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Research paper

Neighborhood Connected Two -Outdegree Equitable Domination of the ϑ-Obrazom Graphs of P_n, G_n and H_n

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Abstract

Let G=(V,E) be a connected graph. A two-outdegree equitable dominating set D of a graph G is called the neighborhood connected two-outdegree equitable dominating set (nc2oe-set) if the induced sub graph <N(D)> is connected. The minimum cardinality of the minimal neighborhood connected two-outdegree equitable dominating set is called neighborhood connected two-outdegree equitable domination number of G and is denoted by $\gamma_{nc2oe}(G)$. In this paper, we initiated the a study of this parameter and obtained $\gamma_{nc2oe}(G)$ for the ϑ-obrazom graphs of P_n, G_n and H_n. Also, we investigated the two-outdegree equitable domination number of the ϑ-obrazom graphs of $W_n$and S_n.

Keywords: *Connected two-outdegree equitable number; Line graph ; Neighborhood connected two-outdegree equitable number ;Two-outdegree equitable domination number.*

1. Introduction

By a graph G=(V,E) we mean a simple finite, undirected and connected graph. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we referred to Chartrand and Lesniak [3].
A non-empty subset D of V is called a dominating set if for every vertex $v \in V - D$, there exists a vertex $u \in D$ such that v is adjacent to u [i.e if N[D]=V]. The minimum cardinality of a minimal dominating set of G is called the domination number of G is denoted by $\gamma(G)$. An excellent treatment of the fundamentals of domination is the book of Haynes et at [4]. A survey of several advanced topics in domination is given in the book edited by Haynes et al [5].
Sambathkumar and Walikar [9] introduced the concept of connected domination in graphs. A dominating set D of G is called the connected dominating set if the induced sub graph <D> is connected. The minimum cardinality of a minimal connected dominating set of G is connected domination number of G and is denoted by $\gamma_c(G)$. S. Arumugam and C.Sivananam [2] introduced the concept of neighborhood connected two-out degree equitable domination in graphs. A dominating set D of a connected graph G is called neighborhood connected dominating set (ncd-set) if the induced sub graph <N(D)> is connected. The minimum cardinality of a minimal ncd set D is called neighborhood connected domination number and is denoted by $\gamma_{nc}(G)$. Ali Sahal and Veena Mathad [8] defined two-out degree equitable dominating set.
ϑ-obrazom of G, denoted as line graph L(G) of a graph G is defined with the vertex set E(G), in which two vertices are adjacent in L(G) if and only if the corresponding edges are adjacent in G.

1.1 Sunlet Graph[12]

The Sunlet graph on $2n$ vertices is defined as $V(S_n) = \{v_1, v_2 \dots v_n\} \cup \{u_1, u_2 \dots u_n\}$ where $v_i's$ are the vertices of cycles taken in cyclic order and $u_i's$ are pendent edges.

1.2 Wheel Graph [7]

The wheel graph W_n on $n+1$ vertices is defined as $W_n = C_n + K_1$ where C_n is n-cycle. Let $V(W_n) = \{v_i: 1 \le i \le n\} \cup \{v\}$ and $E(W_n) = \{e_i = v_i v_{i+1}: 1 \le i \le n, subscripts\ modulo\ n\} \cup \{e_i' = v v_i: 1 \le i \le n$ where v is an external vertex adjacent to every other vertex.

1.3 Gear Graph [7]

The gear graph is a wheel graph with vertices added between pair of vertices of the outer cycle. The gear graph G_n has $2n+1$ vertices and $3n$ edges. Let $V(G_n) = \{v_i : 1 \le i \le n\} \cup \{u_i: 1 \le i \le n\} \cup \{v\}$ and $E(G_n) = \{e_i = v_i u_i : 1 \le i \le n\} \cup \{e_i' = v_i v : 1 \le i \le n\} \cup \{e_i'' = u_i v_{i+1} : 1 \le i \le n$, subscripts modulo n$\}$ where v is an externalvertex adjacent to every other vertex $v_i\ for\ 1 \le i \le n$.

