_2\right) < w_2 a_{22}\bar{x}_2$ *and*

$\frac{1}{2}\sigma_3^2 < a_{33}\bar{y}$

Then the trial solution of (12) is asymptotically mean square stable.

Proof: Let us consider the Lyapunov's function

$$V(u) = \frac{1}{2}\left[w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2\right] \tag{15}$$

where w_i are real positive constants to be chosen in the following. Further more

$$\begin{aligned} LV(u) = {} & w_1\left[-a_{11}\bar{x}_1 u_1 + k_1 u_2 - k_2 u_3\right]u_1 \\ & + w_2\left[k_3 u_1 - a_{22}\bar{x}_2 u_2 - k_4 u_3\right]u_2 \\ & + w_3\left[k_5 u_1 + k_6 u_2 - a_{33}\bar{y}u_3\right]u_3 \\ & + \frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 V(t,u)}{\partial u^2}g(u)\right] \end{aligned} \tag{16}$$

Note that $\frac{\partial^2 V}{\partial u^2} = \begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix}$

And hence

$$g^T(u)\frac{\partial^2 V(t,u)}{\partial u^2}g(u) = \begin{bmatrix} w_1\sigma_1^2 u_1 & 0 & 0 \\ 0 & w_2\sigma_2^2 u_2 & 0 \\ 0 & 0 & w_3\sigma_3^2 u_3 \end{bmatrix}$$

with

$$\frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 V(t,u)}{\partial u^2}g(u)\right] = \frac{1}{2}\left[w_1\sigma_1^2 u_1 + w_2\sigma_2^2 u_2 + w_3\sigma_3^2 u_3\right] \tag{17}$$

If we choose $w_1 k_2 = w_3 k_5$ and $w_3 k_6 = w_2 k_4$ in (16) then from (17) it is easy to see that

$$\begin{aligned} LV(u) = {} & -\left[w_1 a_{11}\bar{x}_1 - \frac{1}{2}\sigma_1^2 w_1 - \frac{\left(w_1 k_1 + w_2 k_2\right)}{2}\right]u_1^2 \\ & -\left[w_2 a_{22}\bar{x}_2 - \frac{1}{2}\sigma_2^2 w_2 - \frac{\left(w_1 k_1 + w_2 k_2\right)}{2}\right]u_2^2 \\ & -\left[w_3 a_{33}\bar{y} - \frac{1}{2}\sigma_3^2 w_3\right]u_3^2 \end{aligned}$$

According to [11], the zeros solution of (12) is asymptotically mean square stable.

6. Numerical Simulations

In this section, we examine the numerical solutions of the dynamic model for giving the following numerical values to the parameters and compute some simulations using those values for this purpose we used the software MATLAB.

$a_{11} = 0.01$, $\varepsilon_1 = 4$, $\Delta_1 = 1$, $\delta_1 = 0.1$, $a_{22} = 0.009$, $\varepsilon_2 = 1$, $\Delta_2 = 0.5$, $\delta_2 = 1$, $\varepsilon_3 = 2$, $a_{33} = 0.01$, $\eta_1 = 3$, $\eta_2 = 1$

