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Research paper

On a Vendor-Buyer Integrated Inventory Model with Different Co-Ordination Levels

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Abstract

An integrated inventory model with different co-ordination levels for a vendor and a buyer is proposed in this paper. The optimal policies for the proposed model are obtained. Numerical example of the developed model is presented and analyzed.

Keywords: *Buyer; Co-ordination level; Integrated inventory model; Optimal policy; Vendor*

1. Introduction

Inventory theory has been one of the most successful areas in optimization to be applied in business, industry and public sector. Inventories are resources of any kind having an economic value which are in idle and have an extremely important role in the economy of an organization. The primary function of inventory is to provide customer service considering factors such as availability of consumer goods at the factual time, in the correct place and at the actual cost. Various costs are associated with inventories that together form the total cost of the inventory system. Inventory models help to determine the optimal stock of items and to decide the optimality of total cost. The main objective of a production planning in an organization is to minimize the total inventory cost. Study in the area of inventory theory has made enormous progress in the last few decades. Due to real life situations, much attention has been paid to the association between the members of the inventory management system. At present in dynamic market conditions, the co-operation of the supplier and buyer is one of the key issues for successful inventory management system. Close co-operation between supplier and buyer can result in more cost effective production and distribution as well as a faster response to customer's demand which creates a beneficial zone for them. Study on the supplier-buyer co-operative inventory model mainly concentrates on the production shipment planning and frequency of shipments.

Many researchers have worked on supplier-buyer co-operative inventory models. Goyal [8] is the first researcher to propose an integrated inventory model for a vendor-buyer inventory problem such that the integrated inventory system has minimum total cost. Authors (Lu Lu [14], Roger [20], Kelle et al. [12] and Sarmah [21]) developed various models to study the benefit of coordinating supply chain inventories. Viswanathan [25] and Docki Saraswati et al. [6] analyzed two different integration inventory approaches. York Y. Woo et al. [26] investigated joint policies for an integrated inventory model with a single vendor and multiple buyers. Hokey and Gengui [10] discussed key challenges and opportunities associated with supply chain modeling.

Chao-Kuei Huang [2] developed and studied a vendor-buyer co-operative inventory model for defective items. Liang-Yuh Ouyang et al. [13] formulated and analysed a vendor-buyer co-operative inventory model with trade credit. Sarmah et al. [22] reviewed literature dealing in vendor–buyer coordination with quantity discount. Ben-Daya et al. [1] provided a comprehensive and latest review of the joint economic lot sizing model. Md. Azizul Baten and Anton Abdulbasah Kamil [15] studied an optimal control approach to production inventory systems with Weibull distributed deterioration. The model for single-vendor and single-buyer co-operative inventory was studied by Chao-Kuei Huang et al. [3] and Chaowalit et al. [4].

Christoph H. Glock [5] suggested two coordination mechanisms in the inventory system and derived analytical and heuristic solutions for both alternatives. Jia-Tzer Hsu and Lie-Fern Hsu [11] established a joined vendor-buyer inventory policy for imperfect quality items with planned backorders. Mona Ahmadi Rad et al. [16] developed and studied a co-operative production-inventory-marketing model for a two-echelon supply chain. Ritha and Francina [19] developed a two echelon inventory model installation under the classical EOQ to study the effects of supply quality on cost performance. Yu-Jen Lin and Hsien-Jen Lin [27] presented a vendor-buyer co-operative inventory model with a combination of price-dependent demand and product recovery. Hemapriya and Uthayakumar [9] investigated the impact of lead time reduction on the continuous review inventory model. Uthayakumar and Ganesh Kumar [23] established co-operative imperfect production inventory problem with stochastic demand. Ritha and Bharathi [18] analysed a co-operative defective inventory model with reworking. A supplier-buyer inventory model with two different types of lead time dependent ordering cost reduction was proposed and studied by Vijayashree and Uthayakumar [24].

