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Research paper

Self Centeredness of Total Graphs W.R.T. D-Distance

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Abstract
In this article using *D*-distance, we study the self-centeredness of total graphs. We end with some open questions.

Keywords: *Total graph, D-distance, self-centered graphs*

1. Introduction

Given a graph $G=(V,E)$, the *total graph* of *G* is a graph whose vertex set consists of all the vertices and edges of *G* i.e., $V\cup E$ and two vertices are adjacent in the new graph if and only if they are adjacent in the graph *G* (see [1]). The total graph of *G* is denoted by $T(G)$ or G^T.

The concept of *D*-distance in a graph *G* was introduced by Reddy Babu and Varma in [3]. The *D*-distance, denoted by $d^D(u,v)$, between two vertices *u,v* of a connected graph *G* is defined as 0 if $u=v$ and $\max\left\{l_s^D(u,v)\right\}$, if $u\neq v$ where the maximum is taken over all $u-v$ paths *s* in *G* and $l^D(u,v)$ denotes the *D*-length of the paths which is given by $l_s^D(u,v)=l(s)$ + sum of the degrees of all vertices present in s. Using this distance we can define *D*-eccentricity of vertices and hence *D*-radius and *D*-diameter in a natural way.
A graph *G* is said to be *D-self centered* if *D*-radius and *D*- diameter of *G* are equal. In otherwords *G* is *D*- self centered if the *D*-eccentricities of all vertices are equal. The *D*-eccentricity of a vertex is denoted by $e^D(v)$ and radius, diameter are denoted by $r^D(G)$ and $dia^D(G)$ respectively.
In the present article we study the problem of a total graph being *D*-self centered. Throughout the article, we denote by x_{ij} the edge joining the vertices v_i and v_j in a graph *G*. All graphs we consider are finite and connected simple graphs.

2. Radius and Diameter of Some Total Graphs

In this section, we study the self – centeredness of total graph w.r.t. *D*-distance. We begin with a theorem.

Theorem 2.1
Consider the star graph $G=St_{n,1}$ with $n+1$ vertices. The *D*-radius and *D*-diameter of $T(St_{n,1})$ are $r^D(G)=2n+6$, $dia^D(G)=3n+2$ for $n\geq 4$. For $n=3$, $r^D(G)=11$ and $dia^D(G)=12$.

Proof: First we consider the case where $n\geq 4$. Consider the star graph with vertex set $\{v_0,v_1,v_2,\cdots v_n\}$ where v_0 is the central vertex and edge set is $\{x_{01},x_{02},\cdots x_{0,n-1},x_{0n}\}$ where $x_{0,i}=v_0v_i$. The degree of each vertex is $d(v_i)=1$ for $i=1,2,\cdots,n$ and $d(v_0)=n$.

In the total graph of *G*, $T\left(St_{n,1}\right)=G^T$. The vertex set is $\left\{\begin{matrix}v_0,v_1,v_2,\cdots,v_n,\\ x_{0,1},x_{0,2},\cdots,x_{0,n-1},x_{0,n}\end{matrix}\right\}$. In G^T, degree $d(v_i)=2$ for $i=1,2,\cdots,n$ and $d(v_0)=2n$ and $d(x_{0,i})=n+1$ for $i=1,2,\cdots,n$. Further we can see that the *D*-distances in G^T are as follows: for $1\leq i,j\leq n$,

$$\max\left\{d^D(v_i,v_j)\right\}=2n+6,$$

$$\max\left\{d^D(v_0,v_i)\right\}=2n+3$$

$$\max\left\{d^D(v_i,x_{0,j})\right\}=2n+6,$$

$$\max\left\{d^D(v_i,x_{0,i})\right\}=n+4$$

$$\max\left\{d^D(x_{0,i},x_{0,j})\right\}=2n+3,$$

$$\max\left\{d^D(v_0,x_{0,i})\right\}=3n+2.$$

From this it is clear that the eccentricities of vertices in G^T, are as follows:

$$e^D(v_i)=2n+6, e^D(v_0)=3n+2,$$
$$e^D(x_{0,i})=3n+2 \text{ for } i=1,2,\cdots,n.$$
$$r^D\{G^T\}=r^D\{T(St_{n,1})\}=2n+6.$$

