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Research paper



(–1, 1) Rings without Nilpotent Elements

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Abstract

A ring of type (γ, δ), was introduced by Albert and Kokoris [1,3] where in they have shown that a simple ring of (γ, δ) type is either associative or contains no idempotent other than 1. In this paper we obtain further results on the residual cases, to prove that a nonassociative (-1,1) rings satisfying $(x, x, y)^2 = 0$, for all elements of the rings imply $(x, x, y) = 0$. But then indeed (-1,1) rings which have no nilpotent elements are associative and there by all such rings are division rings.

Keywords: *Nonassociative ring, (–1, 1) ring, simple ring, idempotent, nilpotent.*

1. Introduction

A very important class of nonassociative algebra are alternative algebra. Algebra of the type almost alternative algebras were studied by Albert [1]. A 2-torsion free right alternative ring is right alternative if and only if it has property (***P***) [2]. Kokoris [3] has shown that a semi simple algebra of (γ, δ) type with $\delta \neq 0$, 1 and with 2, 3, 5 - torsion free has a unity quantity and is a direct sum of simple algebras. Hentzel [4] obtained a series of results on (–1, 1) rings. Suvarna and Subba Reddy [5] have consider a generalization of (1, 1) rings. They proved that if R is a 2-torsion free simple ring satisfying the identities $(x, y, z) = (x, z, y)$ and $(w, (y, x, x), z) = 0$, then R is right alternative. K.Jayalakshmi and K. Hari Babu [6] have prove that Let R be a 2,3- torsion free (-1,1) ring then either right nucleus is equals the center or the commutative center equals the center. If $[N_r, M] = 0$ then $N_r = N$ where M is the alternator ideal. L.Sreenivasulu Reddy, T. Mahesh Kumar, C. Jaya Subba Reddy [8] they proved that Every left nucleus becomes a central nucleus and right nucleus in cyclic ring R is a left nucleus , so $(x, R, R) = 0$ for every x in R. By applying cyclic property to (R, x, R) and (R, R, x) we get the result. K.Madhusudhan Reddy [9] has proved that R be a nonassociative ring of 2 torsion free with unity satisfying $(x^2y^2)\, z^2 - (xy)z \in U$ for all x, y, z in R.Then R is commutative.

2. Main Section:

A ring of type (–1, 1) ring satisfies the identities:

$$A(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y) = 0, \quad (1)$$

$$\text{and } B(x, y, z) = (x, y, z) + (x, z, y) = 0, \quad (2)$$

for all $x, y, z \in R$, where $(x, y, z) = (xy)\, z - x\, (yz)$. A ring is said to be n -torsion free if for any positive integer $nx = 0$, implies $x = 0$.

We begin with a number of identities that are consequences of equations (1) and (2) for details see [4].

$$C(x, y, z) = (x, y, yz) - (x, y, z)y = 0. \quad (3)$$

$$D(x, y, z, w) = (xy, z, w) + (x, y, [z, w]) - x\,(y, z, w) - (x, z, w)y = 0 \quad (4)$$

$$E(x, y, z) = [(y, z, x), x] + [(z, y, x), x] = 0. \quad (5)$$

$$F(x, y, z, w) = (xy, z, w) - (x, yz, w) + (x, y, zw) - x(y, z, w) - (x, y, z)w = 0. \quad (6)$$

Four applications of equation (6) results in

$$0 = F(w, x, y, z) - F(x, y, z, w) + F(y, z, w, x) - F(z, w, x, y)$$

$$G(x, y, z, w) = [(x, y, z), w] - [(y, z, w), x] + [(z, w, x), y] - [(w, x, y), z] = 0. \quad (7)$$

Now $G(x, y, x, x) + (A(x, x, y), x)$ implies $2[(x, y, x), x] = 0$ so that

$$[(x, y, x), x] = 0.$$

But then $[B(x, x, y) - (x, y, x), x] = 0$ implies

$$[(x, x, y), x] = 0. \quad (8)$$

If in equation (8) we replace x by both $x + z$ and $– x + z$, add and divide by 2, we obtain

$$[(x, x, y), z] + [(z, x, y), x] + [(x, z, y), x] = 0. \quad (9)$$

Applying (2) in $G(x, x, y, z) = 0$ we obtain

$$[(y, x, z), x] - [(z, x, y), x] = 0. \quad (10)$$

Now subtracting $[A(z, y, x), x]$ from equation (10) using equation (5) and equation (2) we obtain

$$0 = [(y, x, z), x] - [(z, y, x), x] - [(z, y, x), x] - [(y, x, z), x] - [(x, z, y), x].$$