An integrated inventory model for vendor and buyer provides only acceptable joint policies if buyer and vendor are working in proper (full or equal) co-ordinations. In real situation, this is not possible always. If they are not working in proper co-ordinations, the integrated inventory model for vendor and buyer, does not provide exact and appropriate joint policy. For resolving this issue, a new type model named as proportionate integrated inventory model for a vendor and a buyer which provides appropriate optimal joint policy based on their level co-ordination, is developed and studied.

This paper is structured as: In the Section 2, the notations are presented. Various inventory models of the given inventory problem are given in the Section 3. The Section 4 proposes a proportionate integrated model and a computational method to find an optimal joint policy for the proposed model. A numerical example is showed and analyzed in the Section 5. Finally, Section 6 is dealt with conclusion.

2. Notations

In the developed new model namely, proportionate integrated inventory model, the following notations are being made:

Q : the order quantity for the buyer;

D : per unit time demand;

h_v : per unit per unit time inventory holding cost for the vendor;

h_b : per unit per unit time inventory holding cost for the buyer;

r : the reorder point for the buyer;

A_1 : the ordering cost for the buyer;

A_2 : the setup cost for the vendor;

s : the buyer's shortage cost per unit per time;

n : the size of shipments;

T : the cycle time for the buyer;

L : a continuous random variable denotes the units de -manded during lead time;

$f(l)$: the probability density function of L

where $f(l)=\frac{1}{b-a}$, $a \le l \le b$.

3. An Inventory Model Description

Consider a vendor-buyer continuously reviewed inventory model for a single product with constant demand rate D and infinite vendor's production rate. An order of quantity Q is ordered by the buyer when the inventory reaches the reorder point r. After receiving first order from the buyer, the vendor produces nQ quantity of the requested product with an infinite production rate at one set-up and ships each lot over n times to the buyer. A set-up cost for each production run and an ordering cost for each order of the quantity Q are incurred by the vendor and the buyer respectively. Shortages are completely backordered. The supply lead-time between vendor and buyer is stochastic and follows a uniform distribution with parameters a and b, $L \sim U[a,b]$. The orders do not cross in time and the order quantity received by the buyer is greater than or equal to the maximum demand during lead time. Since the total transportation cost per unit time is constant, we ignore the shipping cost.

The buyer's model, model for the vendor and the vendor-buyer integrated model for the above said model are presented in this section.

3.1. Inventory Model for the Buyer

Now, since shortages arise, $r \le Db$ and the buyer's expected total cost per unit time, $B(r,Q)$ is given as

$B(r,Q)=$ Ordering cost per unit time + Holding cost per unit time + Shortage cost per unit time.

$$= A_1\frac{D}{Q}+\frac{h_b}{T}\int_a^{\frac{r}{D}}\left(r-\frac{Dl}{2}\right)\frac{l}{b-a}dl+\frac{rh_b}{DT}\int_a^b(Q-Dl)\frac{1}{b-a}dl$$

$$+\frac{h_b}{2DT}\int_a^b(Q-Dl)^2\frac{1}{b-a}dl+\frac{r^2h_b}{2DT}\int_{\frac{r}{D}}^b\frac{1}{b-a}dl$$

$$+\frac{s}{2DT}\int_{\frac{r}{D}}^b(Dl-r)^2\frac{1}{b-a}dl.$$

$$=\frac{A_1D}{Q}+\frac{h_b\left(2r^3-3rD^2a^2+D^3a^3\right)}{6DQ(b-a)}+h_br-\frac{h_bDr(b+a)}{2Q}$$

$$+\frac{h_b(Q-Db-Da)}{2}+\frac{h_bD^2\left(b^2+ab+a^2\right)}{6Q}+\frac{h_br^2(Db-r)}{2DQ(b-a)}$$

$$+\frac{s(Db-r)^3}{6DQ(b-a)}.$$

3.2. Inventory Model for the Vendor

Now, when the purchaser order quantity Q is adopted, the orders are received by the vendor at the interval T. If the buyer ordering single items to vendor amounted Q units, the vendor manufactures nQ at one setup with an infinite production rate. The length of system cycle, $T=\frac{nQ}{D}$ since n lots of Q size are delivered from the vendor to the buyer.