Thus $dia^D\{G^T\}=dia^D\{T(St_{n,1})\}=3n+2,$

$\text{for } n\geq 4$

Next we consider the case of star graph with 4 vertices $St_{3,1}$. The *D*-distance between vertices of $T(St_{3,1})$ are as follows:

Table 1: *D*-distances in $T(St_{3,1})$

Vertex	v_0	v_1	v_2	v_3	$x_{0,1}$	$x_{0,2}$	$x_{0,3}$	Ecc
v_0	0	9	9	9	11	11	11	11
v_1	9	0	12	12	7	12	12	12
v_2	9	12	0	12	12	7	12	12
v_3	9	12	12	0	12	12	7	12
$x_{0,1}$	11	7	12	12	0	9	9	12
$x_{0,2}$	11	12	7	12	9	0	9	12
$x_{0,3}$	11	12	12	7	9	9	0	12

From this table it is clear that

$$r^D\{T(St_{3,1})\}=11 \quad \text{and} \quad dia^D\{T(St_{3,1})\}=12.$$

Corollary 2.2

The total graph of $St_{4,1}$ is D-self centered.

Proof: From theorem 2.1 we get that for $n=4$, $r^D\{T(St_{4,1})\}=14=dia^D\{T(St_{4,1})\},$

Hence this graph *D*-self centered.

Remark 2.3:

From the above, it is clear that $St_{n,1}^T$ is not *D*-self centered except for $n=4$. From the proposition 3.12 of [3], we can see that $St_{n,1}$ is also not *D*-self centered, because

$$r^D\{St_{n,1}\}=dia^D\{St_{n,1}\}-2=n+2.$$

Theorem 2.4

The complete graph K_n $(n\geq 3)$ and its total graph $T(K_n)$, both are D-self centered for $n\geq 3$.

Proof: Let *G* be the complete graph K_n for $n\geq 3$. Then G^T denotes the total graph of K_n. We know that *G* has *n* vertices and $\frac{n(n-1)}{2}$ edges. Hence G^T has $\frac{n^2+n}{2}$ vertices and the number of edges in G^T is $\frac{n(n^2-1)}{2}$.

Let $\{v_1,v_2\cdots v_n\}$ be the vertices of $G=K_n$. We can easily see that degree of each vertex in G^T is $2(n-1)$, i.e., $d(v_i)=d(x_{i,j})=2(n-1)$. Further, the *D*-distance between vertices in G^T are as follows: for $1\leq i,j\leq n,$

$$\max\{d^D(v_i,v_j)\}=4n-3,$$
$$\max\{d^D(v_i,x_{i,k})\}=4n-3,$$
$$\max\{d^D(v_i,x_{j,k})\}=6n-4.$$

Therefore the *D*-eccentricities of each vertex is

$e^D(v_i)=e^D(x_{j,k})=6n-4.$ Thus

$r^D\{T(K_n)\}=dia^D\{T(K_n)\}=6n-4.$ Hence $T(K_n)$ is *D*-self centered.

Further from proposition 3.7 of [3] we have, for $n\geq 3$,

$r^D\{K_n\}=dia^D\{K_n\}=2n-1.$ Hence K_n is *D*-self centered.

Next, we look at the path graph.