1.4 Helm Graph [7]

The helm graphH_n is the graph obtained from a n-wheel graph by adjoining a pendent edge at each vertex of the cycle. The helm graph H_n has $2n+1$ vertices and $3n$ edges and $V(G_n) = \{v\} \cup \{v_i: 1 \le i \le n\} \cup \{u_i: 1 \le i \le n\}$ and $E(G_n) = \{e_i = v_i v_{i+1}: 1 \le i \le n-1\} \cup \{e_i' = v_i v: 1 \le i \le n-1\} \cup \{e_i = v_i u_i: 1 \le i \le n-1\}$ where v is an externalvertex adjacent to every other vertex $v_i\ for\ 1 \le i \le n$.

1.5 Line Graph [11]

A ϑ-obrazom of a graph G is a graph on the set of edges of G such that in L(G) there is an edge (a,b) if and only if a ,b are adjacent in G.

2. Two-Outdegree Equitable Domination Number of the ϑ-Obrazom Graphs of W_n and S_n

In this section, we discussed initially the two-out degree equitable domination number of ϑ-obrazom graphs of wheel and sunlet graphs.

Definition 2.1[8]

Let G=(V,E) be a simple, finite, connected and undirected graph. A dominating D of G is said to be a two- outdegree equitable dominating set if for any two vertices $u, v \epsilon D$ such that $|od_D(u) - od_D(v)| \leq 2$ where $od_D(u) = |N(v) \cap V - D|$.The minimum cardinality of a minimal two-outdegree equitable dominating set is called two-outdegree equitable domination number and it is denoted by $\gamma_{2oe}(G)$.

Example 2.2 [7]

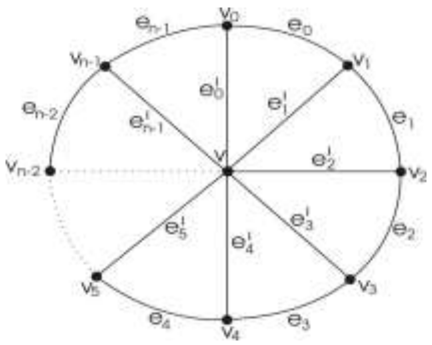

Figure 1: Wheel Graph W_n

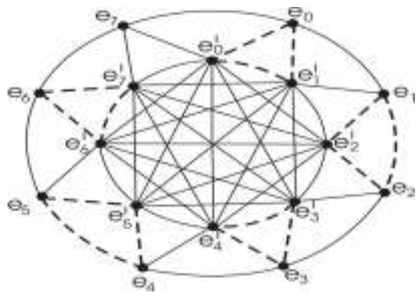

Figure 2: Line Graph of W_8

Example 2.3 [12]

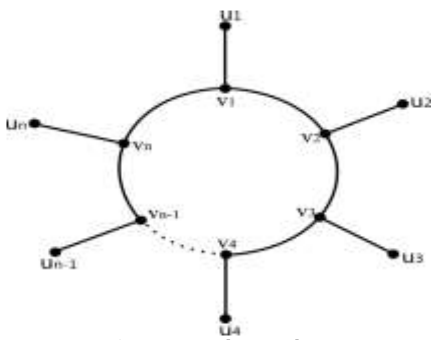

Figure 3: Sunlet Graph S_n

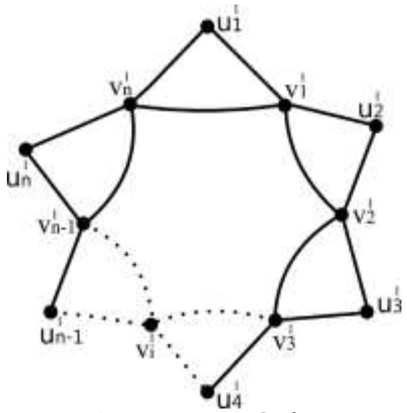

Figure 4: Line Graph of S_n

Theorem 2.4

The two- outdegree equitable domination number of the ϑ-obrazom graph of W_n, $\gamma_{2oe}[L(W_n)] = \left\lceil \frac{n}{2} \right\rceil$ $for\ n \geq 4$

Proof

Let $L(W_n)$ be the ϑ-obrazom graph of W_n of order $2n$. Now $L(W_n)$ contains a complete graphK_{n-1}.

