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Research paper

Hungarian Algorithm using Haar Tuples to Solve Fuzzy Travelling Salesman Problem

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Abstract

Travelling salesman problem(TSP) deals with visiting all the given cities and return back to the starting city with the minimum travelling distance or minimum travelling cost where each city is visited exactly once. The TSP problem is a special kind of an assignment model that excludes sub tours. In this paper we used Haar Hungarian algorithm approach [13] to solve a Fuzzy Travelling Salesman Problem (FTSP) and Numerical examples are given to validate the proposed algorithm.

Keywords: *Triangular fuzzy number; Trapezoidal fuzzy number; Fuzzy arithmetic operations; Fuzzy number; Fuzzy ranking techniques; Fuzzy Travelling Salesman problems; Haar Wavelet;Optimal solution.*

1. Introduction

Travelling salesman problem is a part of computational mathematics which is the collection of computational problems with ingrained difficulties. The Travelling salesman problem is to detect the shortest circuit of cities. Since 18th century this problem has been studied rigorously. In practical situations it is difficult to determine the cost or time as crisp quantity. The fuzzy parameters were used to overcome this uncertainty of fixing the cost or time. The cost or time or distance of the fuzzy TSP is fuzzy numbers.

In recent years, several techniques have been approached to obtain the solutions of fuzzy TSP. Hansen [2] applied tabu search algorithm to solve fuzzy TSP. Jaszkiewicz [3] applied genetic local search algorithm to solve the fuzzy TSP problem. Yan et al., [4] proposed the fuzzified version of evolutionary algorithm to solve fuzzy TSP. Sepideh Fereidouni [5] and Rehemat et al., [8] solved the problem by using multi objective linear programming. Angel et al., [6] proposed dynamic search algorithm to solve fuzzy TSP. Paraquete et al., [7] used the pareto local search algorithm for solving fuzzy TSP. Amit kumar et al., [9] proposed an algorithm for solving fuzzy TSP with LR-fuzzy parameters. Dhanasekar et al., [10] solved fuzzy TSP by using Hungarian algorithm with elementwise subtraction of fuzzy numbers. Since fuzzy TSP is a NP-hard problem, more numbers of algorithms are still proposed to solve the fuzzy TSP. In this paper, the ordering of fuzzy numbers based on Haar wavelet is used [12], to order the fuzzy numbers. The advantage of using this ranking technique is that, it converts a given fuzzy number into average and detailed coefficients using down sampling. The uniqueness of the Haar ranking method [12] is that, the fuzzification from the defuzzied value is very easy to obtain through up sampling. In this paper the Hungarian method using Haar tuples [13] is proposed for solving travelling salesman problem with fuzzy parameters. In section-2 basics are discussed. Section 3 describes the proposed new algorithm. In Section 4 a numerical example is solved by using the proposed method. Conclusion is given in Section-5.

2. Preliminaries

2.1. Fuzzy Set

A Fuzzy set can be obtained by allocating all the elements in the universe of discourse to a value lies in [0,1] by using a membership function.

2.2. Fuzzy Number

A Fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$ satisfies piece wise continuity, convexity and normality.

2.3. Triangular Fuzzy Number

A fuzzy number with membership function in the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, a \le x < b \\ 1, x = b \\ \dfrac{c-x}{c-b}, b < x \le c \end{cases}$$

is called a triangular fuzzy number $\tilde{A} = (a,b,c)$.

2.4. Trapezoidal Fuzzy Number

A fuzzy number with membership function in the form

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a}, a\le x<b\\ 1, b\le x\le c\\ \dfrac{d-x}{d-c}, c<x\le d\end{cases}$$

is called a trapezoidal fuzzy number $\tilde{A}$ = (a,b,c,d).