Theorem 2.5

Let $P_n\,(n\geq 3)$ be the path graph with n vertices. Then we have

$$r^D\{P_n^T\}=\begin{cases}\dfrac{5n+2}{2} & \text{if } n \text{ is even}\\ \dfrac{5n+1}{2} & \text{if } n \text{ is odd}\end{cases} \quad \textit{and}$$

$$dia^D\{P_n^T\}=5(n-1) \text{ for all } n.$$

Proof: Consider the path graph $P_n\,(n\geq 3)$ with vertex set $\{v_1,v_2,\cdots,v_n\}$ and edge set $\{x_{1,2},x_{2,3},\cdots,x_{n-1,n}\}$, degree of each vertex is 2 except end vertices and $d(v_i)=d(v_n)=1$. The total graph of P_n, P_n^T, is of order $(2n-1,4n-5)$. The degrees of $v_2,v_3\cdots v_{n-1}$ are 4, degrees of v_1 and v_n are 2, degrees of $x_{2,3},x_{3,4},\cdots,x_{n-2,n-1}$ are 4 and degrees of $x_{1,2}$ *and* $x_{n-1,n}$ are 3.

To compute the eccentricities of vertices in P_n^T, we consider the following two cases separately, namely, *n* is even and odd.

Case (i)

n is odd, say $n=2m+1$.

In this case the *D*-distances between vertices are presented in the table 3.

From this, we can see that *D*-eccentricities are

$$\{10m,10m-2,10m-7,\cdots,5m+8,$$
$$5m+3,\cdots,10m-2,\ 10m,10m,$$

$10m-4, 10m-9, \cdots, 10m-4, 10m\}$. Thus the maximum D-eccentricity of P_n^T is $10m$ and minimum D-eccentricity is $5m+3$. Hence the D-radius is $5m+3$ and D-diameter is $10m$.

Case (ii):
n is even say $n = 2m$
In this case, the D-distances in P_n^T are as shown in table 4.
From this table , we can see that D-eccentricities of vertices in the graph P_n^T are

$$\left\{\begin{array}{l}10m-5, 10m-7, 10m-12, \cdots, \\ 10m-7, 10m-5, 10m-5, \cdots, \\ 10m-9, 10m-14, \cdots, 5m+1, \\ \cdots, 10m-8, 10m-5\end{array}\right\}.$$

Thus the maximum D-eccentricity of P_n^T is $10m-5$ and minimum D-eccentricity of P_n^T is $5m+1$. Hence the D-radius of P_n^T is $5m+1$ and D-diameter of P_n^T is $10m-5$, if n is even.

Therefore $$r^D\left\{P_n^T\right\} = \begin{cases} 5m+1 & \text{if } n = 2m \\ 5m+3 & \text{if } n = 2m+1 \end{cases}$$

and

$$dia^D\left\{P_n^T\right\} = \begin{cases} 10m-5 & \text{if } n = 2m \\ 10m & \text{if } n = 2m+1 \end{cases}$$

In other words, we have

$$r^D\left\{P_n^T\right\} = \begin{cases} \dfrac{5n+2}{2} & \text{if } n \text{ is even} \\ \dfrac{5n+1}{2} & \text{if } n \text{ is odd} \end{cases}$$

and $dia^D\left\{P_n^T\right\} = 5(n-1)$ for $n \geq 3$.

Proposition 2.6:
We have $r^D(P_2^T) = dia^D(P_2^T) = 5.$

Proof: The vertex set in P_2^T is $\{v_1, v_2, x_{1,2}\}$ and degree of each vertex is 2. The D- distances in P_2^T are as follows :

Table 2: D-distances in P_2^T

Vertex	v_1	v_2	$x_{1,2}$	Ecc
v_1	0	5	5	5
v_2	5	0	5	5
$x_{1,2}$	5	5	0	5

From this table, it follows that the D-eccentricities are $e^D(v_1) = e^D(v_2) = e^D(x_{1,2}) = 5.$

Hence $r^D(P_2^T) = dia^D(P_2^T) = 5$. Therefore P_2^T is D-self centered .

Remark 2.7
From the above it is clear that the total graph of path graph P_n is D -self centered only for n =2. It is not D-self centered for any $n \geq 3$. Also proposition 3.8 of [3] it follows that P_n for $n \geq 3$ is also not D-self centered and P_2 is D-self centered.

3. Conclusions

We begin this section with the example of a graph G for which total graph $T(G)$ is D-self centered but the graph G is not D-self centered.