$\sigma = 0.05$

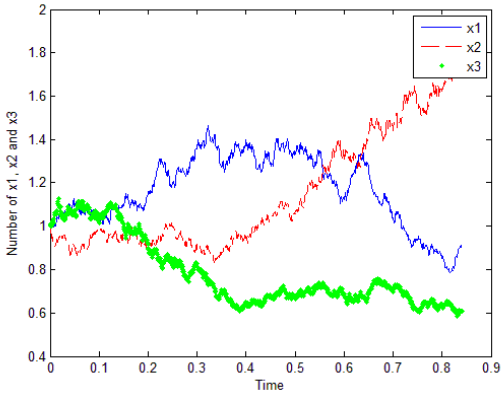


Fig.2: $x_1(1) = 1, x_2(1) = 1, y(1) = 1$

$\sigma = 0.03$

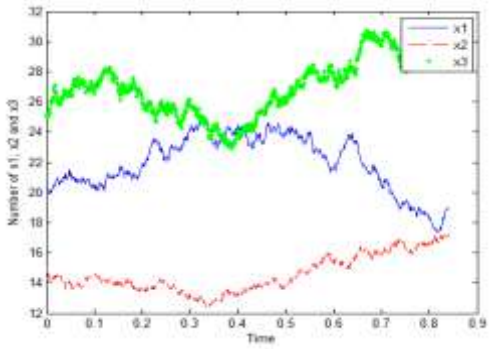


Fig. 3: $x_1(1) = 20, x_2(1) = 15, y(1) = 25$

$\sigma = 0.1$

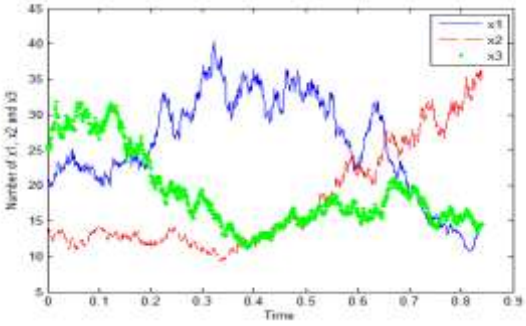


Fig. 4: $x_1(1) = 20, x_2(1) = 15, y(1) = 25$

$\sigma = 0.3$

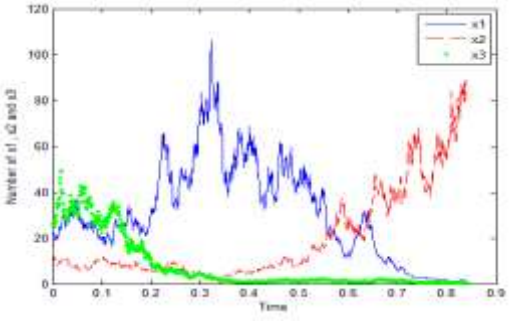


Fig. 5: $x_1(1) = 20, x_2(1) = 15, y(1) = 25$

$\sigma = 0.7$

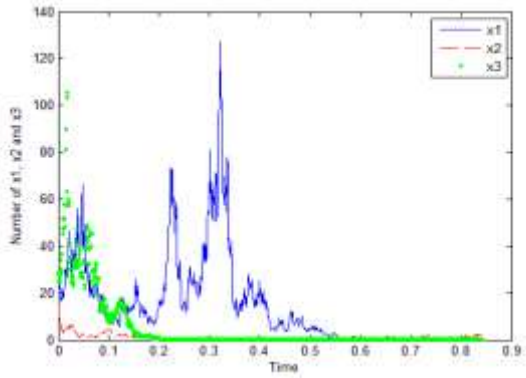


Fig. 6: $x_1(1) = 20, x_2(1) = 15, y(1) = 25$

$\sigma = 0.75$

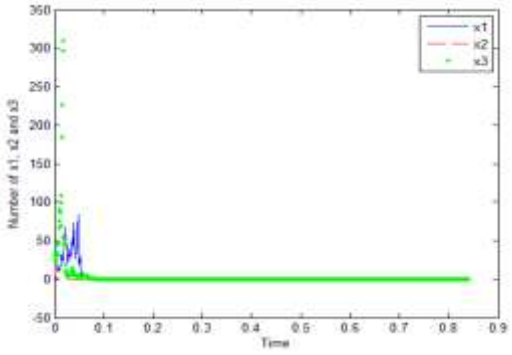


Fig. 7: $x_1(1) = 20, x_2(1) = 15, y(1) = 25$

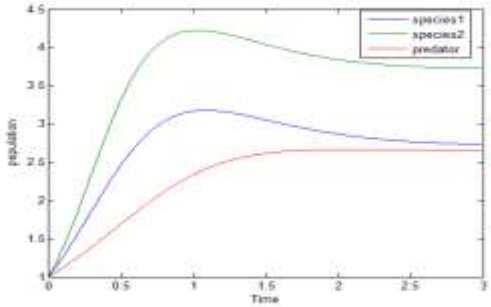


Fig. 8 (a):



Fig. 8 (b):

Figs: 8(a) & 8(b): Variation in the growth rate of the populations and the Phase- Portrait trajectories for the stabilities of the population to the parameter values $a_{11} = 1, \varepsilon_1 = 4, \Delta_1 = 0.5, \delta_1 = 1, a_{22} = 1, \varepsilon_2 = 5, \Delta_2 = 0.5, \delta_2 = 1$, $\varepsilon_3 = 2$, $a_{33} = 1$, $\eta_1 = 0.1$, $\eta_2 = 0.1$, $x_1(0) = 1, x_2(0) = 1, y(0) = 1$

