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Research paper

Approximation of stochastic petri nets by means of continuous petri nets: adaptive approach

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Abstract

Reliability analysis is often based on stochastic discrete event models like Markov models or stochastic Petri nets. For complex dynamical systems with numerous components, analytical expressions of the steady state are tedious to work out because of the combinatory explosion with discrete models. The contribution of this paper is to estimate the asymptotic comportement of stochastic nets by mean of continuous Petri nets according to a no linear modification of the maximal firing speed.

Keywords: *Adaptive Approach; Continuous Petri Nets; Fluidification; Reliability Analysis; Stochastic Petri Nets.*

1. Introduction

Reliability analysis is a major challenge to improve the safety of industrial processes. For complex dynamical systems with numerous interdependent components, such studies are mainly based on stochastic discrete event models like Markov models or Stochastic Petri Nets (SPN) [6], [7]. Such models are mathematically well founded and can be investigated in order to work out either analytical or numerical simulations. In case of large systems, the long time of the convergences considered.
The aim of this paper is to approximate the steady state of SPN or Markov models by mean of Continuous Petri Nets (CPN). The fluidification of discrete Petri nets has been recently investigated mainly for performance evaluation and control applications [3], [5], [8]. It is well known that fluidification leads to some unexpected results and numerous structural and behavioral properties are not preserved with fluidification. In general case, the steady state of a SPN cannot be approximated by the asymptotic behavior of the corresponding continuous Petri net.
The contribution of this paper is according to a modification of the maximal firing speeds, a proportionality relationship exists between both behaviors. This property is then used to accelerate convergence of stochastic simulations. SPN and CPN are combined for that purpose: stochastic simulation is worked out during a short time window of duration T. The asymptotic behavior of CPN is shown to give an improved approximation of the stochastic steady state.

2. Stochastic and continuous petri nets

2.1. Petri nets

A Petri net (PN) is defined as <P, T, W_{PR}, W_{PO}> where P={P_i} is a not empty finite set of n places and T={T_j} is a not empty finite set of q transitions. $W_{PR} = (w^{PR}_{ij}) \in (Z^+)^{n\times q}$ is the pre-incidence matrix (w^{PR}_{ij} is the weight of arc from place P_i to transition T_j) and $W_{PO} = (w^{PO}_{ij}) \in (Z^+)^{n\times q}$ is the post-incidence one (w^{PO}_{ij} is the weight of arc from transition T_j to place P_i) [3]. The PN incidence matrix W is defined as $W = W_{PO} - W_{PR} \in (Z)^{n\times q}$. The PN marking M is an application from the set of places P to the set of non negative integer numbers Z^+ such that, for each place $P_i \in P$, $m_i = M(P_i)$ is the number of tokens in place P_i. M_I is the PN initial marking.

2.2. Stochastic petri nets

A Stochastic PN (SPN) is given by a timed PN whose transitions firing periods are characterized by random distributions. This model has been introduced by Molloy [6], [7], and several extensions have been developed [1],[4] for the reliability analysis of reparable systems.
A SPN, <PN, μ>, is a PN associated with a firing rate vector $\mu = (\mu_j) \in (R^+)^q$. Each transitions T_j of SPNis characterized by the firing rate μ_j so that $\mu_j.dt$ is the probability that the transition T_j will fire between t and t+dt when the transition T_j has been enabled, with degree1at t. The marking process of a SPN will be characterized according to the PN incidence matrices, the initial marking, the firing rates, the firing policy, the server policy and the execution policy [2],[4]. SPN that are considered in this paper satisfy the following assumptions:
(H1) the firing policy is a race policy: the transition whose firing time elapses first is assumed to be the one that will fire next.
(H2) the server policy is of type infinite server: the minimal period of each transition T_j is defined with a stochastic duration which is characterized according to an exponential distribution of varying parameter round $(n_j(M)).\mu_j$. The function $n_j(M)$ is the enabling degree of transition T_j for marking M:

$$n_j(M) = \min (m_i / w^{PR}_{ij}) \text{ for all } P_i \in {}^\circ T_j \quad (1)$$

Where $^\circ T_j$ stands for the set of $T_j$upstream places and "round (.)" is the integer part of (.).

(H3) the execution policy is of type « resampling memory »: at the entrance in a marking, the remaining firing time of all transitions that were enabled is reset.

2.3. Continuous petri nets

Continuous Petri Nets (CPN) has been developed in order to provide a continuous approximation of the discrete behaviors of discrete PN [3], [8]. A CPN is defined as<PN, X_{max}> where PN is a Petri nets and $X_{max} = (x_{max\,j}) \in (R^+)^q$ is the vector of maximal firing speeds. The marking $m_i(t) \in R^+$ of each place P_i is a non-negative real value of the function of time and each transition firing is a flow of marks in continuous PN. $X(t) = (x_j(t)) \in (R^+)^q$ is the firing speeds vector at time t. The marking evolution is given by (2):

$$dM(t) / dt = W.X(t) \quad (2)$$

Finite server (i.e. constant speeds) and infinite server (i.e. variable speeds) semantics exist for CPN. In this paper only the infinite server semantic is considered: X(t) depends continuously on the marking of the places according to (3):

$$x_j(t) = x_{maxj}.n_j(M) \quad (3)$$

3. Fluidification of stochastic petri nets

The aim of this section is to pinpoint the limit of the fluidification of SPNs by CPNs with same structure and same initial marking. The several example for which CPNs do not converge to the asymptotic mean marking of SPNs is presented.

