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Research paper

A Study on Harmonious Coloring of Circulant Networks

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Abstract

Given a simple graph G, a harmonious coloring of G is the proper vertex coloring such that each pair of colors seems to appears together on at most one edge. The harmonious chromatic number of G, denoted by χ_h is the minimal number of colors in a harmonious coloring of G. In this paper we have determined the harmonious chromatic number of some classes of Circulant Networks.

Keywords: *Circulant Network; Diameter; Diametrically uniform graph; Harmonious coloring; Matching;*

1. Introduction

Graph theory is a prospering control containing an assortment of delightful and effective consequences of wide relevance. Its unstable development as of late is for the most part because of its part as a fundamental structure supporting present day connected arithmetic - software engineering, combinatorial advancement, and operations investigate specifically - yet additionally to its expanding application in the more connected sciences. Graph coloring is a unique instance of graph labeling; it is an assignment of labels customarily known as colors to elements of a graph subject to specific imperatives. A k-coloring is an assignment of k colors to the vertices of a graph. A k-coloring is said to be proper if no two adjacent vertices share the same color. The chromatic number of G, denoted by $\chi(G)$ is the minimal number of colors in a k-coloring of G. Distinctive ways are available in coloring a graph. One way of coloring the vertices pleasantly is known as harmonious coloring. This parameter was introduced by Miller and Pritikin. Every graph has a harmonious coloring, because it is adequate to assign each vertex a distinct color; thus $\chi_h(G) \leq |V(G)|$. It was shown by Hopcroft and Krishnamoorthy that deciding the harmonious chromatic number of a graph is NP-hard[1]. Edwards examined the harmonious chromatic number problem for all trees, bounded degree trees, bounded degree graphs and distributed papers relating harmonious chromatic number and the achromatic number[2,4]. He accentuated another upper bound for the harmonious chromatic number, and on the harmonious chromatic number of complete n-ary trees[3]. This problem has potential application in correspondence systems, for instance transportation systems, PC systems, aviation route arrange framework, satellite route framework, radio route framework and so forth. In radio route, it is utilized as a managing framework in terrible climate condition or if there should arise an occurrence of imperceptibility of separation objects. It is likewise used to ascertain the address of the piece in which the coveted record is set.

2. An Overview of the Paper

2.1 Circulant Graphs

Circulant graphs have been used for decades in the design of computer and telecommunication networks due to their optimal fault-tolerance and routing capabilities. The term circulant comes from the nature of its adjacency matrix; a matrix is circulant if all its rows are periodic rotations of the first one. Circulant matrices have also been employed for designing binary codes. Circulant graphs also constitute the basis for designing certain data alignment networks. Most of the earlier research concentrated on using the circulant graphs to build interconnection networks for distributed and parallel systems. The family of circulant graphs includes the complete graph and the cycle among its members. An undirected circulant graph denoted by $G(n; \pm\{1,2,\ldots\ldots j\}), 1 \leq j \leq \left\lfloor\frac{n}{2}\right\rfloor, n \geq 3$ is defined as an undirected graph consisting of the vertex set $V = \{0,1,\ldots\ldots, n-1\}$ and the edge set $E = \{(i,j): |j-i| \equiv s(mod\ n), s \in \{1,2,\ldots\ldots j\}\}$. A circulant network G(11; {1; 2; 3; 4}) is depicted in Figure 1. Additionally we make utilization of an intriguing property of circulant graphs, that it is diametrically uniform.

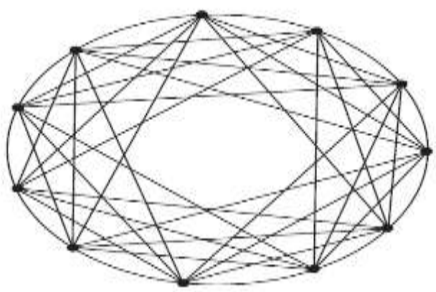

Fig. 1: G(11; {1, 2, 3, 4})

2.2 Diametrically Uniform Graphs:

The eccentricity e(v) of a graph G is the maximum distance between v and some other vertex u of G. The maximal value of eccentricity of vertices of G is called the diameter of G and it is denoted by λ. A vertex v of G is said to be diametrically opposite to a vertex u of G, if $d_G(u,v) = \lambda$. A graph G is said to be diametrically uniform graph if each vertex of G has at least one diametrically opposite vertex. The set of diametrically opposite vertices of x in G is denoted by D(x). Authors Paul et.al observed that any two bfs trees of a diametrically uniform graph are of equal height.

3. Harmonious Chromatic Number of Circulant Networks

Proposition 3.1

The undirected circulant graph $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\lfloor\frac{n}{2}\rfloor\right\}\right)$ is diametrically uniform. Moreover, for any vertex x in $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\lfloor\frac{n}{2}\rfloor\right\}\right)$, the set D(x) of diametrically opposite vertices of x satisfies $|D(x)| = n-1$.