Example 3.1
Consider the (4,5) graph G as shown below

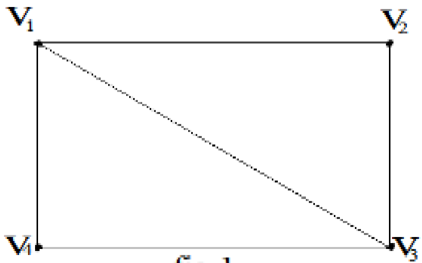


fig 1

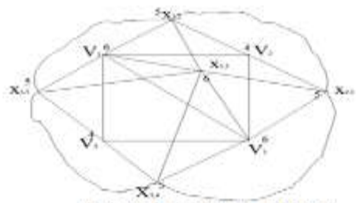
For this graph G , we have the D-eccentricities of vertices are as follows:

Table 5: D-distances in graph G(fig 1)

vertices	v_1	v_2	v_3	v_4	Ecc
v_1	0	6	7	6	7
v_2	6	0	6	9	9
v_3	7	6	0	6	7
v_4	6	9	6	0	9

From the table we can conclude that $r^D(G) = 7$, $dia^D(G) = 9$. Hence G is not D-self centered.

Next , consider the total graph $T(G)$ of G which is of order (9,23). In this the D-distances between vertices are as follows:



Total graph of G
fig 2

Table 6: D-distanecs in graph $T(G)$

$T(G)$	v_1	v_2	v_3	v_4	$x_{1,2}$	$x_{2,3}$	$x_{3,4}$	$x_{1,4}$	$x_{1,3}$	ecc
v_1	0	11	13	11	12	17	17	11	13	17
v_2	11	0	11	16	10	10	16	16	17	17

v_3	13	11	0	11	17	12	12	17	13	17
v_4	11	16	11	0	16	16	10	10	17	17
$x_{1,2}$	12	10	17	16	0	11	17	11	12	17
$x_{2,3}$	17	10	12	16	11	0	11	17	12	17
$x_{3,4}$	17	16	12	10	17	11	0	11	12	17
$x_{1,4}$	11	16	17	10	11	17	11	0	12	17
$x_{1,3}$	13	17	13	17	12	12	12	12	0	17

From this table, it follows that $e^D(v_i) = 17$ for every vertex *v* of $T(G)$ and hence $T(G)$ is *D*- self centered .

Example 3.2
(see example 2.4 of [2]). Consider the (5,6) graph *G* as shown below. This graph *G* is *D*-self centered but its total graph $T(G)$ is not *D*-self centered.

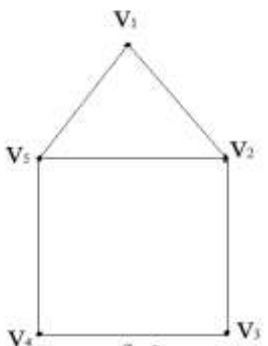


fig 3

From the above examples and the results of section 2, we see that the following various cases are possible:
Case 1: A graph and its total graph both are *D*-self centered.
Case 2: A graph and its total graph both are not *D*-self centered.
Case 3: Among the graph and its total graph only one is *D*-self centered and the other is not.
Thus we conclude the article with the following open problems.

Open Problem 1.
Characterize all the graphs for which the total graph is D-self centered .

Open Problem 2.
Characterize all the graphs for which the graph and its total graph both are D-self centered .