3.1. Continuous approximation of SPNs

Numerous structural and behavioral properties are not preserved with fluidification [8]. The mean markings of a CPN are mainly not identical to the ones of a SPN. Concerning SPNs, the steady state is mainly often different from the one of a CPN with same parameters ($x_{maxj} = \mu_j$, j = 1,...,q). Consider for example the marked SPN described in Figure 1 [5] is has 2 P-semi flows: $y_1 = (1\ 1\ 1\ 0)^T$, $y_2 = (1\ 0\ 4\ 1)^T$. Four regions exist in reachable marking space of PN. The regions are defined by the constraint matrices $A_1$to A_4:

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} A_2 = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} A_3 = \begin{pmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

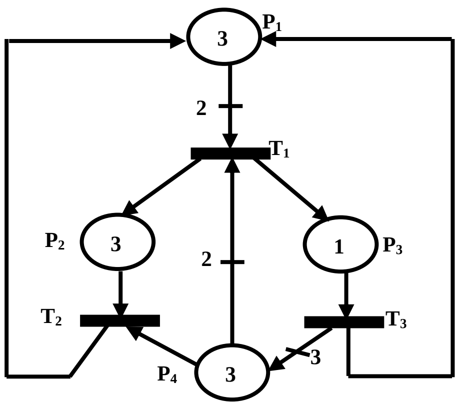

Fig. 1: No Ordinary PN with Conflits and Synchronizations.

The firing speeds are given by the (4):

$$x_{c1}(t) = x_{max1}.\min(m_{c1}(t)/2, m_{c4}(t)/2)$$

$$x_{c2}(t) = x_{max2}.\min(m_{c2}(t), m_{c4}(t)) \quad (4)$$

$$x_{c3}(t) = x_{max3}.m_{c3}(t)$$

According to the minimum functions in the definition of $x_{c1}(t)$and $x_{c2}(t)$, four regions $A_1$to A_4exist. Each region is related to a configuration (Table 1).

Table 1: Regions and Configurations for the CPN of Fig. 1

Regions	Configurations
A_1	$\{(P_4, T_1), (P_4, T_2), (P_3, T_3)\}$
A_2	$\{(P_1, T_1), (P_4, T_2), (P_3, T_3)\}$
A_3	$\{(P_4, T_1), (P_2, T_2), (P_4, T_3)\}$
A_4	$\{(P_1, T_1), (P_2, T_2), (P_4, T_3)\}$

3.2. Trivial of fluidification of SPNs

The main idea of PNs fluidification is to replace a discrete PN by a continuous one with same structure, initial marking and parameter (i.e. standard fluidification). The origin of the approach is that continuous models have been intensively used from the 90th to approximate the behaviour of deterministic discrete event systems, in particular for control issues [10 - 12]. The advantage of fluidification is that the enumeration of discrete states is no longer required with continuous models and that standard tools exist for such model. Unfortunately, numerous structural and behavioural properties are not preserved with fluidification. In particular, the throughput of a CPN is mainly not identical to the throughput of a discrete PN. The example of Figure 1 is considered again and simulated as a CPN. Standard fluidification is used and the results are reported in Table 2.

Table 2: Average Throughputs and Mean Markings of CPN and SPN

	m_1	m_2	m_3	m_4	$x_{1,2,3}$
SPN	3.73	2.09	1.17	1.57	1.17
CPN	1.18	4.06	1.76	1.76	1.76
Computational effort SPN(TU)	0.59	0.59	0.59	0.59	0.59
Computational effort CPN(TU)	0.25	0.25	0.25	0.25	0.25

One can notice that continuous does not converge to stochastic behaviour and the computation effort does not depend on the marking magnitude. The direct fluidification of the SPN in Figure 1 does not lead to the same stationary regime. This is due to the presence of synchronizations and weights. In the next section, we continue investigate how to accelerate convergence of stochastic simulations.

4. Adaptive approach

In this section, continuous Petri nets with adaptive maximal firing speeds are proposed to converge to the asymptotic mean marking of SPNs also in critical regions [9], [10]. The maximal firing speeds are considered as functions of time and updated in order to compensate online the errors on the throughputs and markings regarding the asymptotic stochastic average throughputs and mean markings to be reached. Non linear timed continuous Petri nets (NL-CPNs) with adaptive maximal firing speeds are defined as a consequence.