Theorem 3.2

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\lfloor\frac{n}{2}\rfloor\right\}\right)$ be an undirected circulant graph. Then $\chi_h\left[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor\right\}\right)\right] = |D(x)|+1$.

Proof

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\lfloor\frac{n}{2}\rfloor\right\}\right)$ be an undirected circulant graph. It is clear that $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\lfloor\frac{n}{2}\rfloor\right\}\right)$ is a complete graph K_n, whose diameter is one. A graph with diameter one has harmonious chromatic number $|V(G)|$. Thus $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\lfloor\frac{n}{2}\rfloor\right\}\right)] =$ n. Hence $\chi_h\left[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor\right\}\right)\right] = |D(x)|+1$

Observation 3.3

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ be an undirected circulant graph. Then for every edge (x,y) of G, $D(x)\cup D(y)$ is connected.

Theorem 3.4:

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ be an undirected circulant graph. Then $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)] =$ n.

Proof

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ be an undirected circulant graph. It is clear that $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ is a certain classes of circulant graph, whose diameter is two. The theorem follows from the fact that each vertex must receive a distinct color as it is atmost at a distance 2 from all other vertices. Hence $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)] =$ n.

Theorem 3.5

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)$ be an undirected circulant graph. Then for $n > 4l-1$ $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)] =$ n where $l = 1,2,\ldots\ldots$

Proof

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)$ be an undirected circulant graph. For $n > 4l-1$ $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)$ is a certain classes of circulant graph, whose diameter is two. The theorem follows from the fact that each vertex must receive a distinct color as it is atmost at a distance 2 from all other vertices. Hence $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)] =$ n.

Theorem 3.6

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)$ be an undirected circulant graph. Then for $n < 4l-1$ and $= 2,4,\ldots\ldots$, $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)] = \begin{cases} n-(l-1), & \text{when n is odd} \\ n-l, & \text{when n is even}\end{cases}$ if and only if diameter is 3.

Theorem 3.7

Let $G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)$ be an undirected circulant graph. Then for $n < 4l-1$ and $= 3,5,\ldots\ldots$, $\chi_h[G\left(n;\ \pm\left\{1,2,\ldots\ldots,\left\lfloor\frac{n}{2}\right\rfloor-\ l\right\}\right)] = \begin{cases} n-(l-1), & \text{when n is odd} \\ n-(l-2), & \text{when n is even}\end{cases}$ if and only if diameter is 3.

Theorem 3.8

Let $G(n;\pm\{1,2,\ldots\ldots j\}), 1 \le j \le \left\lfloor\frac{n}{2}\right\rfloor, n \ge 3$ be a circulant graph then G has perfect matching if and only if G doesnot contain an odd cycle, as an induced subgraph.

Corollary 3.9

Let $G(n;\pm\{1,2,\ldots\ldots j\}), 1 \le j \le \left\lfloor\frac{n}{2}\right\rfloor, n \ge 3$ be a circulant graph then G doesnot have perfect matching if and only if G contain an odd cycle, as an induced subgraph.

Observation 3.10

$\lambda[G(n;\pm\{1,3\})] = i+1$ when $n = 6i+j, i \ge 1$ and $0 \le j \le 5$

Proposition 3.11

The undirected circulant graph $G(n;\pm\{1,3\})$ is diametrically uniform. Moreover, for any vertex x in $G(n;\pm\{1,3\})$, the set D(x) of diametrically opposite vertices of x satisfies

1. $|D(x)| = 2$ if $n = 6l+1$
2. $|D(x)| = 3$ if $n = 6l+2$
3. $|D(x)| = 4$ if $n = 6l+3$
4. $|D(x)| = 5$ if $n = 6l+4$
5. $|D(x)| = 6$ if $n = 6l+5$.

The proof of this Proposition follows by considering the breath first search tree rooted at the vertex x of the graph.
It is obvious that $G(4;\pm\{1,3\}) = C_4$ and this is the only graph of $G(n;\pm\{1,3\})$ with $|D(x)| = 1$ and $G(5;\pm\{1,3\}) = K_5$ and this is the only graph of $G(n;\pm\{1,3\})$ with $\lambda = 1$.

Observation 3.12

Let $G(n;\ \pm\{1,3\})$ be an undirected circulant graph. Then $\chi_h[G(n;\ \pm\{1,3\})] =$ n wherever $\lambda[G(n;\pm\{1,3\})] = i+1$ where $n = 6i+j, i = 1$ and $0 \le j \le 5$.

Observation 3.13

Let $G(n;\ \pm\{1,3\})$ be an undirected circulant graph. Then $\chi_h[G(n;\ \pm\{1,3\})] =$ n − |D(x)| wherever $\lambda[G(n;\pm\{1,3\})] = i+1$ where $n = 6i+j, i = 2$ and $0 \le j \le 5$.

4. Conclusion

Thus we have determined the harmonious chromatic number of some classes of Circulant Networks with the help of diameter and the set of diametrically opposite vertices.

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