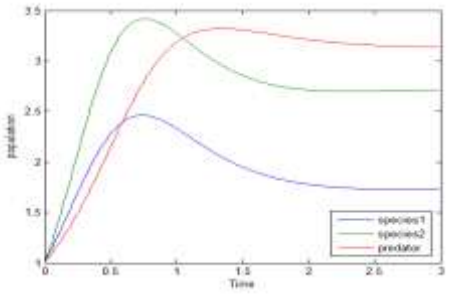


Fig. 9 (a):

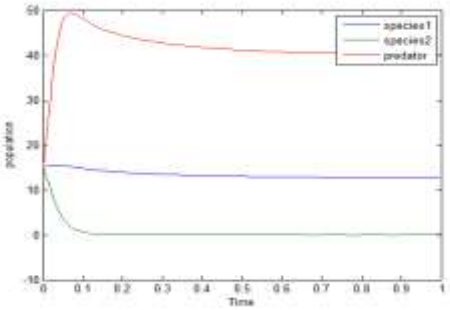


Fig. 9 (b):

Figs. 9(a) & 9(b). Variation in the growth rate of the populations and Phase- Portrait trajectories to the population for $a_{11}=1, \varepsilon_1=4, \Delta_1=0.5, \delta_1=1, a_{22}=1, \varepsilon_2=5, \Delta_2=0.5, \delta_2=1$, $\varepsilon_3=2$, $a_{33}=1$, $\eta_1=0.5$, $\eta_2=0.1$, $x_1(0)=1,\ x_2(0)=1$ and $y(0)=1$

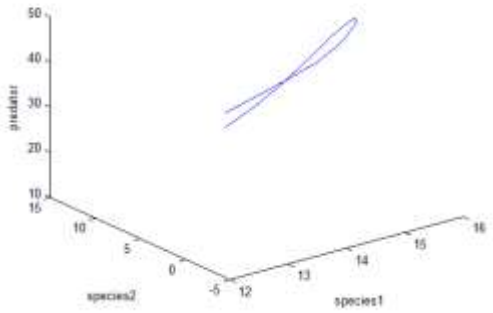


Fig. 10(a):

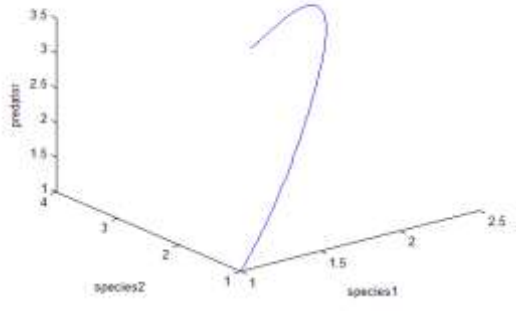


Fig. 10(b)

Figs.10 (a) & 10(b): Variation in the growth rate of the populations for $a_{11}=1$, $\varepsilon_1=4$, $\Delta_1=1$, $\delta_1=0.1$, $a_{22}=1$, $\varepsilon_2=1, \Delta_2=0.9, \delta_2=1, \varepsilon_3=2$, $a_{33}=1, \eta_1=3$, $\eta_2=1$, $x_1(0)=15, x_2(0)=15$ and $y(0)=15$

7. Conclusions

In the present work we discussed about an ecosystem consisting three species in which mutualism interaction between two species and a predator depending up on both mutual species with effect of stochastic perturbation. Initially, we have studied about the model without stochasticity and investigated the existence of equilibrium points; the local stability for each equilibrium point is discussed by employing Routh-Hurwitz criteria where ever necessary, after that the global analysis by constructing Lyapunov function is discussed. Later, we investigated the importance of additive noise to the positive coexistence equilibrium point to our model by evaluating fluctuations of the population densities near their values at that equilibrium point. Also numerical simulations justified the analytical results. Both the analytical results and numerical simulation of present model suggest that the deterministic mode having stable nature always. Besides this for stochastic system, population variances have a great role for the stochastic stability of the system shows in Figs. (2)-(7). The conclusion is that the noise on the equation results in a big variances of fluctuations around the positive equilibrium point. Lastly, the numerical simulations exhibit that the trajectories of the system shows the stability situation in Figs. (8)-(10).

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