2.4. Fuzzy Operations

Addition:
$(a_1,b_1,c_1,d_1)+(a_2,b_2,c_2,d_2)=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$
$(a_1,b_1,c_1) +(a_2,b_2,c_2,) = (a_1+ a_2,b_1+b_2,c_1+c_2)$
Subtraction:
$(a_1,b_1,c_1,d_1) -(a_2,b_2,c_2,d_2)= (a_1- d_2,b_1-c_2,c_1-b_2,d_1-a_2)$
$(a_1,b_1,c_1) -(a_2,b_2,c_2,) = (a_1- c_2,b_1-b_2,c_1-a_2)$

2.5. Haar Ranking

For a given fuzzy number $\tilde{A}=(a,b,c,d)$, the average and detailed coefficients namely the scaling and wavelet coefficients can be calculated using α=((a+b+c+d))/4, β=((a+b)-(c+d))/4, γ=(a-b)/2, δ=(c-d)/4 and call this new 4-tupleas. Define $R(\tilde{A})=(\alpha,\beta,\gamma,\delta)$ if $\tilde{A}\prec\tilde{B}$ the first element of the ordered tuple of $R(\tilde{A})$ is less than the first element of the ordered tuple of $R(\tilde{B})$ .Define if $\tilde{A}\succ\tilde{B}$ the first element of the ordered tuple of $R(\tilde{A})$ is more than the first element of the ordered tuple of $R(\tilde{B})$. Define $\tilde{A}\approx\tilde{B}$ if and only if the elements of $R(\tilde{A})$ and $R(\tilde{B})$ are term wise equal.

2.6. Order Pair Arithmetic

Addition:
$(a_1,b_1,c_1,d_1)+(a_2,b_2,c_2,d_2)=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$
$(a_1,b_1,c_1) +(a_2,b_2,c_2,) = (a_1+ a_2,b_1+b_2,c_1+c_2)$.
Subtraction:
$(a_1,b_1,c_1,d_1) -(a_2,b_2,c_2,d_2)= (a_1- a_2,b_1-b_2,c_1-c_2,d_1-d_2)$
$(a_1,b_1,c_1) -(a_2,b_2,c_2,) = (a_1- a_2,b_1-b_2,c_1-c_2)$.

2.7. Fuzzy Travelling Salesman Problem

The matrix form of the fuzzy TSP is given as follows

	$city-1$	$city-2$	$city-3$	..	$city-n$
$city-1$	∞	$\tilde{c}_{12}$	$\tilde{c}_{13}$	..	$\tilde{c}_{1n}$
$city-2$	$\tilde{c}_{21}$	∞	$\tilde{c}_{23}$	..	$\tilde{c}_{2n}$
…	..	…	…	…	…
$city-n$	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	$\tilde{c}_{n3}$	…	∞

A Fuzzy Travelling salesman problem is defined as

$$Min\tilde{z}=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$$

Subject to

$$\sum_{i=1}^{n}x_{ij}=1,\sum_{j=1}^{n}x_{ij}=1..........(1)$$

$$x_{ij}+x_{ji}\le 1, 1\le i\ne j\le n.......(2)$$

$$x_{ij}+x_{jk}+x_{ki}\le 2, 1\le i\ne j\ne k\le n.....(3)$$

$$x_{ip_1}+x_{p_1p_2}+.....+x_{p_{n-2i}}\le n-2, 1\le i\ne p_1\ne ....\le n.....(4)$$

Constraint (1) makes sure that each city is visited exactly once. Constraint (2) excludes all 2-city sub tours. Constraint (3) excludes all 3-city sub tours. Constraint (4) defies all (n-2) city sub tours.

3. Proposed Algorithm

Consider the matrix form of the fuzzy travelling salesman problem.
Step: 1
Transform the given fuzzy parameters in to Haar parameters.
Step: 2
Fuzzy optimum assignment is obtained by using the Hungarian algorithm.
Step: 3
Inspecting the obtained solution to see whether the solution satisfied route conditions or not.
If it is then it is the optimal solution of fuzzy TSP. If not, take the next best solution.