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Thus $[(z, y, x), x] = 0$. (11)

By linearizing equation (11) we obtain

$H(z, y, x, w) = [(z, y, x), w] + [(z, y, w), x] = 0$. (12)

Now $[(F(w, z, y, x) - F(z, w, y, x) + B(yx, w, z) - [(y, w, z)x, x], x] = 0$ because of equation (11) and equation (1) yields

$I(z, w, y, x) = [z(w, y, x), x] - [w(z, y, x), x] = 0$. (13)

But then $A(x,(z, y, x), x) - (x, (z, y, x), x) = 0$ implies

$(x, x, (z, y, x)) = 0$. (14)

The semi-jacobi identity for (–1, 1) ring is

$J(x, y, z) = [xy, z] - x[y, z] - [x, z]y - (z, x, y) - 2(x, y, z) = 0$. (15)

Replacing $w = x$ in (13) and applying (15) and (14) in sequence we obtain

$K(z, x, y, x) = [z(x, y, x), x] = 0$. (16)

But we define u by $u = (x, y, x)$. Then $H(x, y, x, yx) = 0$ implies $[(y, x, yx), x] + [(x, y, x), yx] = 0 = ((y, x, yx), x) + (u, yx)$.
We now consider $[(2F(x, y, y, x) + xA(y, y, x) + xB(y, y, x) - [[(x, y, y), x], x] + K(y, y, x, x) - I(y, y, x, x) = 0$.
Then equation (11) and equation (2) in this application leads to $2[(x, y, yx), x] = 0$. Consequently $[yx, u] = 0$ because of equation (13) and hence by equation (11) we have $[x, u] = 0$, and
$[y, (x, x, y)] = 0$. But then $[(y, A(y, x, x) - B(x, y, x) + (x, y, x)) = 0$, implies $[y, u] = 0$. Finally $J(x, y, u) = 0$ along with the identity $(x, y, (x, y, x)) = 0$ i.e., $(x, y, u) = 0$ (sec[4]) yields $(u, x, y) = 0$. Thus

$(x, y, u) = (u, x, y) = [y, u] = [x, u] = 0$. (17)

On expanding $J(x, y, u)$ and employing equation (17) we obtain

$[xy, u] = 0$. (18)

Now expanding $0 = -K(xy, x, y, x) - [(x, y, u), x] + J(x, yu, x) + xK(y, x, y, x)$ leads to $(x, yu, x) = 0$.

But then $A(x, x, yu) + (x, x, yu) = (x, x, yu) = 0$.yields (19)

so that expanding $0 = F(x, x, y, u)$, applying equations (19) and (17) we obtain
$(x, xy, u) = (x^2, y, u) - (x, x, y)u$ or

$(x, xy, u) = (x^2, y, u) + (x, y, x)u$. (20)

Now from equation (4) it follows that $(x^2, y, u) = 0$, thus equation (20) reduces to

$(x, xy, u) = (x, y, x)u$ (21)

But then $A(x, xy, u) - B(xy, u, x) + (xy, x, u) = 0$ along with equation (3) results in $(x, xy, u) = 0$.
Applying this in (21), obtained equation is $(x, y, x)u = 0$.

That is $(x, y, x)^2 = 0$. (22)

Also $u(A(x, x, y) - B(x, x, y) = 0$ implies $(x, x, y)^2 = 0$. Thus we are now ready to prove the following theorem.

Theorem:
A 2,3 –torsion free (–1, 1) ring is associative if and only if $(x, y, x)^2 = 0$, implies $(x, y, x) = 0$, for all elements x, y of the ring.

Proof:
Equation (22) implies $(x, y, x) = 0$. Thus linearizing the identity yields $x, y, z) + (z, y, x) = 0$. Now regular applications of equations (1),(2) and the above linearized form proves the only if part. Conversely if the above condition is satisfied then $(x, y, x) = 0$, as a result of (22).
But then also
$(A(x, x, y) + B(x, y, x) - 2(x, y, x)) = 0$ implies

$2(x, x, y) = 0$. Hence $(x, x, y) = 0$ (23)

Again linearizing this identity gives

$(x, z, y) = -(z, x, y)$ (24)

Now from equations (1), (2) and (26) we get $3(x, y, z) = 0$. And thus $(x, y, z) = 0$. For all elements $x, y, z \in R$. The ring is therefore associative. This completes the proof of the theorem.
An immediate application of this theorem is the following.

3. Conclusion

Corollary:
A 2, 3 – torsion free (–1, 1) without nilpotent element is associative.

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