Now, the vendor's set up cost per unit time $=\frac{A_2}{T}=A_2\frac{D}{nQ}$ and the vendor's holding cost per unit time $=\frac{(n-1)Q}{2}h_v$.

Therefore, the vendor expected total cost per unit time $V(n)$ is given as

$V(n)=$ set - up cost per unit time + holding cost per unit time

$$=\frac{DA_2}{nQ}+\frac{(n-1)Qh_v}{2}.$$

3.3. Vendor-Buyer Integrated Model

Suppose that the vendor and buyer agree to cooperate and follow the jointly optimal policy.

Then, the joint expected total cost of the buyer and the vendor per unit time, $I(Q,r,n)$ is given below:

$$I(Q,r,n)=\frac{D(nA_1+A_2)}{nQ}+\frac{Q\left(h_b+(n-1)h_v\right)}{2}+h_br$$

$$+\frac{h_b\left(2r^3-3rD^2a^2+D^3a^3\right)}{6DQ(b-a)}-\frac{h_bDr(b+a)}{2Q}+\frac{h_b(Q-Db-Da)}{2}$$

$$+\frac{h_bD^2\left(b^2+ab+a^2\right)}{6Q}+\frac{h_br^2(Db-r)}{2DQ(b-a)}+\frac{s(Db-r)^3}{6DQ(b-a)}.$$

If the vendor and the buyer do not wish to cooperate, the buyer's model and the vendor's model are studied and determined optimal policies (Q,r) and n independently. In this case, the optimal expected total cost $TC(Q,r,n)$ is equal to the summation of the buyer's and vendor's optimal expected total costs.

If the vendor and the buyer wish to cooperate, the vendor-buyer integrated model is analyzed and obtained a joint optimal policy (Q,r,n). In this case, the optimal joint expected total cost is equal to the optimal value of $I(Q,\mathrm{r},\mathrm{n})$.

In the literature, researchers have shown that the joint optimal policy of the integrated inventory model is more effective than the independent optimal policies of buyer's and vendor's models.

4. Proportionate Integrated Inventory Model for a Vendor and a Buyer

In this section, we develop new type of integrated model for a vendor and a buyer with different co-ordination levels named as a proportionate integrated inventory model for a vendor and a buyer which provides appropriate joint policy based on their level co-ordinations.

Suppose that α % (>0) is the co-ordination level of the buyer and β % (>0) is the co-ordination level of the vendor.

Consider the following non-linear mixed integer programming problem.

(P) Minimize $f(Q,r,n) = \lambda B(r,Q)+(1-\lambda)\mathrm{V}(n)$

Subject to $r \in R^{+}$, $Q \in R^{+}$ and $n \in Z^{+}$

where $\lambda = \dfrac{\alpha}{\alpha+\beta}$, R^{+} is the set all positive real numbers and Z^{+} is the set of all positive integers.

Note that if $\alpha\% = \beta\%$, then the problem (P) becomes the integrated inventory model for a buyer and a vendor.

Now, from the Theorem 2/2.1.in Goeffrion [7]/ Pandian [17], suppose that if (Q^{o},r^{o},n^{o}) is an optimal solution to the problem (P), then (Q^{o},r^{o},n^{o}) is a fair solution of the bi-objective non-linear mixed integer programming problem (MP) where

(MP) Minimize $(\mathrm{B}(r,Q),\mathrm{V}(n))$

Subject to $r \in R^{+}$, $Q \in R^{+}$ and $n \in Z^{+}$

Now, if (Q^{o},r^{o},n^{o}) is an optimal solution of the problem (P), the optimal proportionate joint expected total cost for the proportionate integrated inventory model, $P(Q^{o},\mathrm{r}^{o},\mathrm{n}^{o})$ is defined as

$$P(Q^{\circ},\mathrm{r}^{\circ},\mathrm{n}^{\circ}) = B(r^{\circ},Q^{\circ})+V(n^{\circ}).$$

Note that if $\alpha\% = \beta\%$, then the optimal proportionate joint expected total cost $P(Q^{o},\mathrm{r}^{o},\mathrm{n}^{o})$ is equal to the optimal joint expected total cost by the Lemma 3.1 in Pandian [17] .