Open Problem 3.
Characterize all the graphs for which its total graph is D-self centered implies the graph is also D-self centered

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Table 3: D-distance in total graph of P_n (n odd)

	v_1	v_2	$\cdots$	v_n	v_{n+1}	$\cdots$	v_{2n+1}	$x_{1,2}$	$x_{2,3}$	$\cdots$	$x_{2n-1,2n}$	$x_{2n,2n+1}$	Ecc
v_1	0	7	$\cdots$	5*m*-3	5*m*+2	$\cdots$	10*m*-3	6	11	$\cdots$	10*m*-4	10*m*	10*m*
v_2	7	0	$\cdots$	5*m*-6	5*m*-1	$\cdots$	10*m*-3	8	9	$\cdots$	10*m*-6	10*m*-2	10*m*-2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
v_n	5*m*-3	5*m*-11	$\cdots$	0	9	$\cdots$	5*m*+7	5*m*-2	5*m*-6	$\cdots$	5*m*+4	5*m*+8	5*m*+8
v_{n+1}	5*m*+2	5*m*-1	$\cdots$	9	0	$\cdots$	5*m*+2	5*m*+3	5*m*-1	$\cdots$	5*m*-1	5*m*+3	5*m*+3
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
v_{2n+1}	10*m*	10*m*-3	$\cdots$	5*m*+7	5*m*+2	$\cdots$	0	10*m*	10*m*-4	$\cdots$	11	6	10*m*
$x_{1,2}$	6	8	$\cdots$	5*m*-2	5*m*+3	$\cdots$	10*m*	0	8	$\cdots$	10*m*-7	10*m*-3	10*m*
$x_{2,3}$	11	9	$\cdots$	5*m*-6	5*m*-1	$\cdots$	10*m*-4	8	0	$\cdots$	10*m*-11	10*m*-7	10*m*-4
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
$x_{2n-1,2n}$	10*m*-4	10*m*-6	$\cdots$	5*m*+4	5*m*-1	$\cdots$	11	10*m*-7	10*m*-11	$\cdots$	0	8	10*m*-4
$x_{2n,2n+1}$	10*m*	10*m*-2	$\cdots$	5*m*+8	5*m*+3	$\cdots$	6	10*m*-3	10*m*-7	$\cdots$	8	0	10*m*

Table 4: D-distance in total graph of P_n (n even)

	v_1	v_2	$\cdots$	v_{2n-1}	v_{2n}	$x_{1,2}$	$x_{2,3}$	$\cdots$	$x_{n,n+1}$	$\cdots$	$x_{2n-2,2n-1}$	$x_{2n-1,2n}$	Ecc
v_1	0	7	$\cdots$	10*m*-8	10*m*-5	6	11	$\cdots$	5*m*+1	$\cdots$	10*m*-8	10*m*-5	10*m*-5
v_2	7	0	$\cdots$	10*m*-11	10*m*-8	8	9	$\cdots$	5*m*-1	$\cdots$	10*m*-11	10*m*-7	10*m*-7
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
v_{2n-1}	10*m*-8	10*m*-11	$\cdots$	0	6	10*m*-7	10*m*-11	$\cdots$	5*m*-1	$\cdots$	9	8	10*m*-7
v_{2n}	10*m*-5	10*m*-8	$\cdots$	6	0	10*m*-5	10*m*-9	$\cdots$	5*m*+1	$\cdots$	11	6	10*m*-5
$x_{1,2}$	6	8	$\cdots$	10*m*-7	10*m*-5	0	8	$\cdots$	5*m*-2	$\cdots$	10*m*-2	10*m*-8	10*m*-5
$x_{2,3}$	11	9	$\cdots$	10*m*-11	10*m*-9	8	0	$\cdots$	5*m*-6	$\cdots$	10*m*-16	10*m*-12	10*m*-9
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
$x_{n,n+1}$	5*m*+1	5*m*-1	$\cdots$	5*m*-1	5*m*+1	5*m*-2	5*m*-6	$\cdots$	0	$\cdots$	5*m*-6	5*m*-2	5*m*+1
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
$x_{2n-2,2n-1}$	10*m*-8	10*m*-11	$\cdots$	9	11	10*m*-12	10*m*-16	$\cdots$	5*m*-6	$\cdots$	0	8	10*m*-8
$x_{2n-1,2n}$	10*m*-5	10*m*-7	$\cdots$	8	6	10*m*-6	10*m*-12	$\cdots$	5*m*-2	$\cdots$	8	8	0