Definition 1: A Non Linear Timed Continuous Petri Net: NL CPN = <P, T, W^{PR}, W^{PO}, Ms, Xs> is defined by a set of n + q differential equations defined by (5):

$$dX_{max}/dt = \eta.diag(\mu).((W^T).(M_S-M_C)+(X_S-X_C)) \quad (5)$$

Under the constraint $X_{max} \geq 0$, where η is the adaptation parameter and $X_{max}(0) = diag(\mu) \in (R^+)^{q \times q}$ is the diagonalmatrix of the maximum firing speeds at the initial moment.

In order to motivate the updating (5) of the maximal firing speeds, let consider a transition $T_j$and define w_j as the column j of incidence matrix W.

If at time t, for any places $P_i \in {}^{\circ}T_j$, the marking $m_{ci}(t)$ satisfies $m_{si} - m_{ci}(t) > 0$ (resp. $m_{si} - m_{ci}(t) < 0$) and for any places $P_k \in T_j^{\circ}$, the marking $m_{ck}(t)$ satisfies $m_{sk} - m_k(t) < 0$ (resp. $m_{sk} - m_k(t) > 0$) then $w_j^T(M_s - M_c(t)) < 0$ (resp. $w_j^T(M_s - M_c(t)) > 0$) and x_{maxj} decreases (resp. x_{maxj}increases). The variation of X_{max}isalso driven according to the firing speed $x_{cj}(t)$ of transition T_j: If $x_{sj} - x_{cj}(t) < 0$ (resp. $x_{sj} - x_{cj}(t) > 0$) then x_{maxj} decreases (resp. x_{maxj}increases). But when the preceding conditions are not simultaneously satisfied, the convergence of $M_c(t)$ to M_s is no longer monotonic.

5. Results

In order to have a continuous stationary regime equivalent to that of the stochastic asymptotic regime, we will apply the adaptation law defined by (5) with η = 0.01, on all the maximum firing speeds associated with all the transitions of the CPN of the example. Each firing speed will be corrected by a multiplicative factor. Only the through of the transition T_3 and the marking of the place P_4 will be represented.

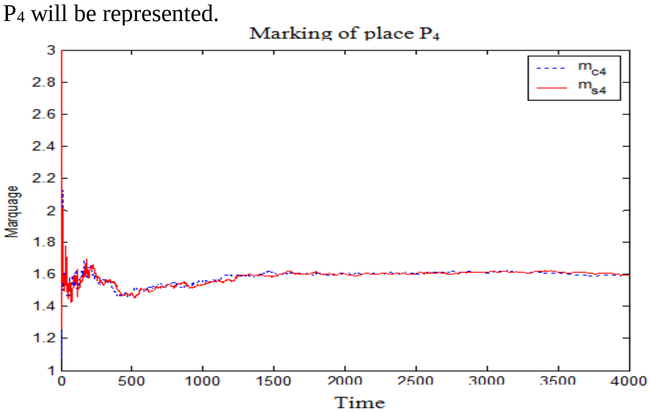

Fig. 2: Marking Evolution of CPN (Dashed Line) and SPN (Solid Line) in Function of Time for Example of Figure 1.

The minimization of the error which is due to the variations of markings and to the variations of speeds with time allows the continuous marking of the place p_4 to converge towards the asymptotic stochastic mean marking.

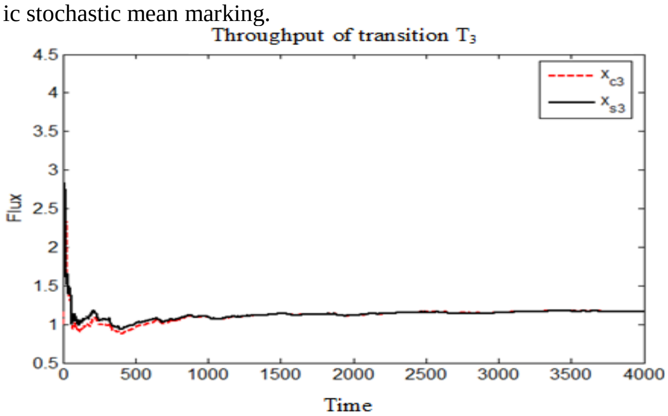

Fig. 3: Throughput Evolution $X_c(T)$ and $X_s(T)$ of CPN (Dashed Line) and SPN (Solid Line) in Function of Time.

The continuous throughput of the transition T_3 converges towards the mean throughput of the stochastic system, after modification of the speed x_{max3} by means of a multiplicative coefficient associated with the maximum speed of the transition T_3 of the CPN.

6. Conclusion

In this paper, the modification of the maximum firing speeds of the continuous transitions using the correction coefficients applied to all the transitions of the CPN makes to ensure the convergence of the markings and the continuous flows towards the average asymptotic values of the SPN. The maximal firing speed vector of CPN has been defined according to the SPN average throughputs and mean markings. In our future works, we will continue our investigation about SPN steady state approximation by means of CPN.

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