By the definition of ϑ-obrazom graph, $V(L(W_n)) = \{e_i = v_i v_{i+1} : 1 \leq i \leq n, subscripts\ modulo\ n\} \cup \{e_i' = vv_i : 1 \leq i \leq n\}$.

Let $D = \{e_{2i-1} : i = 1,2,3 \ldots\}$ be the dominating set of $L(W_n)$ and $V - D = \{e_{2i} \cup (e_i : 1 \leq i \leq n)\}$, where $i = 1,2,3 \ldots$.

Now $e_{2i-1} \in D$ then $od_D(e_{2i-1})$

$= |N(e_{2i-1}) \cap (V - D)|$ $for\ i = 1$

$= |N(e_1) \cap (V - D)|$

$= |\{(e_2, e_n) \cup (e_1', e_2')\} \cap \{e_{2i} \cup (e_i')\}|$

$= |\{e_2, e_1', e_2'\}|$

=3

For $i = 2, e_3 \in D$ then$od_D(e_3) = |N(e_3) \cap (V - D)|$

=4

For $i = 3, e_5 \in D$ then $od_D(e_5) = |N(e_5) \cap (V - D)|$

=4

and so on

Then $|od_D(u) - od_D(v)| \leq 2$, for any $u, v \in D$. Therefore D is the minimum two-out degree equitable dominating set. Hence

$\gamma_{2oe}[L(W_n)] = \left\lceil \frac{n}{2} \right\rceil$

$where\ n \geq 4$.

Theorem 2.5

The two-outdegree equitable domination number of the ϑ-obrazom graph of S_n, $\gamma_{2oe}[L(S_n)] = \left\lceil \frac{n}{3} \right\rceil + 1$.

Proof

Let $V(S_n) = \{v_1, v_2, \ldots v_n\} \cup \{u_1, u_2, \ldots u_n\}$ and $E(S_n) = \{e_i' : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\}$ where e_i is the edge $v_i v_{i+1} (1 \leq i \leq n), e_n$ is the edge $v_n v_1$ and e_i' is the edge $v_i u_i (1 \leq i \leq n)$.

By the definition of ϑ-obrazom graph $V(L(S_n)) = E(S_n) = \{u_i' : (1 \leq i \leq n)\} \cup \{v_i' : \{e_i : 1 \leq i \leq n-1\} \cup \{v_n'\}$

wherev_i' and u_i' represents the edge e_i and e_i' $(1 \leq i \leq n)$ respectively.

Let $= \{v_{2i-1} : i = 1,2,3 \ldots\}$ be the dominating set of $L(S_n)$ and $V - D = \{v_{2i} \cup \{u_i : 1 \leq i \leq n\}\}$ where $i = 1,2,3 \ldots$

Now $v_{2i-1} \in D$ then $od_D(v_{2i-1})$
$= \left| N(v_{2i-1}) \cap (V-D) \right| \; for \; i = 1$
$= \left| N(v_1) \cap (V-D) \right|$
$= \left| \{(v_2', v_3', \ldots v_{n-1}', v_n') \cup (u_1', u_2')\} \cap \{v_{2i} \cup (u_i)\} \right|$
$= \left| \{(v_2', v_4', \ldots, v_n') \cup (u_1', u_2')\} \right|$
$= 3$ or 4.
For $i = 2, v_3 \in D$ then $od_D(v_3) = \left| N(v_3) \cap (V-D) \right|$
$=4$
and so on
Then $\left| od_D(u) - od_D(v) \right| \leq 2$, if $u, v \in D$. So D is the minimum two-outdegree equitable dominating set. Hence $\gamma_{2oe}[L(S_n)] = \left\lceil \frac{n}{3} \right\rceil + 1.$

3. Neighborhood Connected Two-Out Degree Equitable Domination Number of the ϑ-Obrazom Graphs of P_n, G_n And H_n

In this section we discussed the neighborhood connected two -out degree equitable domination number in the ϑ-obrazom graphs of path, gear and helm graphs.