Remark 1:

The value $\lambda = \dfrac{\alpha}{\alpha+\beta}$ is the buyer's joint co-ordination grade and the value $1-\lambda = \dfrac{\beta}{\alpha+\beta}$ is the vendor's joint co-ordination grade in the model.

Remark 2:

(i) If the buyer's and vendor's joint co-ordination grades are the same, the proportionate integrated model coincides with the integrated inventory model.

(ii) If the buyer's and vendor's joint co-ordination grades are not the same, the proportionate integrated model does not coincide with the integrated inventory model.

Now, the proportionate joint optimal policy for the buyer and the vendor and the proportionate joint expected total cost for the integrated inventory system with different co-ordination levels can be obtained by solving the problem (P) using MATLAB.

5. Numerical Example

Consider an inventory system with the following data: D=1000 per year, A_1= \$25 per order, A_2= \$400 per setup, h_b=\$10 per unit per year, h_v= \$8 per unit per year, s= \$30 per unit, a=0 and b=35 days.

For different values of α % and β %, we obtain the following optimal joint policies and its corresponding optimal expected total costs using MATLAB and summarized them as a table form which is given below:

α %	β %	r^{o}	Q^{o}	n^{o}	$B(r^{o},Q^{o})$	$V(n^{o})$	$P(r^{o},Q^{o},n^{o})$
60	90	0	312.57	1	\$1359.5	\$1279.7	\$ 2639.2
50	70	0	312.59	1	\$1359.6	\$1279.6	\$ 2639.2
80	80	0	311.87	1	\$1356.6	\$1282.6	\$ 2639.2
100	100	0	311.87	1	\$1356.6	\$1282.6	\$ 2639.2
90	60	0	311.16	1	\$1353.7	\$1285.5	\$ 2639.2
70	50	0	311.15	1	\$1353.6	\$1285.6	\$ 2639.2

From the above table, we observe in the developed model that if the buyer's coordination level is less / more than the vendor's co-ordination level, the optimal proportionate joint expected total cost is equal to the optimal joint expected total cost but their optimal joint policies are not the same and also, if the buyer's coordination level is equal to the vendor's co-ordination level, the optimal proportionate joint expected total cost coincides with the optimal joint expected total cost (Zhiguang Zhang [28]).

Now, the comparison results with other models are given below:

Model	Optimal policy			Optimal expected total cost
	r^{o}	Q^{o}	n^{o}	
Non-cooperative	27.7	97	3	\$ 2918.1
Integrated	0	311.9	1	\$ 2639.2
Proportionate integrated with α =70 and β =90	0	312.27	1	\$ 2639.2
Proportionate integrated with α =90 and β =60	0	311.16	1	\$ 2639.2

From the above table, we understand that the role of the co-ordination levels of the vendor and the buyer is sensitive in the integrated inventory model and when the vendor and the buyer have different co-ordination levels, analyzing the proportionate integrated inventory model is more beneficial than the integrated inventory model.

6. Conclusion

In this paper, an inventory system for a single vendor and a single buyer is discussed. A new model, namely the proportionate integrated model for a single vendor and a single buyer with different coordination levels is proposed instead of the integrated inventory model. The optimal joint policy for the proposed model is determined which provides an appropriate joint policy for the given inventory system. The proposed inventory model is shown to effect than the integrated inventory model using numerical example. The proposed integrated inventory model for a vendor and a buyer helps an organization to study inventory management system when the buyer and the vendor are in different co-ordination levels. In near future, we have a plan to study an integrated model with a vendor and multiple buyers with different co-ordination levels.

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