4. Numerical Examples

Consider the following fuzzy TSP discussed in \cite{Abha}:

	$city-1$	$city-2$	$city-3$	$city-4$
$city-1$	∞	(1,2,3)	(8,9,10)	(9,10,11)
$city-2$	(0,1,2)	∞	(5,6,7)	(3,4,5)
$city-3$	(14,15,16)	(6,7,8)	∞	(7,8,9)
$city-4$	(5,6,7)	(2,3,4)	(11,12,13)	∞

Convert the given fuzzy numbers in to haar tuples. The TSP with haar tuples as its elements is given as follows

	$city-1$	$city-2$	$city-3$	$city-4$
$city-1$	∞	(2,-.5,-.5,-.5)	(9,-.5,-.5,-.5)	(10,-.5,-5,-.5)
$city-2$	(1,-.5,-.5,-.5)	∞	(6,-.5,-.5,-.5)	(4,-.5,-.5,-.5)
$city-3$	(15,-.5,-5,-.5)	(7,-.5,-.5,-.5)	∞	(8,-.5,-.5,-.5)
$city-4$	(6,-.5,-.5,-.5)	(3,-.5 - .5,-.5)	(12,-.5,-5,-.5)	∞

After applying the first two steps of Hungarian algorithm, we get

	$city-1$	$city-2$	$city-3$	$city-4$
$city-1$	∞	(0,0,0,0)	(2,0,0,0)	(7,0,0,0)
$city-2$	(0,0,0,0)	∞	(0,0,0,0)	(2,0,0,0)
$city-3$	(8,0,0,0)	(0,0,0,0)	∞	(0,0,0,0)
$city-4$	(3,0,0,0)	(0,0,0,0)	(7,0,0,0)	∞

Covering the Haar zeros in a minimum no of lines.

∞	(0,0,0,0)	(2,0,0,0)	(7,0,0,0)
(0,0,0,0)	∞	(0,0,0,0)	(2,0,0,0)
(8,0,0,0)	(0,0,0,0)	∞	(0,0,0,0)
(3,0,0,0)	(0,0,0,0)	(7,0,0,0)	∞

To get the optimal assignment the number of covering lines should be equal to the number of rows. Since it is not in this case, choose the minimum Haar number in the uncovered elements is (2,0,0,0) and Subtract it from all the uncovered Haar elements also add it in the intersection of the lines.

$$\begin{pmatrix} & city-1 & city-2 & city-3 & city-4 \\ city-1 & \infty & (0,0,0,0) & (0,0,0,0) & (7,0,0,0) \\ city-2 & (0,0,0,0) & \infty & (0,0,0,0) & (4,0,0,0) \\ city-3 & (6,0,0,0) & (0,0,0,0) & \infty & (0,0,0,0) \\ city-4 & (1,0,0,0) & (0,0,0,0) & (5,0,0,0) & \infty \end{pmatrix}$$

Again covering the Haar zeros in a minimum no of lines.

$$\begin{pmatrix} \infty & (0,0,0,0) & (0,0,0,0) & (7,0,0,0) \\ \hline (0,0,0,0) & \infty & (0,0,0,0) & (4,0,0,0) \\ \hline (6,0,0,0) & (0,0,0,0) & \infty & (0,0,0,0) \\ \hline (1,0,0,0) & (0,0,0,0) & (5,0,0,0) & \infty \end{pmatrix}$$

Since the number of covering lines is equal to the number of rows, allocations can be made.

$$\begin{pmatrix} & city-1 & city-2 & city-3 & city-4 \\ city-1 & \infty & (0,0,0,0) & (0,0,0,0) & (7,0,0,0) \\ city-2 & (0,0,0,0) & \infty & (0,0,0,0) & (4,0,0,0) \\ city-3 & (6,0,0,0) & (0,0,0,0) & \infty & (0,0,0,0) \\ city-4 & (1,0,0,0) & (0,0,0,0) & (5,0,0,0) & \infty \end{pmatrix}$$

Therefore the optimal tour is 1 → 3→ 4→ 2→ 1. i.e.,A→ C →D → B → A the Haar optimal cost is [21,-2,-2,-2] and the corresponding fuzzy optimal cost is given by (17,21,25).

5. Conclusion

An Algorithm using Haar tuples is proposed for solving fuzzy travelling salesman problem. Since this algorithm is similar to Hungarian algorithm it is effective and easy to compute. This proposed method is effective than the existing methods because the results were obtained in Haar parameters and further it can be transformed in to parameters.

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