Definition 3.1 [6]

A non-empty subset D of a graph G is called the neighborhood connected two-outdegree equitable dominating set (nc2oe-set) if the induced sub graph $< N(D) >$ is connected. The minimum cardinality of a minimal neighbourhood connected two-outdegree equitable dominating set is called the neighbourhood connected two-outdegree equitable domination number of G and is denoted by $\gamma_{nc2oe}(G)$.

Example 3.2 [7,11]

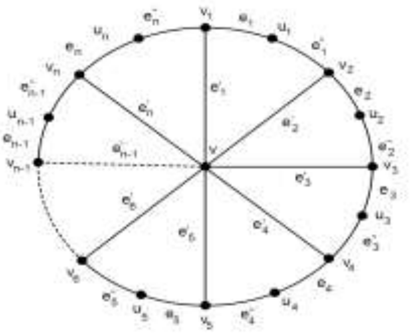

Figure 5: Gear Graph G_n

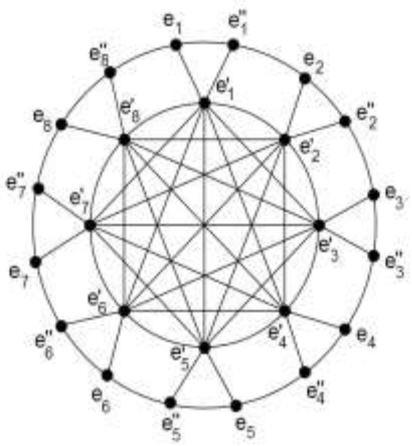

Figure 6: Line Graph of G_6

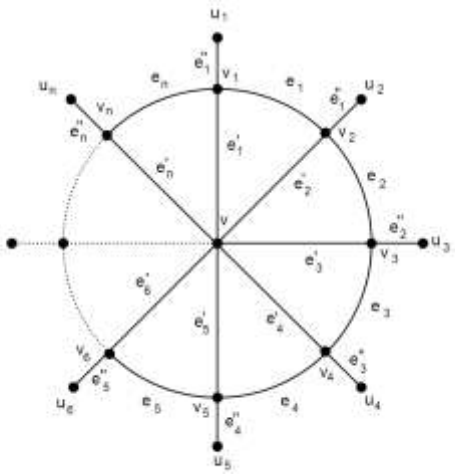

Figure 7::Helm Graph H_n

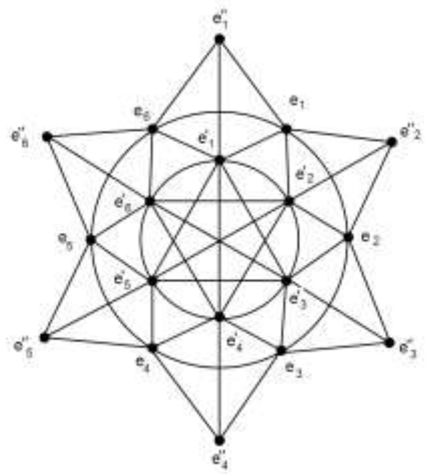

Figure 8: Line Graph of H_6

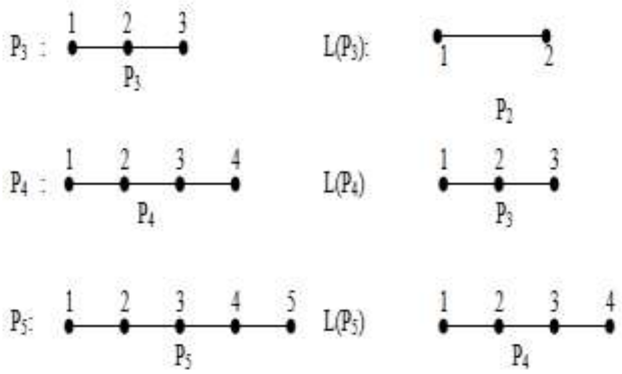

Figure 9:

Theorem 3.3

For any Path $P_n, \gamma_{nc2oe}[L(P_n)] = n - 3, \; for \; n \geq 5.$

Proof

Let P_n have $n$vertices and $n-1$ edges and $L(P_n)$ is P_{n-1} with $n-1$ vertices and $n-2$ edges.
Since $L(P_n)$ is a path, then $\deg(v) \leq 2$. Clearly D is neighborhood connected two -outdegree equitable dominating set and $<N(D)>$ is connected.
Also, let $D = \{v_i : 2 \leq i \leq n-2\}$ be the dominating set of $L(P_n)$ and $V - D = \{v_1, v_{n-1}\}$
Now, $v_i \in D$ then
$od_D(v_i) = \left| N(v_i) \cap V - D \right| for \; i = 2$
$= \left| v_1, v_2, \ldots v_{n-1}) \cap (v_1, v_{n-1}) \right|$

$= \left| v_{n-1} \right|$
=1
For $i = 3$, $v_3 \in D$ then $od_D(v_3) = \left| N(v_3) \cap V - D \right|$
$= \left| v_2, v_4) \cap (v_1, v_{n-1}) \right|$
$=\emptyset$
=0
For $i = n - 2$, $v_{n-2} \in D$ then $od_D(v_{n-2})$
$= \left| N(v_{n-2}) \cap V - D \right|$
=1
Then $\left| od_D(v_i) - od_D(v_j) \right| \leq 2$, for $v_i, v_j \in D$. Therefore D is the two-outdegree equitable dominating set.
Now $N(D) = N(v_i)$
$= N(v_2) \cup N(v_3) \dots . \cup N(v_{n-1})$
$N(D) = V$
Since $< V >$ is connected. Then $<N(D)>$is also connected.
Hence $\gamma_{nc2oe}[L(P_n)] = n - 3, for\ n \geq 5$.

Theorem 3.4
For the Gear graph$G_n, \gamma_{nc2oe}[L(G_n)] = n$

Proof
Let $L(G_n)$ be the ϑ-obrazom graph of G_n of order $3n$. By the definition of ϑ-obrazom graph, $V(L(G_n)) = E(G_n)$ in which the set of vertices of $L(G_n), \{e_i': 1 \leq i \leq n\}$ induces a clique of order n.
Let $\{(e_1, e_2, \dots e_n) \cup (e_1', e_2', \dots e_n') \cup (e_1'', e_2'', \dots e_n''\}$ be the vertices in $L(G_n)$.
Let $D = \{e_i: 1 \leq i \leq n\}$ be the dominating set of $L(G_n)$ and
$V - D = \{(e_i \cup e_i''): 1 \leq i \leq n\}$
Now, $e_i' \in D$ then
$od_D(e_i') = \left| N(e_i') \cap V - D \right| \quad for\ i = 1$
$= \left| \{(e_{i+1}', e_{i+2}', \dots e_n') \cup (e_i, e_i'')\} \cap \{e_i \cup e_i''\} \right|$
$= \left| (e_i, e_i'') \right| \ since\ i = 1$
=2
If $e_i' \in D$, then $od_D(e_i') = \left| N(e_i') \cap V - D \right| for\ i = 2$
$= \left| \{(e_{i-1}', e_{i+1}', \dots e_n') \cup (e_i, e_i'')\} \cap \{e_i \cup e_i''\} \right|$
$= \left| (e_i, e_i'') \right| \ since\ i = 2$
=2
If $e_n' \in D$, then
$od_D(e_n') = \left| N(e_n') \cap V - D \right| for\ i = n$
$= \left| \{(e_1', e_2', \dots e_{n-1}') \cup (e_n, e_n'')\} \cap \{e_i \cup e_i''\} \right|$
$= \left| (e_i, e_i'') \right| \ since\ i = n$
=2
Then $\left| od_D(e_i') - od_D(e_j') \right| \leq 2$, for any $e_i', e_j' \in D$. Therefore D is the two-outdegree equitable dominating set, and $N(e_i') = v\ for\ all\ e_i' \in D$
$N(D) = N(e_i')$
$= N(e_1') \cup N(e_2') \dots . \cup N(e_n')$
$N(D) = V$ and $< V >$ is connected. So $<N(D)>$ is connected. Then $<N(D)>$ is connected two- outdegree equitable dominating set and D is the minimum neighborhood connected two-outdegree equitable dominating set. Hence $\gamma_{nc2oe}[L(G_n)] = n$.

Theorem 3.5
For the ϑ-obrazom graph of helm graph H_n, the neighborhood connected two -out degree equitable domination number is :
$\gamma_{nc2oe}[L(H_n)] = n$

Proof
Let $L(H_n)$ be the ϑ-obrazom graph of H_n of order $3n$ and $\{e_i' \cup e_i \cup e_i'': 1 \leq i \leq n\}$ be the vertices of $L(H_n)$.
Let $D = \{e_i': 1 \leq i \leq n\}$ be a dominating set of $L(H_n)$ which is connected and $V - D = \{(e_i \cup e_i''): 1 \leq i \leq n\}$
Now $e_i' \in D$ then $od_D(e_i')$
$= \left| N(e_i') \cap V - D \right| \quad for\ i = 1$
$= \left| \{(e_{i+1}', e_{i+2}', \dots e_n') \cup (e_i, e_i'', e_n)\} \cap \{e_i \cup e_i''\} \right|$
$= \left| (e_i, e_i'', e_n) \right| \ since\ i = 1$
=3
If $e_i' \in D$, then $od_D(e_i') = \left| N(e_i') \cap V - D \right| for\ i = 2$
$= \left| \{(e_{i-1}', e_{i+1}', e_{i+2}', \dots e_n') \cup (e_{i-1}, e_i, e_i'')\} \cap \{e_i \cup e_i''\} \right|$
$= \left| (e_{i-1}, e_i, e_i'') \right| \ since\ i = 2$
=3
If $e_n' \in D$, then $od_D(e_n') = \left| N(e_n') \cap V - D \right| for\ i = n$
$= \left| \{(e_1', e_2', \dots e_{n-1}') \cup (e_{n-1}, e_n, e_n'')\} \cap \{e_i \cup e_i''\} \right|$
$= \left| (e_{n-1}, e_n, e_n'') \right| \ since\ i = n$
=3
Then $\left| od_D(e_i') - od_D(e_j') \right| \leq 2$, for $e_i', e_j' \in D$. So D is the two-outdegree equitable dominating set and <D> is connected. Clearly <D> is a complete graph and $od_D(e_i) = 3\ for\ 1 \leq i \leq n$.
$N(e_i') = v\ for\ all\ e_i' \in D$
$N(D) = N(e_i')$
$= N(e_1') \cup N(e_2') \dots . \cup N(e_n')$
$N(D) = V$. Since $< V >$ is connected, $<N(D)>$ is connected. Then $<N(D)>$ is connected two -outdegree equitable dominating set and D is the minimum neighborhood connected two-outdegree equitable dominating set. Hence $\gamma_{nc2oe}[L(H_n)] = n$.

5. Conclusion

In this paper, we computed the exact values of the neighborhood connected two-outdegree equitable domination number for the ϑ-obrazom graphs of P_n, G_n and H_n. Also, we have computed the two out degree equitable domination number for the line graphs of $W_n$and S_n. Further, we would like to extend this work for some more special classes of graphs.

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